OPTIMIZATION OF SYNTHETIC APERTURE APPROACH TO RADIOIMAGE PROCESSING WITH HELP OF THE PRONY'S METHOD

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Application of the Prony's method with segmented approach to radioimage analysis is researched. Measuring system with a horn antenna is considered. An optimal geometry of the horn antenna for implementation of Fourier-holography in frequency-time domain is estimated with the help of a big perfect reflector model. The mathematical model of the structure with reflector is constructed and used for initial verification of the method. The model is constructed with the help of the transmission line theory. The Prony's method is then applied to estimation of distances to the points of radioimages constructed with the help of a computer model of an antenna. Distances on the image of a conductive strip obtained by aperture synthesis are estimated with the help of the method. It is found that estimated distance tends to increase with the increase of the lateral distance from the axis of the antenna. This result agrees with the fact that antenna radiates a non-plain wave. It is important for the aperture synthesis calibration procedure with help of the conductive strip.

Keywords: **radioimage, Prony's method, parametric spectral analysis, aperture synthesis, horn antenna**.

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1. Introduction

Radioimgaing is an important branch of engineering which allows using nondestructive analysis to reconstruct the inner structure of dielectric layered objects or to visualize hidden objects. One of the known methods for obtaining of such radioimages is Fourier-holography in frequency-time domain with application of the inverse discrete Fourier transform (IDFT) with aperture synthesis for resolution improvement in lateral direction [1]. The calibration procedure is based on measurements of a single conductive strip thus correct processing of corresponding data is important for successful implementation of the method. Accuracy of the IDFT method is highly dependent on the number of frequency points obtained during the scanning. The distance step of such an image can be artificially decreased with the use of additional points with zero values. The downside of such an approach is the fact that the resulting image is convoluted with the $\mathrm{sinc}(x)$ function. This effect can be mitigated with the help of window functions, but the resultant values and positions of the extrema can remain distorted.

In the current paper as in [1] the Prony's method is considered for the estimation of distances on a radioimage. The Prony's method is a widely used parametric spectral analysis method [2, 3]. Its main advantage in comparison with the IDFT method is the fact that possible "frequencies" of the resultant model are not dependent on the number of points or the frequency discretization step if the maximum signal frequency is below the Nyquist frequency. The main idea of the Prony's method is to approximate the input sequence with the help of the sum of complex exponential discrete functions [4].

The classic implementation of the Prony's method calls for the solution of an overdefined linear system of algebraic equations. In the cases of high SNR ($SNR \geq 30\ \mathrm{dB}$) this can be done by using the Moore-Penrose inversion method [4]. In cases of low SNR values ($SNR \leq 20\ \mathrm{dB}$) such method can lead to incorrect and, frequently for signals with constant component amplitudes, exponentially growing results mainly due to poorly defined matrices. In such cases the eigenvector approach can be used [2, 5]. In order to further improve the eigenvector method, the segmented approach to the Prony's method is proposed [6]. The idea of the approach is to divide the initial signal in a series of segments which may overlap. The calculation procedure is then performed for each segment separately and the resultant vector is constructed as a weighted average of the segment results. It is expected that such approach can reduce the effects of frequency modulation and impulse noise.

The tests were performed on a computer model of a horn antenna. Initial tests were performed on a mathematical model of a 3-layer structure constructed with the help of transmission line theory.

The purpose of the article is to improve the calibration procedure in aperture synthesis with help of a single conductive strip due to more correct estimation the distance to the strip under lateral shifting with help of the modification of the Prony's method.

2. Segmented approach algorithm

The segmented approach requires dividing the signal sequence $\bar{s}$ in segments

$$\bar{s}_n = [s_{(L-K)n} \quad \cdots \quad s_{(L-K)n+L-1}]^T,$$

where L is the length of a segment and K is the number of overlapping points. For each of the segments the solution of the system of equations is found with the use of the eigenvector method. The resultant solution is constructed as a weighted average of the segment solutions. Weight coefficients are chosen as normalized values of SNR [6]. The noise power was found with the help of exceeding the required number of components by 1 and calculating the smallest eigenvalue of the Prony's linear algebraic system [2]

3. Time modulated signal

In order to test the applicability of the segmented Prony's method to signals with time modulation, several test signals were constructed. The results of the analysis were compared to the results obtained with the help of the Skljar's [7] approach to the Prony's method.

