THE GEOMETRY OF THE RINDLER BLACK HOLE

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Using Einstein's equations for gravity for a spherically symmetric field in a static state and the mass function method, a general form of the metric for Schwarzschild-type black holes is proposed. A qualitative analysis of Schwarzschild, Reissner–Nordström and Rindler geometries is performed. In the paper, it is found that Rindler black hole geometry is structurally inverse to Reissner–Nordström geometry, i.e. it has two T-regions and one R-region. In this regard Rindler geometry is like geometry of the Kottler black hole, which does not have flat asymptotics. Potential curves in the gravitational field of the Rindler black hole are constructed, which makes it possible to calculate the limits of finite motion for a test particle in this field. In addition, a qualitative comparison with the known Schwarzschild field metric is performed. Their similarity in behavior at small distances and absolutely different behavior at large scales are shown. Using the known accelerations of the Pioneer spacecraft, limits are numerically found for the T- and R-regions of their motion in the gravitational field of the Solar System.

Keywords: gravitational field, Einstein equation, Schwarzschild, Reissner–Nordström and Rindler metrics, mass function, T-region and R-region, potential energy curves.

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1. Introduction

Black holes are created because of gravitational collapse of massive cosmic objects such as stars. Under the influence of a huge mass, which is concentrated in a small area, spacetime is distorted so that a black hole is formed. At the center of a black hole there may be a mathematical point known as a singularity, where the density and pressure become infinitely large. True nature of this singularity remains unclear, since the classical gravitation theory cannot explain the state at the center of a black hole.

As there is no way to receive a signal from this area, it turns out that we cannot see its internal structure. To describe such structures in the theory of gravitation, one utilizes the square of an interval, that is, the metric (the interval is an analogue of distance in four-dimensional spacetime).

In the theory of relativity, the concept of the square of the interval is an important tool for defining distances and events in four-dimensional spacetime. The key idea is that the square of the interval is invariant, that is, it remains unchanged for any two events, regardless of the reference system where they are observed.

The metric is utilized to describe black holes using different solutions of Green–Einstein field equations that describe the spacetime structure around black holes. Different types of black holes are characterized by different metrics that represent the geometric and physical properties of these objects.

The paper is concerned with describing the metric for a static spherically symmetric field. We compare two well-known cases for the external vacuum field of a black hole (the Schwarzschild solution) and the external field for a charged black hole (the Reissner–Nordström solution) with a relatively new, but quite popular in modern research papers, field at large distances (the Rindler solution).

Using the metrics allows us to understand the gravitational and geometrical properties of black holes, as well as to predict observable effects such as gravitational waves and light distortion, which can give indication of the presence of black holes in space.

2. Prerequisites for introducing the Rindler metric

Flat rotation curves around the outermost galaxies are one of the most amazing astrophysical discoveries of the latter part of the 20th century. These cases could be attributed to unobservable dark matter, which still lacks a satisfactory candidate.

In the general theory of relativity, there is quite an interesting approach, which is developing models with a constant centrifugal force. One such attempt was formulated by Grumiller in [1, 2] with the definition of the centrifugal force as $F=-\left(\frac{m}{r^2}+a\right)$, where m represents the mass (both regular and dark ones), while the parameter a is a positive constant called the Rindler acceleration [3] created by a constant force of gravity. The Newtonian potential in such a field can be represented as $\Phi(r)\sim-\frac{m}{r}+ar$, and when $r\to\infty$ the term $\Phi(r)\sim ar$ becomes dominant. As in the case of Newtonian circular motion for the mass m the tangential speed $v(r)$ and the radius r at large distances are related as $v(r)\sim r^{1/2}$, this fact in general brings us closer to the concept of flat curves of rotation. In this model, a few simplifications are introduced, and the exact rotation curves are somewhat complicated in nature. Physically, the parameter a becomes significant when dealing with an accelerated frame in flat space, known as Rindler frame, and accordingly the term “Rindler acceleration” is used.

