EXACT GREEN'S FUNCTION OF PHOTON AND MEDIUM EFFECTS IN QED$_{2+1}$

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In (2+1)-dimensional Maxwell-Chern-Simons quantum electrodynamics, we derive the structure of the exact polarization operator in the presence of medium characterized by a chemical potential μ. We show that the transverse part of the operator is the sum of four tensors. These tensors and unit one form an algebra with respect to the commutation operation. Green's function of photons at zero temperature is derived on the basis of calculations of the one-loop form factors. The spectrum of modes is investigated. We find that the transverse and longitudinal modes exist in medium. This result differs from that of other authors. Dependence of the photon Debye mass on the form factors is investigated and a static electric potential is calculated.
Keywords: photon polarization operator, Green's function, spectrum, screening.

1. Introduction

Maxwell-Chern-Simons (MCS) QED$_{2+1}$ attracts major attention as a theory which describes various condensed matter physics effects, such as high temperature superconductivity [1–4] and the quantum Hall effect [5]. The photon polarization operator $\Pi_{\mu\nu}$ ($\mu, \nu = 1, 2, 3$) in the MCS theory acts as major object. It is responsible both for magnetic and Debye screening, determines the photon energy spectrum and properties of the thermodynamic system. Its structure was investigated from various perspectives for applications [6–10]. However, a set of conclusions from the photon dispersion relation in [10] ended up inaccurate because of the algebraically closed structure of this tensor has not been considered.

Our goal here is to derive the photon exact Green's function in medium from the polarization operator. For doing this we derive the complete tensor structure from the transversality condition $\Pi_{\mu\nu} k_\mu = 0$. Then we calculate the form factors in one- and two-loop approximations at a finite medium density.

The photon spectrum is determined from the poles of the obtained Green's function. As it will be shown, in the presence of medium, photon has two longitudinal elliptical modes. This result is consistent with that of obtained in [10]. Based on the completeness of the tensor structure, photon Green's function and the dispersion laws are derived.

For the component $\Delta_{33}(k_3 = 0, \mathbf{k})$ of Green's function, we calculate a static electric potential and determine the dependence of Debye's mass on the form factors.

The paper is organized as follows. Section 2 explores the general structure of the polarization operator and Green's function of photon. Section 3 calculates the form factors of the polarization operator. The energy spectrum of photon is studied in Section 4. Debye's mass and potential between charges are calculated in Section 5. The results obtained are discussed in Section 6.

2. Exact photon polarization operator and Green's function

Consider the MCS QED$_{2+1}$ using Euclidean variables $x_\mu = (\mathbf{x}, x_3)$ in the presence of a medium characterized by a chemical potential μ. The Lagrangian is

$$L = \frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}(\hat{\partial} + m)\psi - ie\bar{\psi}\hat{A}\psi - \frac{1}{2\xi}(\partial_\mu A^\mu)^2, \tag{1}$$

where ξ is a gauge parameter. We are not adding the P-odd Chern-Simons term to the (1)

$$L_{CS} \sim \varepsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda \tag{2}$$

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where $\varepsilon^{\mu\nu\lambda}$ is the fully asymmetric unit tensor. Such term is generated by radiation corrections.

Covariant photon propagator with momentum of k_μ and covariant electron propagator with momentum of p_μ are respectively:

$$D_{\mu\nu}(k) = \frac{1}{k^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \frac{\xi}{k^2}\frac{k_\mu k_\nu}{k^2}, \tag{3}$$

$$G(p) = \frac{-i\hat{p} + m}{p^2 + m^2}. \tag{4}$$

Our aim is to derive exact Green's function of photon $\Delta_{\mu\nu}$, based on Schwinger–Dyson's equation [12]

$$\Delta_{\mu\nu} = (D_{\mu\nu}^{-1} - \Pi_{\mu\nu})^{-1}. \tag{5}$$

To evaluate it we have to construct a closed tensor structure for $\Pi_{\mu\nu}$. Polarization operator has been studied already in [6–9]. However such structure was not derived.