Fig. 1 shows the comparison between results obtained with the help of the segmented approach and Skljar's method for a signal with linear time modulation ($s(f) = \cos(2\pi(2.5 + 0.01f)f)$).

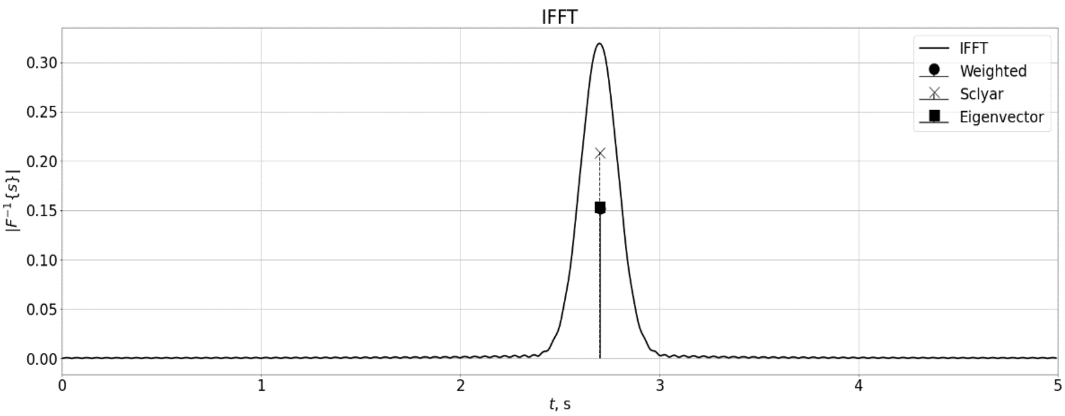

Fig. 1. Results of time estimation for the linearly modulated signal.

It can be seen that in this case the segmented approach yields results similar to ones obtained with the help of Skljar's method.

The method was also tested for a signal with harmonic modulation pattern ($s(f) = \cos(2\pi(2.5 + 0.5\,cos(2\pi \cdot 0.05f))f)$). The results of the testing can be seen in Fig. 2.

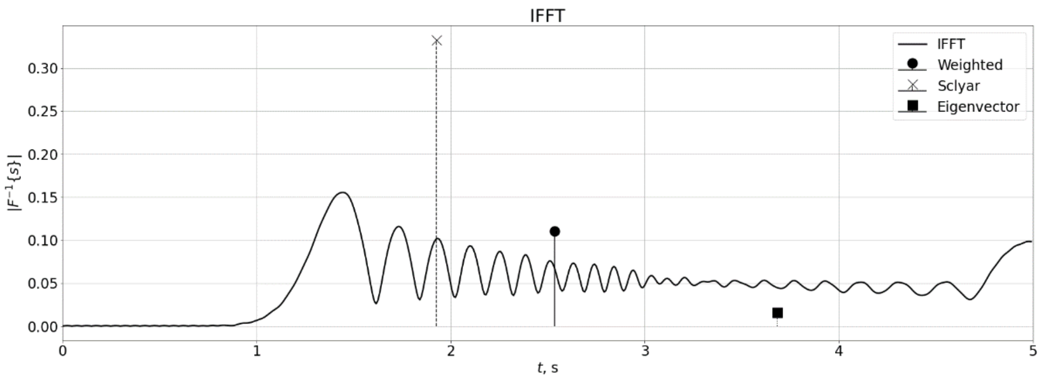

Fig. 2. Results of the processing of a harmonically modulated signal.

As can be seen from the figure, weighted method was successfully able to estimate the mean frequency better, than the Skljar's method. Due to these results, it was decided to use the segmented method to analyze the results of the scanning.