Rindler coordinates were first introduced by Wolfgang Rindler to describe the spacetime of a distant observer. The inertialess Rindler frame consists of constantly and uniformly accelerated clocks. Accelerations are adjusted in such a way that if two clocks are connected by a solid rod, no tension occurs in the rod during motion. In other words, in principle, it is possible to implement the Rindler frame for material bodies. The Rindler system is very useful as a relativistic model of a constant uniform gravitational field, often used as a test site for all kinds of theories.

The Rindler metric is a mathematical description of spacetime around a uniformly accelerated object in flat spacetime. It is commonly used in the special theory of relativity to describe the effect of acceleration on the perception of time and space. It is also used in theoretical physics to study the properties of black holes and other objects with strong gravitational fields.

In Ref. [4], the influence of Rindler-type acceleration on the Oort cloud was investigated, and in Refs. [5, 6] the restrictions of the Solar System on Rindler acceleration were investigated, whereas in [7] the light distortion in gravitational models at long distances proposed by Grumiller [1, 2] was studied. Finding the physical meaning of the Rindler acceleration term in the spacetime metric has been a challenge in recent years [8, 9]. An anisotropic fluid field was first considered by Grumiller [1, 2], while nonlinear electromagnetism was proposed as an alternative source [10]. Currently, the Rindler metric is widely used to describe the internal geometry of a black hole (an anisotropic fluid model with positively defined energy and negatively defined pressure).

Summarizing the experimental data, it can be concluded that the Grumiller effect is manifested at large distances, that is, the Schwarzschild metric is modified by a special parameter a that resembles the Rindler acceleration. Thus, under certain conditions, a successful explanation of the Pioneer anomaly (slowing down the movement of artificial space objects over long distances) was proposed in the static central field. The Pioneer anomaly is an unmodelled deviation of the Pioneer 10 and Pioneer 11 spacecrafti from the predicted accelerations towards the Sun. This acceleration was not caused by known gravitational or other physical processes. Now the most suitable conclusion is that the Pioneer anomaly can be explained by the loss of mass due to thermal radiation, but there are also other hypotheses [11]. Grumiller showed a correlation between the parameter a and the Pioneer anomaly.

3. A general representation for the black hole field metric of the Schwarzschild type

The square of the interval (the interval is an analogue of the distance between two points in four-dimensional spacetime) is generally written as

$$ds^2 = e^{\nu(R,t)}dt^2 - e^{\lambda(R,t)}dR^2 - e^{\mu(R,t)}d\Omega^2. \tag{1}$$

In the case of static (independence on time) and spherical symmetry, we have:

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2, \tag{2}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ defines a metric for a two-dimensional sphere.

In order to find the physical meaning of the introduced metric (2), it is necessary to find values for metric coefficients in an explicit form, that is, in our case, the values $e^{\nu(r)}$ and $e^{\lambda(r)}$. To do this, we need to write down Einstein's equation, and introduce several definitions that are specific for this particular case.

Einstein's equation in the general case has the form:

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = \frac{8\pi\gamma}{c^4}t_\mu^\nu, \tag{3}$$

where R_μ^ν is the Ricci tensor, R is the scalar curvature, γ is the gravitational constant, t_μ^ν is the energy–momentum tensor of the gravitational field. Considering static and spherical symmetry, equation (3) remains in its form with the following components:

$$\begin{cases} -e^{-\lambda(r)}\left(-\dfrac{\lambda'}{r} + \dfrac{1}{r^2}\right) + \dfrac{1}{r^2} = T_0^0; \quad -e^{-\lambda(r)}\left(\dfrac{\nu'}{r} + \dfrac{1}{r^2}\right) + \dfrac{1}{r^2} = T_1^1; \\ -\dfrac{1}{2}e^{-\lambda(r)}\left(\nu'' + \dfrac{(\nu')^2}{2} + \dfrac{\nu' - \lambda'}{r} - \dfrac{\nu'\lambda'}{2}\right) = T_2^2 = T_3^3. \end{cases} \tag{4}$$

Here and further below, for the simplicity and shorthand, the energy–momentum tensor is written in the form: $\dfrac{8\pi\gamma}{c^4}t_\mu^\nu = T_\mu^\nu$. Hereinafter we consider this notation by default for compactness and to save the dimension.