The condition must work for the transverse part of the operator

$$k_\mu \Pi_{\mu\nu} = 0. \tag{6}$$

Within the MCS theory, there are four objects defining the tensor structure of the operator: $k_\mu, \varepsilon_{\mu\nu\lambda}$, also a speed of medium u_μ and Kronecker's $\delta_{\mu\nu}$. All possible products of the initial objects give us 11 second rank tensors:

$$T'^1_{\mu\nu} = \delta_{\mu\nu}, \ \ T'^2_{\mu\nu} = k_\mu k_\nu, \ \ T'^3_{\mu\nu} = k_\mu u_\nu,$$
$$T'^4_{\mu\nu} = u_\mu k_\nu, \ \ T'^5_{\mu\nu} = u_\mu u_\nu,$$
$$T'^6_{\mu\nu} = u_\mu \varepsilon_{\nu\alpha\beta} k_\alpha u_\beta, \ \ T'^7_{\mu\nu} = k_\mu \varepsilon_{\nu\alpha\beta} k_\alpha u_\beta,$$
$$T'^8_{\mu\nu} = \varepsilon_{\mu\nu\lambda} u_\lambda, \ \ T'^9_{\mu\nu} = \varepsilon_{\mu\nu\lambda} k_\lambda,$$
$$T'^{10}_{\mu\nu} = u_\nu \varepsilon_{\mu\alpha\beta} k_\alpha u_\beta, \ \ T'^{11}_{\mu\nu} = k_\nu \varepsilon_{\mu\alpha\beta} k_\alpha u_\beta. \tag{7}$$

Tensors (7) define polarization operator in general case:

$$\Pi_{\mu\nu} = \sum_{i=1}^{11} T'^i_{\mu\nu} \Pi'_i, \tag{8}$$

where Π'_i are the corresponding form factors.

Let us perform the transformations

$$T^i = M^i_j T'^j, \tag{9}$$

where $T'^j, (j = 1..11)$ is a set of tensors (7), $T^i, (i = 1..11)$ – new set and M^i_j – transformation matrix. With this transformation, we obtain new form factors, which are the linear combinations of form factors in (8). A new set of tensors has to follow the linear independency condition

$$det(M) \neq 0. \tag{10}$$

Let us choose a new tensor basis in which all tensors are either symmetric or antisymmetric with respect to the permutation indices. Discarding structures that do not satisfy the transver-

sality condition (6), we get

$$T^1_{\mu\nu} = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2},$$

$$T^2_{\mu\nu} = \frac{k_\mu k_\nu}{k^2} - \frac{k_\mu u_\nu + k_\nu u_\mu}{(uk)} + \frac{u_\mu u_\nu}{(uk)^2} k^2,$$

$$T^3_{\mu\nu} = \varepsilon_{\mu\nu\lambda} k_\lambda,$$

$$T^4_{\mu\nu} = \varepsilon_{\mu\alpha\beta} k_\alpha u_\beta \left(k_\nu - \frac{k^2}{(uk)} u_\nu \right) +$$

$$+ \varepsilon_{\nu\alpha\beta} k_\alpha u_\beta \left(k_\mu - \frac{k^2}{(uk)} u_\mu \right). \tag{11}$$

The first two tensors are known from (3+1)-dimensional thermal QED. The third is from the vacuum MCS QED$_{2+1}$. All the tensors, together with the unit one $T^0_{\mu\nu} = \delta_{\mu\nu}$, form the algebra with respect to a commutation operation with structural constants C^{ij}_k

$$\left[T^i, T^j \right] = C^{ij}_k T^k. \tag{12}$$

Non-zero components C^{ij}_k are

$$C^{23}_5 = -C^{32}_5 = -\frac{1}{(uk)},$$

$$C^{24}_5 = -C^{42}_5 = C^{25}_4 = -C^{52}_4 = 1 - \frac{k^2}{(uk)^2},$$

$$C^{35}_1 = -C^{53}_1 = 2k^2(uk) \left(\frac{k^2}{(uk)^2} - 1 \right),$$

$$C^{35}_2 = -C^{53}_2 = -4k^2(uk),$$

$$C^{45}_1 = -C^{54}_1 = 2k^2(uk)^2 \left(\frac{k^2}{(uk)^2} - 1 \right)^2,$$

$$C^{45}_2 = -C^{54}_2 = -4k^2(uk)^2 \left(\frac{k^2}{(uk)^2} - 1 \right). \tag{13}$$

Therefore, the transverse part of the full polarization operator has the form

$$\Pi_{\mu\nu} = \sum_{i=1}^{4} T^i_{\mu\nu} \Pi_i, \tag{14}$$

where $T^i_{\mu\nu}$ is a set of tensors (11) and Π_i are corresponding form factors. The longitudinal part is proportional to the unit tensor $\delta_{\mu\nu}$.