3. Antenna model

Antenna model used during the research was a horn antenna with horn length $l_h = 30$ mm. The model also included the feed line in the form of a rectangular waveguide. The area of the aperture of the antenna was varied in range between 5.2×5.2 mm^2 and 46×46 mm^2. The throat of the antenna had the area of 5.2×2.6 mm^2. The outside calculation region was shaped as a rectangular prism with the aperture of the antenna belonging to one of the sides. Where possible, the distance between the edges of the outside computation domain and the conductive surfaces was no less than 2λ to reduce the reflections caused by computation errors. All the open surfaces of the external computational domain except the one in front were given the free space matching boundary condition. In order to simulate the presence of a big reflector in front of the antenna, side parallel to the aperture was given a perfect electrical conductor boundary condition for the appropriate experiments.

To eliminate the reflections due to errors of computation in experiments where open space in front of the antenna is required, the side in front of the aperture was replaced with a spherical surface. The center of the surface was positioned in the center of the throat.

The experiment was carried out in the frequency range between 38 GHz and 52 GHz. In this range only the main mode of the waveguide is traveling. To improve the speed of the calculation, for most experiments the frequency step was chosen equal to 0.25 GHz, which is sufficient for the Prony's method application.

During the research both the pure complex value of the reflection coefficient and the square of the absolute value were used for the testing. The latter value corresponds to results which can be obtained during a real experiment with application the Fourier-holography approach in frequency-time domain.

4. Mathematical model

It is important to note that the problem of radioimaging in frequency domain is difficult for the methods such as the classic Prony's method due to frequency of components depending on the number of the signal point. To test the applicability of the Prony's method to the problem of distance estimation on radioimages, a mathematical model of the structure was constructed. The model was based on the known transmission line formulas for loaded 2-port elements and waveguide wavelength for different frequencies. In the used model the physical length of the waveguide was 50 mm, length of the horn was 30 mm and the distance from the aperture to the reflector was 23 mm. Resultant model was then processed with the help of the segmented Prony's method (Fig. 3).

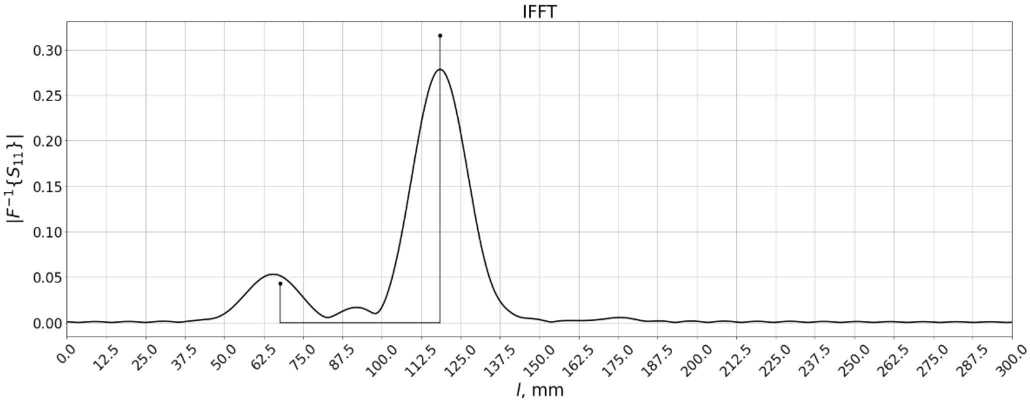

Fig. 3. Results of IDFT processing for the model signal.

The results of the processing were 67.71 mm and 118.34 mm for the feed line connection and the reflector respectively. The mean effective length of the waveguide and distance to the reflector during the experiment were 68.45 mm and 121.45 mm respectively. It can be seen that the method was able to accurately estimate the absolute distance to the reflector and the electrical length of the feed line. The distance from the aperture to the reflector was estimated to be 20.63 mm.

5. Optimal antenna estimation

To conduct the experiment, it was required to find an optimal surface of the aperture for the specified constraints of the design. The optimal antenna was defined as one which would give the highest primary reflection in comparison with the secondary reflections. In order to prevent the overlapping of the peaks on the image, the reflecting boundary was placed at 23 mm from the aperture. Initial results were obtained with the help of the IDFT method. The results of the testing can be seen in Fig. 4. The obtained image is based on the processing of the complex reflection coefficient.

As can be seen from the image, the best antenna aperture surface was estimated to be 23.(3)×23.(3) mm^2.