For our case, the components of the energy–momentum tensor

$$T_\nu^\mu = (\varepsilon + p)U^\mu U_\nu - \delta_\nu^\mu p$$

are determined as:

$$T_0^0 = \varepsilon = T_1^1 = -p_r, \quad T_2^2 = T_3^3 = -p_\perp, \tag{5}$$

where U^μ is the four-velocity vector, ε is energy density, p_r is the radial component of pressure, $p_\perp$ is the tangential component of pressure.

Solving the system of equations (4), we equate the components $T_0^0 = T_1^1$ and find that $\nu' = -\lambda'$. And, therefore, $e^{\nu(r)}$ and $e^{\lambda(r)}$ are related to each other. There are different types of relationships between these functions, but we will use their inverse one, i.e.:

$$e^{\nu(r)} = e^{-\lambda(r)}. \tag{6}$$

In the general theory of relativity, when writing Einstein's equations by components, a special mass function was introduced, which in some cases (in particular, ours) significantly simplifies the form of the equations [12-14]. It represents the total energy (including gravitational energy) in units of length. Then, considering (4) and (5), we have:

$$m(r)=r(1-e^{-\lambda(r)}). \tag{7}$$

Taking into account relation (7), the metric (2) can be represented as:

$$ds^2=\left(1-\frac{m(r)}{r}\right)dt^2-\frac{1}{1-\frac{m(r)}{r}}dr^2-r^2d\Omega^2. \tag{8}$$

This equation describes Schwarzschild-type black holes. This model neglects the rotation (spin) of the black hole and additional electric charges. The main characteristics of a Schwarzschild-type black hole are determined by the mass parameter and its spherically symmetric distribution.

To find the mass function, we use the expression for T_2^2 from (4), with accounting for the condition $\nu'=-\lambda'$ and Eq. (7). As $e^{\nu(r)}=1-\frac{m(r)}{r}$, then $e^{\nu(r)}\nu'=-\frac{m'(r)}{r}+\frac{m(r)}{r^2}$. Therefore, the expression for T_2^2 can be rewritten as:

$$T_2^2=T_3^3=-\frac{1}{2}\left(e^{\nu(r)}\nu'^2(r)+e^{\nu(r)}\nu''(r)+e^{\nu(r)}\nu'(r)\frac{2}{r}\right)=\frac{1}{2}\frac{m''(r)}{r}. \tag{9}$$

As it is known, there must be a relationship between the components T_1^1 and T_2^2 that determines the type of metric:

$$T_2^2=k\cdot T_1^1, \tag{10}$$

where *k* is a coupling, then we have:

$$\frac{m''(r)}{2r}=k\frac{m'(r)}{r^2}. \tag{11}$$

Whence, after integration, we can get the expression $m'=cr^{2k}$, which enables to represent the mass function in the form:

$$m(r)=r_g+\frac{r^{2k+1}}{r_0^{2k}}, \tag{12}$$

where r_g is the gravitational radius, r_0 is an arbitrary constant.

If there are several non-interacting fields, then

$$m(r)=r_g+\sum_{k=-1}^{1}\frac{r^{2k+1}}{r_0^{2k}}, \tag{13}$$

where the sum is performed for coupling coefficients from –1 to 1 over non-interacting fields.