In [7], in the one-loop approximation at a finite temperature, the polarization operator was calculated componentwise. However, its tensor structure has not been studied. In papers [8–10], in the presence of medium, the tensor structure was obtained. In our present calculations, it follows from (14) when the form factor Π_4 is equalled to zero,

$$\Pi^{one-loop}_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Pi_1 + \left(\frac{k_\mu k_\nu}{k^2} - \right.$$

$$\left. - \frac{k_\mu u_\nu + k_\nu u_\mu}{(uk)} + \frac{u_\mu u_\nu}{(uk)^2} k^2 \right) \Pi_2 + \varepsilon_{\mu\nu\lambda} k_\lambda \Pi_3. \tag{15}$$

In [10], the complete polarization operator is the following

$$\Pi_{\mu\nu} = \Pi_T P_{\mu\nu} + \Pi_L Q_{\mu\nu} + \varepsilon_{\mu\nu\lambda} k_\lambda \Pi_3 \tag{16}$$

where Π_T and Π_L are transverse and longitudinal form factors, respectively. The type and properties of the projectors $P_{\mu\nu}$ and $Q_{\mu\nu}$ are defined in paper [11].

In the next section, we show that in the two-loop approximation, in comparison with the expression for the one loop (15), the tensor $T^4_{\mu\nu}$ is added with the form factor Π_4. Therefore, the tensor basis $P_{\mu\nu}$, $Q_{\mu\nu}$ and $\varepsilon_{\mu\nu\lambda}k_\lambda$ is complete only for the polarization operator in the one-loop approximation, and it is impossible to construct an exact Green's function on its basis. It can be built based on the full set (14).

Note also that in Refs. [8, 9] only the form factor Π_3 was calculated within the approximation of zero photon momentum.

In the next section, we calculate all the form factors of the polarization operator (15) at finite density and zero temperature for arbitrary k_3. This generalizes the results of [6], where the calculations were performed in the limit of static fields ($k_3 = 0$).

It should be noted that discarding all terms with antisymmetric unit tensor, the expression (14) reduces to the already obtained for the usual Maxwell QED [13]:

$$\Pi_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Pi_1 +$$
$$+ \left(\frac{k_\mu k_\nu}{k^2} - \frac{k_\mu u_\nu + k_\nu u_\mu}{(uk)} + \frac{u_\mu u_\nu}{(uk)^2}k^2\right)\Pi_2. \tag{17}$$

To derive photon Green's function from equation (5) the full polarization operator (14) must be used. Taking into account the tensor algebra (11) and photon propagator in the Feynman gauge (3), after inversion the Green's function has the form

$$\Delta_{\mu\nu} = \sum_{i=1}^{4} T^i_{\mu\nu}\Delta_i + \frac{1}{k^2}\frac{k_\mu k_\nu}{k^2} \tag{18}$$

where T^i are tensors from the set (11) and Δ_i are the corresponding form factors,

$$\Delta_1 = \left(k^2 - \Pi_1 - \left(\frac{k^2}{(uk)^2} - 1\right)\Pi_2\right)M,$$
$$\Delta_2 = \Pi_2 M,$$
$$\Delta_3 = \Pi_3 M,$$
$$\Delta_4 = \Pi_4 M,$$
$$M = \left[\left(k^2 - \Pi_1\right)\left(k^2 - \Pi_1 -\right.\right.$$
$$- \left(\frac{k^2}{(uk)^2} - 1\right)\Pi_2\right) + k^2\left(\Pi_3^2 - \left(\frac{k^2}{(uk)^2} - 1\right)^2(uk)^2\Pi_4^2\right)\right]^{-1}. \tag{19}$$

The resulting full Green's function contains a form factor at the fourth tensor from the complete set. It was missed in [10] because of incompleteness of the used tensor basis. In fact, its absence does not affect the dispersion equation based on the one-loop approximation

$$\left(k^2 - \Pi_1\right)\left(k^2 - \Pi_1 - \left(\frac{k^2}{(uk)^2} - 1\right)\Pi_2\right) + k^2\Pi_3^2 = 0. \tag{20}$$

However, in general case starting from the two-loop and higher orders, this is incorrect. If Π_3 is equal to zero, the Green's function (18) coincides with Green's function for Maxwell (3+1)-dimensional QED [13]. For $\Pi_2 = 0$ it becomes well-known vacuum one-loop Green's function

for the MCS theory (see, for example, Ref. [14]).

