LOW-ENERGY EFFECTIVE LAGRANGIAN OF THE TWO-HIGGS-DOUBLET MODEL

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We consider a decoupling scenario within the two-Higgs-doublet model (2HDM) with small CP-violation. Mass eigenstates of this model include one neutral scalar field with the mass of the Standard model (SM) Higgs boson and four other scalars, which decouple at low energies. We derive the effective operators of interactions between the SM fermions and the lightest scalar particle of the model. The coefficients at these operators are expressed in terms of the two-Higgs-doublet model parameters. The scattering processes affected by this effective Lagrangian are identified.

Keywords: two-Higgs-doublet model, low-energy effective Lagrangian, decoupling.

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1. Introduction

Nowadays, the Standard model is the best experimentally proven theoretical description of interactions between elementary particles. However, there are physical phenomenons which could not be explained within the SM, such as baryon asymmetry in the Universe, neutrino masses, dark matter, etc. To address these problems, many different models were proposed, which extend the SM with different new particles. Observable predictions of these models had been tested in experiments, but no new states beyond the SM were found so far. This could happen because of different reasons. In our paper, we consider the case when masses of new particles are much bigger than the collision energies used in the experiments. Hence, their contributions to the scattering amplitudes could be small because of decoupling, and the non-resonant search methods become relevant [1].

It is convenient then to describe the interactions of new particles with the low-energy effective Lagrangian (EL) of the SM fields, which consists of high-dimensional operators. Then contributions of these operators could be constrained by experiment. The low-energy effective Lagrangians of the new physics models are also different – some types of operators are suppressed or enhanced in a particular model. Thus, it is necessary to obtain the experimental constraints for the effective Lagrangian of each model, to improve the experimental reach [2]. In our paper, we derive the low-energy effective Lagrangian of the two-Higgs-doublet model (2HDM). A detailed review of this model could be found in [4, 6–9].

Here we consider the 2HDM as one of the extensions of the SM, which introduces a wide variety of new phenomena. For instance, one of Sakharov's baryogenesis conditions could be fulfilled within the SM extended with one scalar doublet [5]. As it is known, the minimal SM does not have this feature [13].

The 2HDM predicts that there exist five "physical" scalar particles, while only one has been experimentally observed as the Higgs boson. We investigate the case when the SM Higgs boson is the lightest state of the 2HDM, and the other four scalar particles are heavy. We integrate over these heavy scalar bosons and obtain a low-energy EL of the 2HDM. We find the analytical expressions for the corrections coming from interactions with heavy scalars to the Yukawa couplings of the SM and parameters of the tree-level potential of the SM Higgs boson. Then we derive new effective operators of the dimensions 5 and 6. They are introduced by interactions with the heavy 2HDM bosons, and find the analytical expressions for the couplings of these operators. All the corrections we provide up to the order of Λ^{-2}, where Λ is a mass scale of the heavy bosons. We point out decoupling phenomenon in the considered model.

The scenario where some or all of the scalar bosons become heavy was considered in [3,7,11]. CP-conserving potential of the 2HDM which is symmetric under the change of sign of one of the doublets was discussed in these papers. Expressions for the couplings of "physical" scalars to other fields were obtained in [7]. In that research, a scenario where couplings between the non-minimal scalars and the SM particles are small for some values of the model parameters was discussed. Low-energy effective Lagrangian of the 2HDM was obtained in [11] for the case when all physical 2HDM particles are beyond the reach of the modern colliders. However, the discovery of the $125\,GeV$ Higgs boson makes this hypothesis questionable, so we do not proceed with it. Authors of [3] have obtained the low-energy EL for the 2HDM where one of the scalars is light and the others are heavy. As it was shown there, such variant of the 2HDM does not fit good enough to the LHC Higgs data, and some modifications of the model are required. In our paper, we choose the more general potential, discussed in [6, 8], which also allows for a small violation of the CP-symmetry, and obtain the low-energy EL for such a model.

This paper is organized as follows. In section 2 we discuss the particle spectrum of the model and analyze properties of the particles. Section 3 contains the low-energy effective Lagrangian of the 2HDM. In that section we figure out corrections to the parameters of the SM and couplings of the effective operators introduced by the 2HDM. Section 4 summarises our results. We provide analytical expressions for the mass matrices of the scalar particles and for the terms of Yukawa interaction between the 2HDM scalars and the SM fermions in the Appendix.

2. Two-Higgs-doublet model potential

We start with the Lagrangian of the 2HDM scalar fields $\mathcal{L}_s$:

$$\mathcal{L}_s = \sum_{i=1,2} \left(D^\mu \phi_i\right)^\dagger D_\mu \phi_i - V(\phi_1;\phi_2), \quad iD_\mu = i\partial_\mu + \frac{1}{2} g\sigma_a W^a_\mu + \frac{1}{2} g' B_\mu, \quad a = \overline{1;3}. \quad (1)$$

Here ϕ_1 and ϕ_2 denote two scalar doublets. $V(\phi_1;\phi_2)$ is a potential of the scalar fields. There is also a Lagrangian $\mathcal{L}_Y$ of Yukawa interaction between the scalar doublets and the SM fermions, which we discuss in the next section. In our investigation, we consider only effective vertexes with the SM Higgs h and/or fermions in the initial and final states. Contributions of the weak gauge bosons to these vertexes are of the next-to-leading order, so we neglect them and omit gauge fields in the kinetic term in (1).

There are many possible types of interactions between particles which could be introduced by a general potential of the two-Higgs-doublet model. In our paper, we choose the specific potential:

$$\begin{aligned} V(\phi_1;\phi_2) = {} & m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - (m_{12}^2 \phi_1^\dagger \phi_2 + m_{12}^{2*} \phi_2^\dagger \phi_1) + \\ & + \frac{1}{2}\lambda_1 (\phi_1^\dagger \phi_1)^2 + \frac{1}{2}\lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \\ & + \frac{1}{2}\left[\lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_5^* (\phi_2^\dagger \phi_1)^2\right], \end{aligned} \quad (2)$$

$$\phi_i = \begin{pmatrix} a_i^+ \\ \phi_i^0 \end{pmatrix}, \quad \phi_i^0 = \frac{1}{\sqrt{2}}\left(v_i + b_i + i c_i\right). \quad (3)$$

Here a_i^+, b_i and c_i are charged, neutral CP-even and neutral CP-odd components of the doublet ϕ_i, respectively. Neutral components of the doublets have real vacuum expectation values (VEVs) $\frac{1}{\sqrt{2}}v_1$ and $\frac{1}{\sqrt{2}}v_2$, $v_1 > v_2$. All parameters in the potential (2) are real, except m_{12}^2 and

λ_5. Because of this, there are neutral scalars with unspecified CP-parity among the mass eigenstates of the model. Yukawa interaction of these states with fermions violates CP-parity [6], and the magnitude of CP-violation is regulated by Im λ_5.