Now, let us present the basic, observationally verified forms for metrics that arise for this type of coupling and are successfully used in the theory of gravity:

the Schwarzschild solution:

$$e^{\nu}=1-\frac{r_g}{r},\quad k=0; \tag{14.1}$$

the de Sitter solution:

$$e^{\nu} = 1 - \frac{r_g}{r} - \frac{r^2}{r_0^2}, \quad k = 1; \tag{14.2}$$

the Reissner–Nordström solution:

$$e^{\nu} = 1 - \frac{r_g}{r} - \frac{r_0^2}{r^2}, \quad k = -1; \tag{14.3}$$

the Rindler solution:

$$e^{\nu} = 1 - \frac{r_g}{r} - \frac{r}{r_0}, \quad k = \frac{1}{2}. \tag{14.4}$$

4. Geometry of gravitational fields

To describe the features of spacetime in the theory of gravity, the method of R- and T-regions is used. The method is finding singular points of the metric, where the metric coefficient for the time component of the square of the interval is zero, and, accordingly, for the spatial component, is equal to infinity. These points are called coordinate singularities. At these points, the signature changes and they are the boundaries of different regions in spacetime. It is accepted that the R-region is the space, in which all the observed matter is located, it is defined by the direct signature "+– – –", while the T-region is the inner part of the black hole, which is defined by the inverse signature "– + – –".

The general form for the Schwarzschild-type metric is presented in Eq. (8), for which $m(r)$ – is a mass function that is different for various objects of description.

The expression for the square of the interval describing the geometry around the Schwarzschild black hole (for the case (14.1)) has the form:

$$ds^2 = \left(1 - \frac{r_g}{r}\right)dt^2 - \frac{1}{\left(1 - \frac{r_g}{r}\right)}dr^2 - r^2 d\Omega^2, \tag{15}$$

for which the mass function is $m(r) = r_g$ and the coordinate singularity is observed at the point corresponding to the gravitational radius $r = r_g$. When $r > r_g$, we have the R-region, when $r < r_g$, respectively, the T-region.

The Reissner–Nordström metric around a charged black hole in the general form from (14.3) is

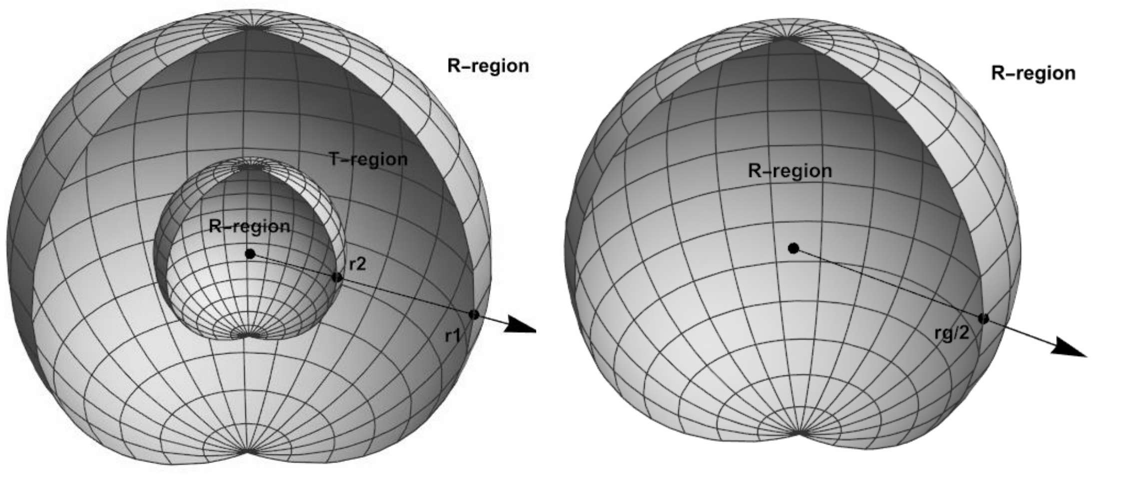
$$ds^2 = (1 - \frac{r_g}{r} + \frac{q^2}{r^2})dt^2 - \frac{1}{(1 - \frac{r_g}{r} + \frac{q^2}{r^2})}dr^2 - r^2 d\Omega^2, \tag{16}$$