3. Calculation of form factors

In this section, we carry out calculations within an imaginary time formalism. At zero temperature and finite density, the substitution for the electron momentum is

$$p_\mu \to p_\mu^* = \begin{cases} p_\mu, & \mu = 1,2 \\ p_3 - i\mu, & \mu = 3 \end{cases} \tag{21}$$

where μ is chemical potential.

One-loop polarization operator has the form

$$\Pi_{\mu\nu}^{one-loop} = \frac{e^2}{(2\pi)^3} Tr \int d^3p\, \sigma_\mu G(p^*)\sigma_\nu G(p^* + k), \tag{22}$$

where σ_μ are Pauli's matrices.

After the trace calculating, we get

$$\Pi_{\mu\nu}^{one-loop} = \frac{2e^2}{(2\pi)^3} \int d^3p \left[m\varepsilon_{\mu\nu\lambda}k_\lambda + \delta_{\mu\nu}(p^{*2} + \right.$$
$$+ p^*k + m^2) - (2p_\mu^* p_\nu^* + p_\mu^* k_\nu + p_\nu^* k_\mu) \big] \times$$
$$\times \left(p^{*2} + m^2 \right)^{-1} \left((p^* + k)^2 + m^2 \right)^{-1}. \tag{23}$$

Considering the expression (23), we come to conclusion that Π_4 and Π_5 form factors equal zero, in one-loop approximation. Therefore, the tensor structure of the polarization operator is defined by the expression (15).

After integration, we obtain the well-known result [7, 14] for renormalized vacuum polarization operator

$$\Pi_{R\mu\nu}^{vac} = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Pi_1(k) + \varepsilon_{\mu\nu\lambda}k_\lambda \Pi_3(k),$$
$$\Pi_1(k) = -\frac{e^2}{8\pi} \left(2m + \frac{k^2 - 4m^2}{|k|} \times \right.$$
$$\times \arcsin\left(\frac{|k|}{\sqrt{k^2 + 4m^2}} \right) \right),$$
$$\Pi_3(k) = \frac{me^2}{2\pi|k|} \arcsin\left(\frac{|k|}{\sqrt{k^2 + 4m^2}} \right), \tag{24}$$

with renormalization condition

$$\Pi_1(0) = 0. \tag{25}$$

Calculation of the statistical parts in the long-wavelength limit $\mathbf{k} \ll k_3$ gives

$$\Pi_1^{stat} = \frac{e^2}{(2\pi)^2} \theta(\mu^2 - m^2) \left[-J + \frac{\mathbf{k}^2}{k_3^2} L_1(k_3) + \right.$$
$$+ i \left(L_1(k_3) - \frac{\mathbf{k}^2}{k_3^2} \left(\frac{1}{2} L_3(k_3) + L_4(k_3) \right) \right) \right], \tag{26}$$

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$$\Pi_2^{stat} = \frac{e^2}{(2\pi)^2}\theta(\mu^2 - m^2)\left[\frac{\mathbf{k}^{-2}}{k_3^{-2}}\left(J - \left(1 + \frac{4m^2}{k_3^2}\right) \times \right.\right.$$

$$\left.\left. \times L_1(k_3)\right) - \frac{8m^2}{k_3^2}L_1(k_3) - L_5(k_3) + iL_3(k_3)\right], \tag{27}$$

$$\Pi_3^{stat} = -\frac{e^2}{(2\pi)^2}\theta(\mu^2 - m^2)m\left[\frac{2L_1(k_3)}{k_3^2} - \right.$$

$$\left. -\frac{\mathbf{k}^2}{k_3^2}\left(\frac{L_1(k_3)}{k_3^2} + L_2(k_3)\right)\right], \tag{28}$$

where $\theta(\mu^2 - m^2)$ is Heaviside's step function, and the energy functions are

$$L_1(k_3) = \pi k_3 \arctan\frac{2k_3(\mu-m)}{k_3^2+4\mu m},$$

$$L_2(k_3) = 2\pi\frac{m}{k_3^2+4m^2}\left(\frac{\mu}{m}\left(\frac{k_3^2+4m^2}{k_3^2+4\mu^2}\right)^2 - 1\right),$$