Vacuum state of the model minimizes the potential (2):

$$\frac{\partial V}{\partial \phi_1}\Big|_{vac} = 0, \quad \frac{\partial V}{\partial \phi_2}\Big|_{vac} = 0.$$

From these equalities we find relations between some of the model parameters:

$$\begin{aligned} m_{11}^2 &= \frac{v_2}{v_1}\mathrm{Re}\, m_{12}^2 - \frac{1}{2}\left[\lambda_1 v_1^2 + v_2^2\left(\lambda_3 + \lambda_4 + \mathrm{Re}\,\lambda_5\right)\right], \\ m_{22}^2 &= \frac{v_1}{v_2}\mathrm{Re}\, m_{12}^2 - \frac{1}{2}\left[\lambda_2 v_2^2 + v_1^2\left(\lambda_3 + \lambda_4 + \mathrm{Re}\,\lambda_5\right)\right], \\ \mathrm{Im}\, m_{12}^2 &= \frac{1}{2} v_1 v_2 \mathrm{Im}\,\lambda_5. \end{aligned}$$

We investigate the scenario when one of the mass eigenstates has the same mass as the SM Higgs boson, and four another are very heavy, so they decouple at energies of order $O(v)$, where v is the SM Higgs VEV. This scenario could be realized if we put $\mathrm{Re}\, m_{12}^2$ to be very big [3, 7, 9]. In our research we consider $\mathrm{Re}\, m_{12}^2$, v_1, v_2 and scalar self-couplings λ_i, $i = \overline{1;5}$ as free parameters of the model. For simplicity we also assume that Im λ_5 is small.

The mass matrices of the scalar fields are given by coefficients in the quadratic terms of the Taylor series expansion of (2) near its minimum,

$$V(\phi_1;\phi_2) = V(\phi_1;\phi_2)\Big|_{vac} + \begin{pmatrix} a_1^+ \\ a_2^+ \end{pmatrix}^T M_a^2 \begin{pmatrix} a_1^- \\ a_2^- \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b_1 \\ b_2 \\ c_1 \\ c_2 \end{pmatrix}^T M_{bc}^2 \begin{pmatrix} b_1 \\ b_2 \\ c_1 \\ c_2 \end{pmatrix} + O(\phi^3).$$

In this equation, M_a^2 and M_{bc}^2 are the mass matrices of the particles a_i^+, b_i and c_i, respectively [6, 8]. The expressions for them are given in the Appendix. Eigenstates of the matrix M_a^2 are the charged Goldstone boson G^+ and the massive particle H^+:

$$\begin{aligned} &H^+ = -a_1^+ \sin\beta + a_2^+ \cos\beta, \quad G^+ = a_1^+ \cos\beta + a_2^+ \sin\beta, \\ &\tan\beta = \frac{v_2}{v_1}. \end{aligned} \tag{4}$$

One of the eigenvalues of M_{bc}^2 is zero, so that there are three massive scalars h_1, h_2, h_3 and one Goldstone boson G_0:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ G_0 \end{pmatrix} = R \begin{pmatrix} b_1 \\ b_2 \\ c_1 \\ c_2 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 0 \\ 0 & R_\beta \end{pmatrix}, \quad R_\beta = \begin{pmatrix} -s_\beta & c_\beta \\ c_\beta & s_\beta \end{pmatrix},$$
$$s_\beta = \sin\beta, \quad c_\beta = \cos\beta, \quad t_\beta = \tan\beta.$$

The mass matrix of the massive neutral scalars h_1, h_2 and h_3 is

$$M_h^2 = \begin{pmatrix} \lambda_1 v^2 c_\beta^2 + t_\beta \mathrm{Re}\, m_{12}^2 & v^2 s_\beta c_\beta \lambda_{345} - \mathrm{Re}\, m_{12}^2 & -\frac{1}{2}\mathrm{Im}\,\lambda_5 v^2 s_\beta \\ v^2 s_\beta c_\beta \lambda_{345} - \mathrm{Re}\, m_{12}^2 & \lambda_2 v^2 s_\beta^2 + \frac{1}{t_\beta}\mathrm{Re}\, m_{12}^2 & -\frac{1}{2}\mathrm{Im}\,\lambda_5 v^2 c_\beta \\ -\frac{1}{2}\mathrm{Im}\,\lambda_5 v^2 s_\beta & -\frac{1}{2}\mathrm{Im}\,\lambda_5 v^2 c_\beta & \frac{1}{s_\beta c_\beta}\mathrm{Re}\, m_{12}^2 - \mathrm{Re}\,\lambda_5 v^2 \end{pmatrix},$$
$$v^2 = v_1^2 + v_2^2, \quad \lambda_{345} = \lambda_3 + \lambda_4 + \mathrm{Re}\,\lambda_5.$$

We diagonalize this matrix via 3 the rotations of the basis $\{h_1; h_2; h_3\}$ [6]. The rotation matrixes are

$$R_1 = \begin{pmatrix} c_{\alpha_1} & s_{\alpha_1} & 0 \\ -s_{\alpha_1} & c_{\alpha_1} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_2 = \begin{pmatrix} c_{\alpha_2} & 0 & s_{\alpha_2} \\ 0 & 1 & 0 \\ -s_{\alpha_2} & 0 & c_{\alpha_2} \end{pmatrix}, \quad R_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\alpha_3} & s_{\alpha_3} \\ 0 & -s_{\alpha_3} & c_{\alpha_3} \end{pmatrix},$$
$$\alpha = \alpha_1 - \frac{\pi}{2}. \tag{5}$$