where q is the charge function. Any charged black hole has two coordinate singularities:

$$r_{1,2} = \frac{r_g}{2} \pm \sqrt{\frac{r_g^2}{4} - q^2}. \tag{17}$$

Different types of the relationship (17) existence are presented in Fig. 1. The Reissner–Nordström geometry for the case $\frac{r_g^2}{4} > q^2$ ensures the existence of two real roots. For cases $r < r_2$ and $r > r_1$ we have two R-regions; for the case $r_2 < r < r_1$, respectively, the T-region

(Fig. 1, a). For the case $\frac{r_g^2}{4} = q^2$ the Reissner–Nordström geometry guarantees the existence of only one real root $r = \frac{r_g}{2}$. With $r < \frac{r_g}{2}$ and $r > \frac{r_g}{2}$ we have two R-regions (Fig. 1, b). For the case $\frac{r_g^2}{4} < q^2$ there are no regions at all.



a **b**

Fig. 1. Schematic representation of the Reissner–Nordström geometry for the case $\frac{r_g^2}{4} > q^2$ (a) and $\frac{r_g^2}{4} = q^2$ (b).

The square of the interval for the gravitational field at large distances, which is the Rindler solution from Eq. (14.4), is expressed by the formula:

$$ds^2 = (1 - \frac{r_g}{r} - 2ar)dt^2 - \frac{1}{(1 - \frac{r_g}{r} - 2ar)} dr^2 - r^2 d\Omega^2, \tag{18}$$

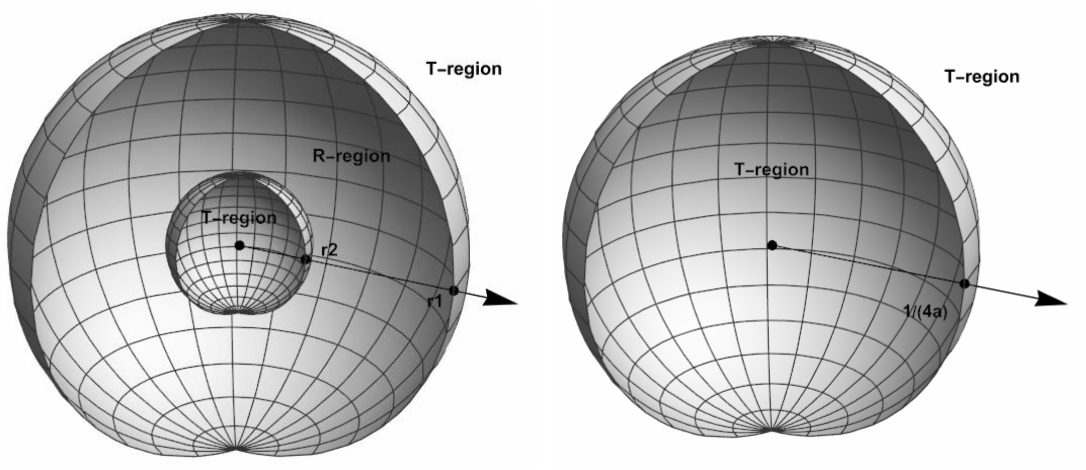
where a is Rindler acceleration. We also have two singular points for this field:

$$r_{1,2} = \frac{1}{4a}\left(1 \pm \sqrt{1 - 8ar_g}\right). \tag{19}$$

Different types of dependence (19) existence are presented in Fig. 2. The Rindler geometry for the case $8ar_g < 1$ provides the existence of two real roots. For cases $r < r_2$ and $r > r_1$ we have two T-regions; for the case $r_2 < r < r_1$, correspondingly, R-region (Fig. 2, a). For the case $8ar_g = 1$ the Rindler geometry gives the existence of only one real root $r = \frac{1}{4a}$. With $r < \frac{1}{4a}$ and $r > \frac{1}{4a}$ we have two T-regions (Fig. 2, b). For the case $8ar_g > 1$ no area exists.