$$L_3(k_3) = \frac{\pi k_3}{2}\ln\left(\frac{k_3^2+4\mu^2}{k_3^2+4m^2}\right),$$

$$L_4(k_3) = \pi k_3(\mu^2 - m^2)(3k_3^4 + 4(\mu^2 + m^2)k_3^2 - $$

$$-16\mu^2 m^2)(k_3^2 + 4m^2)^{-1}(k_3^2 + 4\mu^2)^{-2},$$

$$L_5(k_3) = 2\pi(\mu - m)(k_3^4 + 4\mu(\mu - m)k_3^2 + $$

$$+8\mu(\mu^3 - m\mu^2 - m^2\mu - m^3))(k_3^2 + 4\mu^2)^{-2},$$

$$J = 2\pi(\mu - m). \tag{29}$$

In the high-density limit, $\mu \gg m$, we obtain

$$L_1(k_3) = \pi\left(k_3 \arctan\frac{2\mu}{k_3} - 2m\right),$$

$$L_2(k_3) = 2\pi\mu\left(\frac{k_3^2}{(k_3^2+4\mu^2)^2} - \right.$$

$$\left. -\frac{1}{k_3^2}\frac{m}{\mu} + \frac{4\mu^2}{(k_3^2+4\mu^2)^2}\frac{m^2}{\mu^2}\right),$$

$$L_3(k_3) = \pi\left(\frac{k_3}{2}\ln\left(1 + \frac{4\mu^2}{k_3^2}\right) - \frac{2\mu^2}{k_3}\frac{m^2}{\mu^2}\right),$$

$$L_4(k_3) = \frac{\pi\mu^2}{(k_3^2+4\mu^2)^2}\left(k_3(3k_3^2 + 4\mu^2) - \right.$$

$$\left. -\frac{1}{k_3}(32\mu^4 + 3k_3^4 + 12\mu^2 k_3^2)\frac{m^2}{\mu^2}\right),$$

$$L_5(k_3) = 2\pi\mu\left(\frac{k_3^4+4\mu^2 k_3^2+8\mu^4}{(k_3^2+4\mu^2)^2} - \right.$$

$$\left. -\frac{m}{\mu} + \frac{4\mu^2 k_3^2}{(k_3^2+4\mu^2)^2}\frac{m^2}{\mu^2}\right). \tag{30}$$

The imaginary parts in expressions (26), (27) and (28) appear for nonzero photon energies ($k_3 \neq 0$). In Ref. [6] the polarization operator was calculated in the static limit and the imaginary parts have been missed. It should also be noted that the obtained form factors correspond to Green's function at finite temperature.

A nonzero Π_3 form factor indicates that the density of the Lagrangian function (1) incap-

sulates Chern-Simons term (2). Thus we observe the P-symmetry dynamic violation, that is an essential feature of planar theories.

Two-loop calculations of the form factors are too large, so, we present only the integral expression for the Π_4

$$\Pi_4 = \frac{me^4}{(2\pi)^6} \int \frac{d^3p\, d^3q(q_3^* - 3p_3^*)}{\left((p^* + k)^2 + m^2\right)\left((q^* + k)^2 + m^2\right)\left(p^{*2} + m^2\right)\left(q^{*2} + m^2\right)\left(p^* - q^*\right)^2},$$
(31)

which, as a result of a numerical integration, turns out to be nonzero.

4. Photon spectrum in medium

As it is known [4], in the planar MCS QED there are two massive photon modes. Indeed, from the expression for the exact Green's function in the one-loop approximation (18), by turning on no-environment mode ($\Pi_2 = 0$ and $\Pi_4 = 0$) and equating the denominator to zero, we obtain the equation

$$\left(k^2 - \Pi_1\right)^2 + k^2 \Pi_3^2 = 0.$$
(32)

Its solutions determine the energy spectrum of two states.