Here we follow the notation of [6] and use the angle α instead of α_1. Non-diagonal element M_{h12}^2 of M_h^2 vanishes after the rotation R_1:

$$M_h^{2\prime} = R_1 M_h^2 R_1^T = \begin{pmatrix} S+\Delta & 0 & -\frac{1}{2}\mathrm{Im}\,\lambda_5 v^2 c_{\alpha+\beta} \\ 0 & S-\Delta & \frac{1}{2}\mathrm{Im}\,\lambda_5 v^2 s_{\alpha+\beta} \\ -\frac{1}{2}\mathrm{Im}\,\lambda_5 v^2 c_{\alpha+\beta} & \frac{1}{2}\mathrm{Im}\,\lambda_5 v^2 s_{\alpha+\beta} & \frac{1}{s_\beta c_\beta}\mathrm{Re}\, m_{12}^2 - \mathrm{Re}\,\lambda_5 v^2 \end{pmatrix},$$

where S, Δ and α are defined as

$$S = \frac{1}{2}\left[\frac{1}{s_\beta c_\beta}\mathrm{Re}\, m_{12}^2 + v^2(\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2)\right],$$
$$\Delta = \frac{1}{2\cos 2\alpha}\left[\frac{2}{t_{2\beta}}\mathrm{Re}\, m_{12}^2 - v^2(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2)\right], \tag{6}$$

$$\tan 2\alpha = t_{2\beta}\frac{1 - \frac{1}{2}\varepsilon\lambda_{345}s_{2\beta}}{1 - \frac{1}{2}\varepsilon t_{2\beta}(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2)}, \quad \varepsilon = \frac{v^2}{\mathrm{Re}\, m_{12}^2}. \tag{7}$$

The angle α is such that $\cos 2\alpha > 0$ by definition. We diagonalize $M_h^{2\prime}$ with rotations R_2 and R_3. Since $|\mathrm{Im}\,\lambda_5| \ll 1$ and $\alpha_2 \sim \mathrm{Im}\,\lambda_5$, $\alpha_3 \sim \mathrm{Im}\,\lambda_5$, the corresponding rotation angles α_2 and α_3 are small, too. So the following approximations for R_2 and R_3 are valid,

$$R_2 \approx \begin{pmatrix} 1 & 0 & \alpha_2 \\ 0 & 1 & 0 \\ -\alpha_2 & 0 & 1 \end{pmatrix}, \quad R_3 \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \alpha_3 \\ 0 & -\alpha_3 & 1 \end{pmatrix}.$$

We neglect all the terms of the second and higher orders in $\mathrm{Im}\,\lambda_5$, and obtain the following approximations for α_2 and α_3:

$$\alpha_2 \approx \frac{\mathrm{Im}\,\lambda_5 v^2 \cos(\alpha+\beta)}{2M_{h33}^2 - 2(S+\Delta)},$$
$$\alpha_3 \approx -\frac{\mathrm{Im}\,\lambda_5 v^2 \sin(\alpha+\beta)}{2M_{h33}^2 - 2(S-\Delta)}. \tag{8}$$

Here M^2_{h33} denotes the third diagonal element of the mass matrix M^2_h. Finally, we obtain the neutral mass eigenstates H, h and A_0

$$\begin{pmatrix} H \\ h \\ A_0 \end{pmatrix} = R_3 R_2 R_1 \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} -b_1 s_\alpha + b_2 c_\alpha + \alpha_2 \left(c_2 \cos\beta - c_1 \sin\beta\right) \\ -b_1 c_\alpha - b_2 s_\alpha + \alpha_3 \left(c_2 \cos\beta - c_1 \sin\beta\right) \\ b_1 \left(\alpha_3 c_\alpha + \alpha_2 s_\alpha\right) + b_2 \left(-\alpha_2 c_\alpha + \alpha_3 s_\alpha\right) + c_2 \cos\beta - c_1 \sin\beta \end{pmatrix},$$
$$G_0 = c_1 \cos\beta + c_2 \sin\beta. \tag{9}$$

As one can seen from these expressions, the neutral fields h, H and A_0 do not have definite CP-parities, because they are the linear combinations of the CP-even fields b_1, b_2 and CP-odd fields c_1 and c_2. This mixing is proportional to $\mathrm{Im}\,\lambda_5$. So, when $\mathrm{Im}\,\lambda_5 = 0$, h and H become CP-even, and A_0 becomes CP-odd. Simultaneously, CP-parity of the Goldstone boson G_0 does not depend on $\mathrm{Im}\,\lambda_5$, and this particle always remains CP-odd.

The masses of the particles are given in the table 1.

Table 1

Masses of the 2HDM bosons

$H^\pm$	$m^2_{H^+} = \frac{1}{s_\beta c_\beta}\mathrm{Re}\, m^2_{12} - \frac{1}{2}v^2(\lambda_4 + \mathrm{Re}\,\lambda_5)$
A_0	$m^2_A = \frac{1}{s_\beta c_\beta}\mathrm{Re}\, m^2_{12} - \mathrm{Re}\,\lambda_5 v^2$
H	$m^2_H = S + \Delta$
h	$m^2_h = S - \Delta$

In the 2HDM, the masses of the weak gauge bosons are introduced by interaction with the scalar doublets, and they are proportional to v. Hence, v is equal to the VEV of the Higgs field in the minimal SM – $v \approx 250\,GeV$. In the limit when $\mathrm{Re}\, m^2_{12} \gg v^2$ and the all scalar self-couplings are $\sim O(1)$, particles $H^\pm$, H and A_0 become heavy and nearly degenerate in masses, as it is shown in the table 2.

Table 2

Masses of the Higgs bosons in the limit Re $m^2_{12} \gg v^2$

$H^\pm$	$m^2_{H^+} \approx \frac{1}{s_\beta c_\beta}\mathrm{Re}\, m^2_{12}$
A_0	$m^2_A \approx \frac{1}{s_\beta c_\beta}\mathrm{Re}\, m^2_{12}$
H	$m^2_H \approx \frac{1}{s_\beta c_\beta}\mathrm{Re}\, m^2_{12}$
h	$m^2_h \approx \frac{1}{2}\lambda_1 v^2 c^2_\beta \left(1 + \frac{1}{\cos 2\beta}\right) + \frac{1}{2}\lambda_2 v^2 s^2_\beta \left(1 - \frac{1}{\cos 2\beta}\right)$

This limit also implies that $\tan 2\alpha \to \tan 2\beta$. Mass of the scalar boson h is then $O(v)$, so this quantity could be close to that of the SM Higgs boson.

Hereafter we use the following parametrization for the scalar doublets:

$$\phi_1 = U \begin{pmatrix} -H^+ s_\beta \\ \frac{1}{\sqrt{2}}\left[v_1 + (A_0\alpha_3 - h)c_\alpha + (A_0\alpha_2 - H)s_\alpha - i(A_0 + H\alpha_2 + h\alpha_3)s_\beta\right] \end{pmatrix},$$
$$\phi_2 = U \begin{pmatrix} H^+ c_\beta \\ \frac{1}{\sqrt{2}}\left[v_2 + (A_0\alpha_3 - h)s_\alpha - (A_0\alpha_2 - H)c_\alpha + i(A_0 + H\alpha_2 + h\alpha_3)c_\beta\right] \end{pmatrix},$$
$$U = \exp\left[-i\frac{(\overline{G}\overline{\sigma})}{v}\right], \quad (\overline{G}\overline{\sigma}) = \sigma^1 G_1 + \sigma^2 G_2 + \sigma^3 G_3,$$
$$G^\pm = \frac{1}{\sqrt{2}}(G_2 \mp iG_1), \quad G_0 = G_3. \tag{10}$$

Here σ_a, $a = \overline{1;3}$ denote Pauli's matrices. The original parametrization (3) could be obtained from (10) if one neglects the terms which are quadratic in fields [11]. In the unitary gauge (10), the inessential Goldstone degrees of freedom do not enter the potential (2), and $V(\phi_1;\phi_2)$ is represented in terms of the "physical" scalar fields, only.