The R-region of the Rindler metric lies between the spheres of radii r_2 and r_1 (Fig. 2, b). This means that a particle in the gravitational field cannot leave it and reach infinity.

Any non-classical model must have asymptotics, i.e. take a classical form at infinity. Similarly, for the metric, there must be a real domain at infinity, in which the classical laws hold. But, as we can see, the R-region of the Rindler metric is restricted on both sides by T-regions, which is like to the structurally similar Kottler metric, that is the solution of the Einstein equation with a positive cosmological constant. This means the absence of flat asymptotics [15].



a **b**

Fig. 2. Schematic representation of the Rindler geometry for the cases $8ar_g < 1$ (a) and $8ar_g = 1$ (b).

5. Potential curves

The field potential–coordinate curve is constructed to determine the regions of finite motion (movement along closed curves) of a particle in this field for given conditions.

Consider the equation of motion in the Schwarzschild field:

$$\frac{dr}{dt} = \pm c\left(1-\frac{r_g}{r}\right)\sqrt{1-\left(1-\frac{r_g}{r}\right)\left(\frac{m^2c^4}{E^2}+\frac{M^2c^2}{E^2r^2}\right)}, \tag{20}$$

where E is the total energy, M is an angular momentum.

Eq. (20) can be obtained from the equation for geodesic lines, or by direct derivation from the Hamilton–Jacobi equation. Let's rewrite (20) as follows:

$$\frac{1}{\left(1-\frac{r_g}{r}\right)}\frac{dr}{cdt} = \pm\frac{1}{E}\sqrt{E^2-U^2}, \tag{21}$$

where $U(r)$ is the effective potential energy, which can be given in the form:

$$U^2(r) = m^2c^4\left(1-\frac{r_g}{r}\right)\left(1+\frac{M^2}{m^2c^2r^2}\right). \tag{22}$$

The allowed range of the particle motion is determined by the condition: $E^2 \geq U^2$. To simplify the solving, we introduce such dimensionless variables:

$$\frac{r}{r_g} = y, \quad \frac{M^2}{m^2c^2r_g^{\ 2}} = A, \quad \frac{U}{mc^2} = \omega. \tag{23}$$

Then, with accounting for (23), the expression (22) takes the form:

$$\omega(y) = \frac{U(r)}{mc^2} = \sqrt{\left(1 - \frac{1}{y}\right)\left(1 + \frac{A}{y^2}\right)}. \tag{24}$$

The extrema of this function are found from the equality of the first derivative to zero (24):

$$y_m = A\left(1 \pm \sqrt{1 - \frac{3}{A}}\right), \tag{25}$$

where the "+" sign is responsible for stable circular motion, i.e. is the minimum of the function, while "–" describes unstable motion.

A set of curves for different values of the momentum has the form presented in Fig. 3.