In the medium presence, the expression (18) gives the following dispersion equation

$$\left(k^2 - \Pi_1\right)\left(k^2 - \Pi_1 - \left(\frac{k^2}{(uk)^2} - 1\right)\Pi_2\right) + k^2\left(\Pi_3^2 - \left(\frac{k^2}{(uk)^2} - 1\right)^2 (uk)^2 \Pi_4^2\right) = 0,$$
(33)

Note that the authors of [10] have obtain photon Green's function from an incomplete tensor basis for the polarization operator. This results in the absence of the Π_4 form factor in (33). Such type equation is valid only in one-loop approximation. He also noted that the form factor (Π_3) leads to mixing of transverse $\left(k^2 - \Pi_1\right)$ and longitudinal $\left(k^2 - \Pi_1 - \left(\frac{k^2}{(uk)^2} - 1\right)\Pi_2\right)$ modes. As a result, it was concluded that there are two longitudinally elliptical massive states of photon

$$m_{(1,2)}^2 = -\Pi_1 - \frac{\mathbf{k}^2}{2k_3^2}\Pi_2 \pm \frac{1}{2k_3^2}\sqrt{\mathbf{k}^4\Pi_2^2 - 4k_3^4(k_3^2 + \mathbf{k}^2)\Pi_3^2}.$$
(34)

For the $\Pi_3 = 0$ they, the masses for the transverse and longitudinal states become known, respectively

$$m_t^2 = -\Pi_1,$$
(35)

$$m_l^2 = -\Pi_1 - \frac{\mathbf{k}^2}{k_3^2}\Pi_2.$$
(36)

The authors also noted that in the limit of high densities and temperatures, the Chern-Simons form factor can be neglected. And that leads to the transition from longitudinal elliptical modes to the combination of transverse and longitudinal ones.

Our calculations confirm that Chern-Simons form factor Π_3 in the $\mu \gg m$ limit is negligible small compared to Π_1 and Π_2. The form factor Π_4 has a similar behaviour. However, based on the fact that the tensor set is closed for the polarization operator (11), we obtain the dispersion equation (33), which determines the spectrum of two massive longitudinal elliptic states.

We obtain solutions of (33) in the limit of dense medium, $\mu \gg (m, |k_3|)$. In this case,

equation splits in two dispersion equations for the longitudinal and the transverse modes, respectively

$$1 - \frac{Re\Pi_{33}(\mathbf{k}, \omega(\mathbf{k}))}{\mathbf{k}^2} = 0, \tag{37}$$

$$\mathbf{k}^2 - \omega^2 - Re\Pi_1(\mathbf{k}, \omega(\mathbf{k})) = 0, \tag{38}$$

where $\omega^2 = -k_3^2$ and $\Pi_{33} = \frac{\mathbf{k}^2}{\mathbf{k}^2 + k_3^2}\left(\Pi_1 + \frac{\mathbf{k}^2}{k_3^2}\Pi_2\right)$. For the form factors in the last equations, the standard analytic continuation to the retarded Green's function is performed [13]

$$\Pi^{ret} = \Pi(k_3 = i(k_0 + i\varepsilon)). \tag{39}$$

Taking into account the above and the explicit form of the form factors (26), (27), (28), (29) and (30), we obtain two branches of transverse oscillations

$$\omega^2 = \frac{1}{2}\mathbf{k}^2 + \frac{e^2}{8\pi}(\mu - m) \pm$$

$$\pm\sqrt{\frac{e^4}{16\pi^2}(\mu - m)^2 + \frac{3e^2}{2\pi}\mathbf{k}^2(\mu - m) + \mathbf{k}^4}, \tag{40}$$

with low attenuation

$$Im\Pi_1 = -2\frac{\mu^2}{\omega^2}. \tag{41}$$

For the longitudinal mode, we obtain a quintic equation for ω variable

$$\omega^5 = \frac{e^2}{4\pi}\left(2(\mu - m)\omega^3 - 2\mu^2\omega^2 - 4\mu^2\mathbf{k}^2\right), \tag{42}$$

which can be solved numerically if the charge e is replaced by effective charge $\bar{e}(\mu)$. It is important to note that the longitudinal spectrum is stable even in the more general case ($\mu \sim \omega, \mu \gg m$):

$$Im\Pi_{33} = 0. \tag{43}$$

5. Debye's screening

The potential between two charges q_1 and q_2 is determined by the time components of Green's function (18)

$$V(R) = q_1 q_2 \int \frac{d^2k}{(2\pi)^2}\Delta_{33}(k_3 = 0, \mathbf{k})e^{i\vec{k}\cdot\vec{R}} =$$

$$= q_1 q_2 \int \frac{d^2k}{(2\pi)^2}\frac{(\mathbf{k}^2 - \Pi_1)e^{i\vec{k}\cdot\vec{R}}}{(\mathbf{k}^2 - \Pi_1)(\mathbf{k}^2 - \Pi_{33}) + \mathbf{k}^2\Pi_3^2}, \tag{44}$$

where $\vec{R}$ is a distance between charges.