3. Low-energy effective Lagrangian of the 2HDM

We assume in our treatment that the SM Higgs is the lightest scalar boson of the Standard model with two scalar doublets, and it is described with the h field. Then the high-energy dynamics of the 2HDM and fermions is described by the following Lagrangian:

$$\mathcal{L} = \mathcal{L}_s + \mathcal{L}_Y. \tag{11}$$

The second term of (11), $\mathcal{L}_Y$, is the Lagrangian of Yukawa interaction:

$$\mathcal{L}_Y = -\sum_{f;f'}\sum_{i=1,2}\left\{ y^{i(1)(q)}_{ff'}(\overline{Q}^{(f)}_L\phi_i)d^{(f')}_R + y^{i(2)(q)}_{ff'}(\overline{Q}^{(f)}_L\phi^c_i)u^{(f')}_R + \right.$$
$$\left. + y^{i(1)(l)}_{ff'}(\overline{L}^{(f)}_L\phi_i)e^{(f')}_R + y^{i(2)(l)}_{ff'}(\overline{L}^{(f)}_L\phi^c_i)\nu^{(f')}_R + h.\,c.\right\},$$
$$f;f' = \overline{1;3}, \quad \phi^c_i = -i\sigma_2\phi^*_i, \quad Q^{(f)}_L = \begin{pmatrix} u^{(f)}_L \\ d^{(f)}_L \end{pmatrix}, \quad L^{(f)}_L = \begin{pmatrix} \nu^{(f)}_L \\ e^{(f)}_L \end{pmatrix}. \tag{12}$$

In this expression $y^{(1)(q)}_{ff'}$, $y^{(2)(q)}_{ff'}$, $y^{(1)(l)}_{ff'}$ and $y^{(2)(l)}_{ff'}$ are the Yukawa couplings. Superscripts (q) and (l) denote couplings which describe interactions with quarks and leptons, respectively. ϕ^c_i is the doublet which is charge-conjugated to ϕ_i, $Q^{(f)}_L$, and $L^{(f)}_L$ are the doublets of the left-handed quarks and leptons of the generation f, respectively. For instance, $u^{(1)}_L$ is a left-handed u-quark, $d^{(2)}_R$ is a right-handed s-quark etc. Similarly, $\nu^{(1)}_L$ is a left-handed electron neutrino, and $e^{(3)}_L$ is a left-handed tau-lepton. Fermion doublets in (12) are parametrized in such a gauge that Goldstone's bosons do not enter $\mathcal{L}_Y$. Besides, the all fermionic fields in (12) are the symmetry eigenstates.

As it is known from the experimental data, there are no tree-level flavour-changing interactions between charged leptons or quarks of the same charge, in the considered range of energies. This fact could be taken into account with the specific choice of the pattern or values of the Yukawa couplings. However, the main results of our investigation do not depend on such constraints. So we use the general expression for the Yukawa Lagrangian (12).

In terms of the mass eigenstates of the 2HDM, the Yukawa Lagrangian (12) reads:

$$\mathcal{L}_Y = -J^+H^- - J^-H^+ - J_H H - J_A A_0 - J_h h. \tag{13}$$

In this equation, J^+, J^-, J_H, J_A and J_h denote contributions of the fermionic fields of the SM:

$$J^\pm = J^{\pm(q)} + J^{\pm(l)}, \quad J_H = J_H^{(q)} + J_H^{(l)}, \quad J_A = J_A^{(q)} + J_A^{(l)}, \quad J_h = J_h^{(q)} + J_h^{(l)}. \tag{14}$$

Here operators $J^{-(q)}$, $J^{+(q)}$, $J_H^{(q)}$, $J_A^{(q)}$ and $J_h^{(q)}$ contain quark fields, while $J^{-(l)}$, $J^{+(l)}$, $J_H^{(l)}$, $J_A^{(l)}$ and $J_h^{(l)}$ consist of leptonic fields. These terms are given in Appendix.

Lagrangian (1) in terms of the mass eigenstates reads:

$$\mathcal{L}_s = \frac{1}{2}\sum_{a=1}^{3}(\partial_\mu G_a)^2 + \partial^\mu H^+ \partial_\mu H^- + \frac{1}{2}(\partial_\mu A_0)^2 + \\ + \frac{1}{2}(\partial_\mu H)^2 + \frac{1}{2}(\partial_\mu h)^2 - V(H^\pm; A_0; H; h). \tag{15}$$

Hereafter we enumerate fields H^+, H^-, H and A_0 with one index "a":

$$\{H^+; H^-; H; A_0\} = \{H^a\}, \quad a = \overline{1;4}.$$

Now we derive the effective action Γ_{eff} of the light particles of the theory. Γ_{eff} describes the interactions of the light particles in the processes where the non-minimal Higgs bosons $H^\pm$, H and A_0 do not appear in the initial or final states. Instead, they participate in the interactions as the virtual states only, and contribute the low-energy dynamics via the effective operators of the SM fields. We integrate over the non-minimal scalar bosons and derive Γ_{eff}:

$$e^{i\Gamma_{eff}} = \int \mathcal{D}H\mathcal{D}A_0\mathcal{D}H^+\mathcal{D}H^- \exp\left(i\int d^4x\mathcal{L}\right) \tag{16}$$

Since $\mathcal{L}$ contains the terms which are cubic and quartic in the scalar fields, we calculate Γ_{eff} in the Gaussian approximation. That is, we expand action of the scalar fields near some classical field configuration H^a_{class},

$$S[h; H^a] = \int d^4x\mathcal{L} = S[h; H^a_{class}] + \int d^4x \frac{\delta S}{\delta H^a(x)}\Big|_{H^a = H^a_{class}} \Delta H^a(x) + \\ + \frac{1}{2}\int d^4x_1 d^4x_2 \frac{\delta^2 S}{\delta H^a(x_1)\delta H^b(x_2)}\Big|_{H^a = H^a_{class}} \Delta H^a(x_1)\Delta H^b(x_2) + O((\Delta H^a)^3), \\ \Delta H^a(x) = H^a(x) - H^a_{class}(x). \tag{17}$$

The fields H^a_{class} are such that S has a minimum at H^a_{class}, and we find this configuration as the solution to the classical motion equations:

$$\frac{\delta S}{\delta H^a(x)}\Big|_{H^a = H^a_{class}} = 0 \Rightarrow \partial^2 H^a_{class} + \frac{\partial V}{\partial H^a}\Big|_{H^a = H^a_{class}} - \frac{\partial \mathcal{L}_Y}{\partial H^a}\Big|_{H^a = H^a_{class}} = 0. \tag{18}$$

Simultaneously, we neglect all of the terms which contain $\Delta H^a(x)$ in powers which are bigger than two in the expansion (17). In this way, effective action Γ_{eff} accounts only for the contributions of small quantum fluctuations over the classical background H^a_{class}.