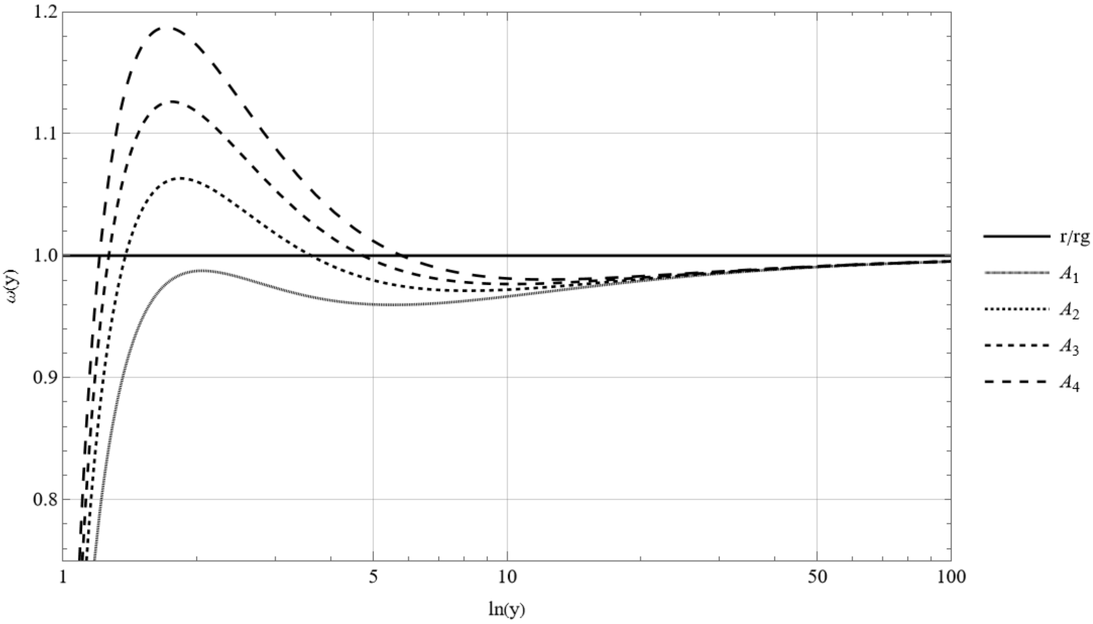


Fig. 3. Potential curves in the Schwarzschild field. Different curves correspond to different values of the momentum of the particle (corresponding to different values of the parameter A). The dashed line separates the regions of the black hole: $r < r_g$ – the interior, $r > r_g$ – the region of the outer field.

In the area between the two maxima (see Fig. 3), the particle moves along closed trajectories, at the point of the minimum we have motion in circle, in the area $r > r_g$ – the particle travels freely along non-closed trajectories.

Let us perform analogous calculations for the Rindler-type metric. In this case, the square of the effective potential will have an additional term, according to (14.4):

$$U^2(r) = m^2c^4\left(1 - \frac{r_g}{r} - 2ar\right)\left(1 + \frac{M^2}{m^2c^2r^2}\right). \tag{26}$$

With the substitutions (23), equation (26) can be rewritten in the form:

$$\omega(y) = \frac{U(r)}{mc^2} = \left(1 - \frac{1}{y} - 2r_g ay\right)\left(1 + \frac{A}{y^2}\right). \tag{27}$$

The curve for the potential of type (27) will have four roots and its shape in the region of the positive radius vector is shown in Fig. 4.

As we can see in Fig. 4, the potential in the gravitational field of the Rindler metric at small distances has a similar structure to the Schwarzschild potential, only the gravitational radius is slightly shifted due to the introduction of acceleration. But at large distances we have utterly different behavior. In the Schwarzschild case, this potential at infinity acquires a finite value, which cannot be said about the Rindler metric, for which the potential turns to zero at a given point in the space for sufficiently large distances. The obtained potential curve in Fig. 4 is a logical proof of the geometry constructed in the previous section.