Divide the numerator and the denominator of the integrand by $\mathbf{k}^2 - \Pi_1$. We get

$$V(R) = q_1 q_2 \int \frac{d^2k}{(2\pi)^2}\frac{e^{i\vec{k}\cdot\vec{R}}}{\mathbf{k}^2 + m^2} = q_1 q_2 K_0(m_l R), \tag{45}$$

where

$$m^2 = -\Pi_1 - \frac{\mathbf{k}^2}{k_3^2}\Pi_2 + \frac{(\mathbf{k}^2 + k_3^2)\Pi_3^2}{\mathbf{k}^2 + k_3^2 - \Pi_1} \qquad (46)$$

and K_0 is Macdonald function with $\lim_{r\to\infty} K_0(r) \sim \sqrt{\frac{\pi}{2r}}e^{-r}$. Therefore

$$V(R) = \frac{q_1 q_2}{2\sqrt{2\pi}}\frac{e^{-m_D R}}{\sqrt{m_D R}}, \qquad (47)$$

where m_D is the Debye mass of photon, which is the limit of m at $k_3 = 0, \mathbf{k} \to 0$:

$$m_D^2 = -\Pi_{33}(k_3 = 0, \mathbf{k} \to 0) + \frac{\mathbf{k}^2\Pi_3^2(k_3 = 0, \mathbf{k} \to 0)}{\mathbf{k}^2 - \Pi_1}. \qquad (48)$$

The expression (47) is the same as derived for zero density and finite temperature [15]. The form factors of the polarization operator in the static limit are (3):

$$\Pi_{33}(k_3 = 0, \mathbf{k} \to 0) = -\frac{e^2}{2\pi}\theta(\mu^2 - m^2)(\mu - m),$$
$$\Pi_3(k_3 = 0, \mathbf{k} \to 0) = 0. \qquad (49)$$

Formulas (47), (48) and (49) show that the topological Chern-Simons form factor is not present in the photon Debye mass and does not affect the screening. Therefore, one can use the standard definition

$$m_D^2 = -\Pi_{33}(k_3 = 0, \mathbf{k} \to 0) = \frac{e^2}{2\pi}\theta(\mu^2 - m^2)(\mu - m). \qquad (50)$$

Expression (50) shows the linear grows of Debye's mass with increasing of density. Similar situation takes also place in (3+1)-dimensional QED [13].

6. Discussion and conclusion

(2 + 1) - dimensional Maxwell-Chern-Simons theory in the presence of medium density was considered. From the transversality condition, which is a consequence of the Ward identities for photon Green's function, the closed tensor set of the polarization operator was constructed. The operator is formed out the sum of four tensors with the corresponding form factors (14). Tensors together with the unit tensor $\delta_{\mu\nu}$ form the algebra with respect to the commutation operation. This structure generalizes the results of [6], where the form factors have been calculated in the one-loop approximation at a finite density in the limit $|k_3| \gg \mathbf{k}$. We showed that the first three terms from the general expression (14) are nonzero in one-loop approximation. The Π_4 form factor has a nonzero value in two-loop approximation. From explicit calculations in the high-density limit, we also have obtained that the Chern-Simons form factors Π_3 and Π_4 are negligible small compared to the others.

The photon exact Green's function (18) has been derived from the Schwinger–Dyson equation by using the calculated polarization operator. Its pole structure confirms the existence of two longitudinal elliptical modes. This result is consistent with obtained in [10]. However, the dispersion equation turned out to be different because of the incompleteness of the basis set used in that paper. In the $\mu \gg (m, \omega)$ limit, the longitudinal elliptic modes migrate to transverse and longitudinal ones. We have derived the dispersion laws for each of these modes. We found that the transverse mode has a weak damping, and the longitudinal mode is stable even in more

general case $\mu \sim \omega$.

We also investigated the Debye's screening effect as a function of medium density. The static electric potential was calculated. Its structure is similar to the one with zero density and finite temperature [15]. We found that the Chern-Simons term is not involved into the photon Debye mass. Therefore, the standard definition can be used $m_D^2 = -\Pi_{33}(k_3 = 0, \mathbf{k} \to 0)$. The Debye shielding radius diminishes with increasing of medium density.

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