Classical motion equations (18) are non-linear, and we solve them approximately, similarly to [3]. In the zeroth order in the scalar self-couplings and for energies $|p^2| \ll \mathrm{Re}\, m_{12}^2$, $|p^2| = O(v^2)$, the solutions are

$$\begin{aligned}
H^{\pm}_{class} &\approx -\frac{s_\beta c_\beta}{\mathrm{Re}\, m_{12}^2} J^{\pm},\\
A_{0class} &\approx -\frac{\alpha_2 s_{2(\alpha-\beta)} + \alpha_3 (1 + c_{2(\alpha-\beta)})}{2c^2_{\alpha-\beta}} h - \frac{s_{2\alpha} + 2s_{2\beta} - s_{2(\alpha-2\beta)}}{8c^2_{\alpha-\beta}\mathrm{Re}\, m_{12}^2} J_A - \\
&- \frac{\alpha_2 (s_{2\alpha} - s_{2(\alpha-2\beta)} - 2s_{2\beta}) + \alpha_3 (c_{2\alpha} - c_{2(\alpha-2\beta)})}{8c^2_{\alpha-\beta}\mathrm{Re}\, m_{12}^2} J_H,\\
H_{class} &\approx \frac{s_{2(\alpha-\beta)}}{2c^2_{\alpha-\beta}} h + \frac{s_\beta c_\beta (\alpha_3 s_{2(\alpha-\beta)} - \alpha_2 c_{2(\alpha-\beta)}) + \alpha_2 s_\beta c_\beta}{2c^2_{\alpha-\beta}\mathrm{Re}\, m_{12}^2} J_A - \frac{s_{2\beta}}{2c^2_{\alpha-\beta}\mathrm{Re}\, m_{12}^2} J_H.
\end{aligned} \tag{19}$$

Here we neglected the kinetic terms in the equations (18) within the low-energy approximation

$$|p^2| \ll |\mathrm{Re}\, m_{12}^2| \Rightarrow |\partial^2 H^{\pm}| \ll \mathrm{Re}\, m_{12}^2 |H^{\pm}|, \quad |\partial^2 H| \ll \mathrm{Re}\, m_{12}^2 |H|, \quad |\partial^2 A_0| \ll \mathrm{Re}\, m_{12}^2 |A_0|.$$

We insert the solutions (19) into the Lagrangians $\mathcal{L}_s$ and $\mathcal{L}_Y$, and get

$$\begin{aligned}
\mathcal{L}_s[h; H^a_{class}] &= \frac{1}{2}\sum_{a=1}^{3} (\partial_\mu G_a)^2 + \frac{1}{2c^2_{\alpha-\beta}} (\partial_\mu h)^2 - \frac{1}{2}\mu^2 h^2 - \lambda^{(3)} h^3 - \lambda^{(4)} h^4 - \\
&- \varepsilon h (C_1 J_H + C_2 J_A) - \frac{\varepsilon}{v} h^2 (C_3 J_H + C_4 J_A) - \frac{\varepsilon}{v^2} h^3 (C_5 J_H + C_6 J_A) - \\
&- \frac{\varepsilon s_{2\beta} (\alpha_3 + \alpha_2 t_{\alpha-\beta})}{2v^2 c^2_{\alpha-\beta}} J_A \partial^2 h + \frac{\varepsilon s_{2\beta} t_{\alpha-\beta}}{2v^2 c^2_{\alpha-\beta}} J_H \partial^2 h,
\end{aligned} \tag{20}$$

$$\begin{aligned}
\mathcal{L}_Y[h; H^a_{class}] &= -h \left(J_h - \frac{\alpha_2 s_{2(\alpha-\beta)} + 2\alpha_3 c^2_{\alpha-\beta}}{2c^2_{\alpha-\beta}} J_A + \frac{s_{2(\alpha-\beta)}}{2c^2_{\alpha-\beta}} J_H \right) - \\
&- \frac{\varepsilon (s_{2\alpha} + 2s_{2\beta} - s_{2(\alpha-2\beta)})}{8v^2 c^2_{\alpha-\beta}} J_A^2 - \frac{\varepsilon s_{2\beta}}{2v^2 c^2_{\alpha-\beta}} J_H^2 + \\
&+ \frac{\varepsilon s_{2\beta}}{v^2} J^+ J^- - \frac{\varepsilon s_{2\beta} s_{\alpha-\beta}}{v^2 c^2_{\alpha-\beta}} J_A J_H (\alpha_3 c_{\alpha-\beta} + \alpha_2 s_{\alpha-\beta}).
\end{aligned} \tag{21}$$

Here we have taken into account only the operators of up to sixth order and neglected the others, which are suppressed by the factors $(\mathrm{Re}\, m_{12}^2)^{-d}$, $d \geq 2$. μ^2, $\lambda^{(3)}$, $\lambda^{(4)}$ and C_i, $i = \overline{1;6}$ are constants. Their values are as follows

$$\begin{aligned}
&\mu^2 = \kappa v^2, \quad \lambda^{(3)} = -\frac{\kappa v}{2c_{\alpha-\beta}}, \quad \lambda^{(4)} = \frac{\kappa}{8c^2_{\alpha-\beta}},\\
&\kappa = \frac{1}{8c^2_{\alpha-\beta}} \left[\lambda' + 4\lambda_{12}^{(-)} c_{2\beta} + \lambda'' c_{4\beta} \right],\\
&\lambda_{12}^{(\pm)} = \lambda_1 \pm \lambda_2, \quad \lambda' = 3\lambda_{12}^{(+)} + 2\lambda_{345}, \quad \lambda'' = \lambda_{12}^{(+)} - 2\lambda_{345},
\end{aligned} \tag{22}$$