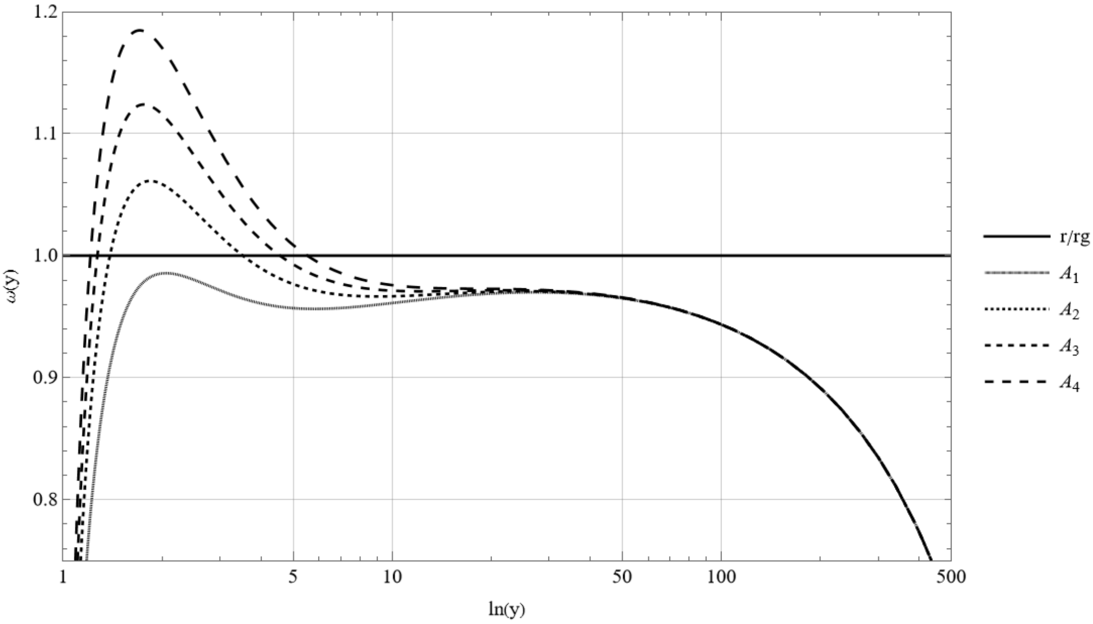


Fig. 4. Potential curves in the Rindler field. Parameter value $a = 10^{-10}\,м / c^2 \approx a_{Rindler}$. Different curves correspond to different values of the momentum of the particle (corresponding to different values of the parameter A).

Taking into account the acceleration for the Pioneers, $a = (8,74 \pm 1,25) \times 10^{-10}\ \mathrm{m}/\mathrm{s}^2$, let's find the domain of the potential, that is, the limits of the R-region in the gravitational field of the Sun and the Earth:

$$r_2 \approx 10^{-2}\ \mathrm{m}, \quad r_1 \approx 5 \times 10^{27}\ \mathrm{m}.$$

As we can see, these characteristics give wide boundaries of the R-region, which enables to describe the objects of this gravitational field using the Rindler metric. The maximum possible accelerations in the gravitational field of the Sun $a_{\mathrm{S}} = 1,29 \times 10^{20}\ \mathrm{m}/\mathrm{s}^2$ and of the Earth $a_{\mathrm{E}} = 3,8 \times 10^{14}\ \mathrm{m}/\mathrm{s}^2$. Therefore, the possible values for acceleration of macroscopic objects can be considered unlimited, since the acceleration of the found orders cannot be experienced by real macroscopic bodies.

6. Conclusions

In the paper the general form of the Schwarzschild-type metric is found. Considering the dependences of the Reissner–Nordström and Rindler metrics on the coordinates, we can conclude that they are structurally inverted, although they describe different gravitational fields. That is, when replacing the R-region with the T-region, and, conversely, the T-region with the R-region, in each of the three cases of the Reissner–Nordström metric, we will get the structure of the Rindler metric, and vice versa. Finding the boundaries of the R-region, we actually find the boundaries of the world we live in (without bare singularities), i.e., by substituting these boundaries for real objects, we can find out their boundaries for research. It can also be seen that the Rindler metric is like the Kottler metric, in which the R-region is bounded, and the particle cannot reach spatial infinity, i.e. there is no flat asymptotics.

Potential curves in the Schwarzschild and Rindler fields are constructed, the regions of the finite and infinite motion of a particle in each gravitational field are found.

Using the known anomaly for the acceleration of artificial space objects Pioneers, the validity of the introduced metric is numerically confirmed.

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