$$C_1 = -\frac{s_{2\beta}}{16c^3_{\alpha-\beta}}\left[\lambda^{(-)}_{12}(s_{\alpha-3\beta}+3s_{\alpha+\beta})+\lambda' s_{\alpha-\beta}+\lambda'' s_{\alpha+3\beta}\right],$$
$$C_2 = \frac{s_{2\beta}}{32c^3_{\alpha-\beta}}\left[\lambda''(\alpha_3 c_{\alpha-5\beta}+\alpha_3 c_{\alpha+3\beta}+2\alpha_2 s_{\alpha+3\beta})+2\lambda'(\alpha_3 c_{\alpha-\beta}+\alpha_2 s_{\alpha-\beta})-\right.$$
$$-2\,\mathrm{Im}\,\lambda_5(s_{2\alpha}+2s_{2\beta}-s_{2(\alpha-2\beta)})+$$
$$\left.+2\lambda^{(-)}_{12}(\alpha_2 s_{\alpha-3\beta}+3\alpha_2 s_{\alpha+\beta}+2\alpha_3 c_{\alpha-3\beta}+2\alpha_3 c_{\alpha+\beta})\right], \tag{23}$$
$$C_3 = -\frac{3}{2c_{\alpha-\beta}}C_1, \quad C_5 = \frac{1}{2c^2_{\alpha-\beta}}C_1, \quad C_4 = -\frac{3}{2c_{\alpha-\beta}}C_2, \quad C_6 = \frac{1}{2c^2_{\alpha-\beta}}C_2. \tag{24}$$

As one can see, the corrections to the 2HDM are linearly-dependent. Only C_1, C_2, κ and $\cos(\alpha-\beta)$ could be independently measured at low energies, in the processes with fermions and the SM Higgs boson in the external states.

Now we turn to the contribution of the quadratic terms in the Gaussian approximation (17). Second functional derivatives in the expansion (17) could be represented in the following matrix form:

$$\frac{\delta^2 S}{\delta H^a(x_1)\delta H^b(x_2)}\Big|_{H^a=H^a_{class}}\Delta H^a(x_1)\Delta H^b(x_2) = \begin{pmatrix}\Delta H^+\\ \Delta H^-\\ \Delta H\\ \Delta A_0\end{pmatrix}^T M_S \begin{pmatrix}\Delta H^+\\ \Delta H^-\\ \Delta H\\ \Delta A_0\end{pmatrix},$$
$$M_S = M_S^{(0)}+\delta M_S, \quad M_S^{(0)} = -\begin{pmatrix}0 & \partial^2+m^2_{H^+} & 0 & 0\\ \partial^2+m^2_{H^+} & 0 & 0 & 0\\ 0 & 0 & \partial^2+m^2_H & 0\\ 0 & 0 & 0 & \partial^2+m^2_A\end{pmatrix},$$
$$\delta M_S = -\begin{pmatrix}0 & \delta^{+-} & \delta^+_H & \delta^+_A\\ \delta^{+-} & 0 & \delta^-_H & \delta^-_A\\ \delta^+_H & \delta^-_H & \delta_{HH} & \delta_{HA}\\ \delta^+_A & \delta^-_A & \delta_{HA} & \delta_{AA}\end{pmatrix}. \tag{25}$$

As we can see here, the matrix $M_S^{(0)}$ contains the inverse propagators of free fields $H^\pm$, H and A_0. The functional integral of exponent of the quadratic terms is

$$\int \mathcal{D}H\mathcal{D}A_0\mathcal{D}H^+\mathcal{D}H^- \exp\left[\frac{i}{2}\int d^4x_1 d^4x_2 \frac{\delta^2 S}{\delta H^a(x_1)\delta H^b(x_2)}\Big|_{H^a=H^a_{class}}\times\right.$$
$$\left.\times\Delta H^a(x_1)\Delta H^b(x_2)\right] = (\det M_S)^{-\frac{1}{2}} = \exp\left[-\frac{1}{2}\,\mathrm{Tr}\ln M_S^{(0)} - \frac{1}{2}\,\mathrm{Tr}\ln\left(1+M_S^{(0)-1}\delta M_S\right)\right],$$
$$M_S^{(0)} = \begin{pmatrix}0 & G^{\pm-1} & 0 & 0\\ G^{\pm-1} & 0 & 0 & 0\\ 0 & 0 & G_H^{-1} & 0\\ 0 & 0 & 0 & G_A^{-1}\end{pmatrix}. \tag{26}$$

In this equation, trace is computed over both spatial and discrete indices of the matrix M_S. The term $\mathrm{Tr}\ln M_S^{(0)}$ does not contain any fields and is constant, so we omit it. The matrix δM_S consists of the terms which come from the quartic part of the potential (2). So the components of this matrix are proportional to the scalar self-couplings. Hence, the Taylor series expansion of the logarithm in (26) in powers of $M_S^{(0)-1}\delta M_S$ is equivalent to a perturbative series expansion. In the first order in the scalar self-couplings, the last term in the square brackets in (26) reads

$$\exp\left[-\frac{1}{2}\,\mathrm{Tr}\ln\left(1+M_S^{(0)-1}\delta M_S\right)\right]\approx\exp\left[-\frac{1}{2}\,\mathrm{Tr}\left(M_S^{(0)-1}\delta M_S\right)\right]=$$
$$=\exp\left[\frac{1}{2}\int d^4x\left(2G^{\pm}(x;x)\delta^{+-}(x)+G_H(x;x)\delta_{HH}(x)+G_A(x;x)\delta_{AA}(x)\right)\right]. \qquad (27)$$

Here $\delta^{\pm}(x)$, $\delta_{HH}(x)$ and $\delta_{AA}(x)$ contain only the terms which are proportional to $J^{\pm}$, J_H, J_A, $J^{\pm}h$, $J_H h$ and $J_A h$. $G^{\pm}(x;x)$, $G_H(x;x)$, and $G_A(x;x)$ are constants which describe contributions of heavy scalar loops. We include these terms into the renormalization of fermionic masses and the corresponding Yukawa couplings. So their contributions are not observable.

Finally, the effective Lagrangian of the 2HDM is

$$\begin{aligned}
\mathcal{L}_{eff}=&\frac{1}{2}\sum_{a=1}^{3}(\partial_\mu G_a)^2+\frac{1}{2c^2_{\alpha-\beta}}(\partial_\mu h)^2-\frac{1}{2}\mu^2h^2-\lambda^{(3)}h^3-\lambda^{(4)}h^4-\\
&-h\left[J_h+\left(\varepsilon C_2-\frac{\alpha_2 s_{2(\alpha-\beta)}+2\alpha_3 c^2_{\alpha-\beta}}{2c^2_{\alpha-\beta}}\right)J_A+\left(\varepsilon C_1+\frac{s_{2(\alpha-\beta)}}{2c^2_{\alpha-\beta}}\right)J_H\right]-\\
&-\frac{\varepsilon}{v}h^2(C_3J_H+C_4J_A)-\frac{\varepsilon}{v^2}h^3(C_5J_H+C_6J_A)-\\
&-\frac{\varepsilon s_{2\beta}(\alpha_3+\alpha_2 t_{\alpha-\beta})}{2v^2c^2_{\alpha-\beta}}J_A\partial^2h+\frac{\varepsilon s_{2\beta}t_{\alpha-\beta}}{2v^2c^2_{\alpha-\beta}}J_H\partial^2h-\\
&-\frac{\varepsilon(s_{2\alpha}+2s_{2\beta}-s_{2(\alpha-2\beta)})}{8v^2c^2_{\alpha-\beta}}J_A^2-\frac{\varepsilon s_{2\beta}}{2v^2c^2_{\alpha-\beta}}J_H^2+\\
&+\frac{\varepsilon s_{2\beta}}{v^2}J^+J^--\frac{\varepsilon s_{2\beta}s_{\alpha-\beta}}{v^2c^2_{\alpha-\beta}}J_AJ_H\left(\alpha_3c_{\alpha-\beta}+\alpha_2s_{\alpha-\beta}\right).
\end{aligned} \qquad (28)$$

This description of interactions between the SM and the heavy scalars of the 2HDM is valid for the energies which are much less than $\mathrm{Re}\,m^2_{12}$. The operators $J_H h$ and $J_A h$ in (28) correspond to the Yukawa interactions between h and the fermions. These terms modify the Yukawa couplings of the SM. As it is shown in (28), these corrections to the Yukawa couplings are either proportional to $(\mathrm{Re}\,m^2_{12})^{-1}$, $\sin(2(\alpha-\beta))$ or α_3, so they become small when $\varepsilon\ll 1$. In particular, in this limit we have the following relations for the mixing angles, using definitions (7) and (8):

$$\lim_{\varepsilon\to 0}\tan 2\alpha=\tan 2\beta\Rightarrow\alpha=\beta,\quad \sin 2(\alpha-\beta)\to 0,\quad \lim_{\varepsilon\to 0}\alpha_3=0. \qquad (29)$$

Lagrangian (28) also contains new effective vertexes, which are introduced by interactions with the additional scalar particles. These are non-renormalizable contact four-fermion interactions J^+J^-, J_AJ_H, J_A^2 and J_H^2, and contact interactions of the SM Higgs and fermions J_Hh^2, J_Ah^2, J_Hh^3, J_Ah^3, $J_H\partial^2h$ and $J_A\partial^2h$. These new effective vertexes are suppressed by the factor $(\mathrm{Re}\,m^2_{12})^{-1}$.

When $\varepsilon \ll 1$ we have the following expressions for the constants in (22), (23) and (24):

$$\lim_{\varepsilon\to 0}\mu^2 = \kappa v^2, \quad \lim_{\varepsilon\to 0}\lambda^{(3)} = -\frac{1}{2}\kappa v, \quad \lim_{\varepsilon\to 0}\lambda^{(4)} = \frac{\kappa}{8},$$
$$\lim_{\varepsilon\to 0}\kappa = \frac{1}{8}\left[\lambda' + 4\lambda_{12}^{(-)}c_{2\beta} + \lambda'' c_{4\beta}\right],$$
$$\lim_{\varepsilon\to 0}C_1 = -\frac{s_{2\beta}}{16}\left[2\lambda_{12}^{(-)}s_{2\beta} + \lambda'' s_{4\beta}\right],$$
$$\lim_{\varepsilon\to 0}C_2 = \frac{s_{2\beta}}{32}\left[2\alpha_2\lambda'' s_{4\beta} - 8s_{2\beta}\mathrm{Im}\,\lambda_5 + 4\alpha_2\lambda_{12}^{(-)}s_{2\beta}\right],$$
$$\lim_{\varepsilon\to 0}C_3 = -\frac{3}{2}C_1, \quad \lim_{\varepsilon\to 0}C_5 = \frac{1}{2}C_1, \quad \lim_{\varepsilon\to 0}C_4 = -\frac{3}{2}C_2, \quad \lim_{\varepsilon\to 0}C_6 = \frac{1}{2}C_2.$$

As it follows from these equalities, in the decoupling limit the relations between $\lambda^{(4)}$, $\lambda^{(3)}$ and μ^2 are the same as those in the potential of the minimal SM Higgs boson, up to the sign of the h field.

However, transformation properties of h are not identical to those of the SM Higgs boson when $\varepsilon \to 0$. In this limit, h does not become a CP-even field, as in the one-Higgs-doublet SM. Even when additional scalar bosons become heavy, the mixing angle α_2 does not vanish, so h contains the contribution of the CP-odd states c_1 and c_2, which is proportional to α_2,

$$\lim_{\varepsilon\to 0}\alpha_2 = \frac{\mathrm{Im}\,\lambda_5 c_{2\beta}s_{2\beta}}{t_{2\beta}(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) - 2s_{2\beta}\mathrm{Re}\,\lambda_5 - s_{2\beta}(\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2)}. \tag{30}$$

Hence, some effects of CP violation could be detected in processes with the h boson.

4. Discussion and conclusions

In the previous sections we discussed the scenario when one of the 2HDM scalar particles has a mass similar to that of the SM Higgs boson, and the other model states are heavy. We have obtained the analytical expressions for the effective operators describing interactions between the SM fermions and the lightest particle of the two-Higgs-doublet model, in terms of the 2HDM parameters. The considered potential of the scalar fields also introduces a small CP-violation. It was shown that the angle α_2, which describes mixing of the scalars with opposite CP-parity, does not vanish in the limit when heavy scalars decouple, and the lightest neutral mass eigenstate of the model is not the eigenstate of the CP transformation. Hence, CP-violation in the 2HDM is potentially visible in the experiments at low energies, and additional interactions within the scalar sector could be identified. Besides, we found that parameters α_2 and α_3 also contribute the effective vertexes in the low-energy EL (28).

At a tree level, the 2HDM introduces reactions mediated by the charged scalars $H^\pm$, which are absent in the SM with the one Higgs doublet. In the low energy region, these processes are described by the effective operators $J^{+(q)}J^{-(q)}$, $J^{+(l)}J^{-(l)}$, $J^{+(q)}J^{-(l)}$ and $J^{+(l)}J^{-(q)}$. Similar processes take place in the SM, too, but they are mediated only by the vector bosons $W^\pm$.

Effective Lagrangian (28) also introduces the new vertexes, which describe annihilation of the fermion-antifermion pair and the subsequent production of one, two or three Higgs bosons.

Numerical predictions of the model with the EL (28) are left beyond the scope of this paper, and will be studied in a separate paper.

Appendix

The mass matrices of the scalar fields in 2HDM are

$$M_a^2 = \left[\frac{\text{Re}m_{12}^2}{v_1 v_2} - \frac{1}{2}\left(\lambda_4 + \text{Re}\lambda_5\right)\right]\begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix},$$

$$M_{bc}^2 = \begin{pmatrix} \frac{v_2}{v_1}\text{Re}\, m_{12}^2 + \lambda_1 v_1^2 & -\text{Re}\, m_{12}^2 + \lambda_{345} v_1 v_2 & \frac{v_2}{v_1} M_{bc23}^2 & -M_{bc23}^2 \\ -\text{Re}\, m_{12}^2 + \lambda_{345} v_1 v_2 & \frac{v_1}{v_2}\text{Re}\, m_{12}^2 + \lambda_2 v_2^2 & M_{bc23}^2 & -\frac{v_1}{v_2} M_{bc23}^2 \\ \frac{v_2}{v_1} M_{bc23}^2 & M_{bc23}^2 & M_{bc33}^2 & -\frac{v_1}{v_2} M_{bc33}^2 \\ -M_{bc23}^2 & -\frac{v_1}{v_2} M_{bc23}^2 & -\frac{v_1}{v_2} M_{bc33}^2 & \frac{v_1^2}{v_2^2} M_{bc33}^2 \end{pmatrix},$$

$$M_{bc23}^2 = \frac{1}{2}\text{Im}\,\lambda_5 v_1 v_2, \quad M_{bc33}^2 = \frac{v_2}{v_1}\text{Re}\, m_{12}^2 - \text{Re}\,\lambda_5 v_2^2. \tag{31}$$

Yukawa's interactions of the 2HDM mass eigenstates with the SM fermions is described by the terms in (13). The contributions of quarks $J^{\pm(q)}$, $J_H^{(q)}$, $J_A^{(q)}$ and $J_h^{(q)}$ are as follows:

$$J^{-(q)} = \sum_{f;f'}\left[\left(y_{ff'}^{2(1)(q)} c_\beta - y_{ff'}^{1(1)(q)} s_\beta\right)\overline{u}_L^{(f)} d_R^{(f')} + \left(y_{ff'}^{1(2)(q)*} s_\beta - y_{ff'}^{2(2)(q)*} c_\beta\right)\overline{u}_R^{(f')} d_L^{(f)}\right], \tag{32}$$

$$J_H^{(q)} = \frac{1}{\sqrt{2}}\sum_{f;f'}\left\{\left[-y_{ff'}^{1(1)(q)} s_\alpha + y_{ff'}^{2(1)(q)} c_\alpha + i\alpha_2\left(-y_{ff'}^{1(1)(q)} s_\beta + y_{ff'}^{2(1)(q)} c_\beta\right)\right]\overline{d}_L^{(f)} d_R^{(f')} + \right.$$
$$\left. + \left[-y_{ff'}^{1(2)(q)} s_\alpha + y_{ff'}^{2(2)(q)} c_\alpha + i\alpha_2\left(y_{ff'}^{1(2)(q)} s_\beta - y_{ff'}^{2(2)(q)} c_\beta\right)\right]\overline{u}_L^{(f)} u_R^{(f')} + h.\,c.\right\}, \tag{33}$$

$$J_A^{(q)} = \frac{1}{\sqrt{2}}\sum_{f;f'}\left\{y_d \overline{d}_L^{(f)} d_R^{(f')} + y_u \overline{u}_L^{(f)} u_R^{(f')} + h.\,c.\right\},$$
$$y_d = i\left[-y_{ff'}^{1(1)(q)} s_\beta + y_{ff'}^{2(1)(q)} c_\beta + i\left(-y_{ff'}^{1(1)(q)}(\alpha_3 c_\alpha + \alpha_2 s_\alpha) + y_{ff'}^{2(1)(q)}(\alpha_2 c_\alpha - \alpha_3 s_\alpha)\right)\right],$$
$$y_u = i\left[y_{ff'}^{1(2)(q)} s_\beta - y_{ff'}^{2(2)(q)} c_\beta + i\left(-y_{ff'}^{1(2)(q)}(\alpha_3 c_\alpha + \alpha_2 s_\alpha) + y_{ff'}^{2(2)(q)}(\alpha_2 c_\alpha - \alpha_3 s_\alpha)\right)\right], \tag{34}$$

$$J_h^{(q)} = \frac{1}{\sqrt{2}}\sum_{f;f'}\left\{\left[-y_{ff'}^{1(1)(q)} c_\alpha - y_{ff'}^{2(1)(q)} s_\alpha + i\alpha_3\left(-y_{ff'}^{1(1)(q)} s_\beta + y_{ff'}^{2(1)(q)} c_\beta\right)\right]\overline{d}_L^{(f)} d_R^{(f')} + \right.$$
$$\left. + \left[-y_{ff'}^{1(2)(q)} c_\alpha - y_{ff'}^{2(2)(q)} s_\alpha + i\alpha_3\left(y_{ff'}^{1(2)(q)} s_\beta - y_{ff'}^{2(2)(q)} c_\beta\right)\right]\overline{u}_L^{(f)} u_R^{(f')} + h.\,c.\right\}, \tag{35}$$

The contributions of leptons are analytically the same. They could be found if one substitutes u-type quarks with neutrinos and d-type quarks with electrons, of the corresponding generation.

From the Yukawa Lagrangian (12) we also have the mass terms for the fermion fields

$$-\mathcal{L}_{mass} = \frac{1}{\sqrt{2}}\sum_{i=1,2} v_i \sum_{f;f'}\left[y_{ff'}^{i(1)(q)}\overline{d}_L^{(f)} d_R^{(f')} + y_{ff'}^{i(2)(q)}\overline{u}_L^{(f)} u_R^{(f')} + \right.$$
$$\left. + y_{ff'}^{i(1)(l)}\overline{e}_L^{(f)} e_R^{(f')} + y_{ff'}^{i(2)(l)}\overline{\nu}_L^{(f)} \nu_R^{(f')} + h.c.\right]. \tag{36}$$

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