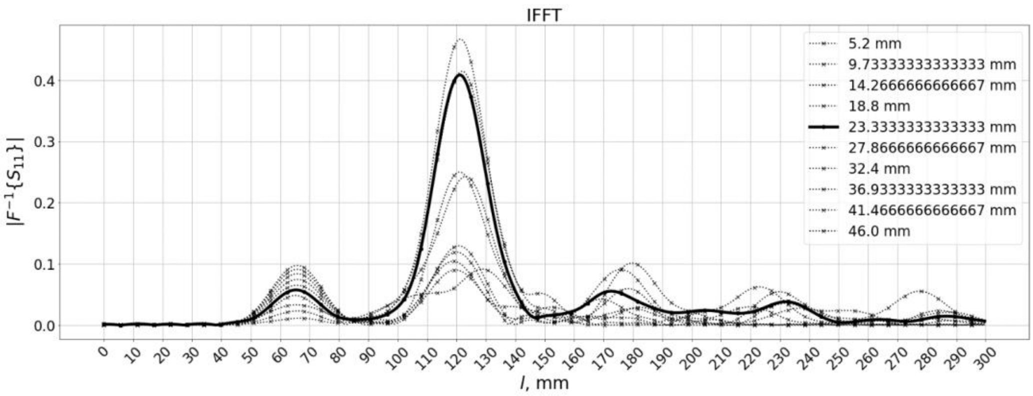

Fig. 4. Results of IDFT processing for different aperture areas.

6. Prony's method application to the simulated data

In order to calibrate the optimal antenna, simulations with different distances to the reflector were conducted. The results of distance estimation are presented in Table 1.

Table 1

Results of distance estimation with the Prony's method

l, mm	l_{est}, mm	l, mm	l_{est}, mm
10	7.24	22	23.48
12	13.88	24	26.29
14	14.46	26	27.24
16	18.45	28	30.61
18	18.98	30	31.84
20	22.19		

These results were obtained by subtracting the estimated electrical distance to the throat and the nominal length of the horn from the estimated distance to the reflector. The estimated electrical length of the horn is difficult to obtain by using such methods due to the reflection coefficient having a very low magnitude.

7. Aperture synthesis calibration procedure

The calibration process for aperture synthesis is based on measurements of reflection from conductive strip. After the calibration process, the Prony's method was implemented to estimate the distances to an image. In this experiment the far surface of the outside computation domain was replaced with a spherical surface centered on the throat of the antenna and given a free space boundary condition. The reflector was made in the shape of a conductive copper strip with surface area of 10×50 mm^2 and thickness of 1 mm. Due to the symmetry of the problem only half of the range of motion was considered. The reflector was placed at the distance of 23 mm from the aperture and the transverse position was chosen in the range from 0 mm to 10 mm with the step of 2.5 mm. Results of the processing with the help of the Prony's method are shown in Fig. 5 and corresponding distances are represented in Table 2.

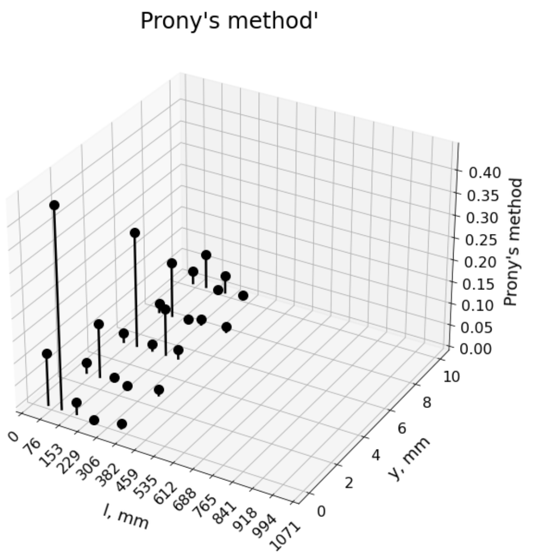

Fig. 5. Results of the aperture synthesis image processing.

Table 2

Estimated distances to the reflector

y, mm	l_{est}, mm
0.0	25.00
2.5	25.68
5.0	25.46
7.5	25.67
10.0	26.77

It can be seen that the estimated distance to the reflector tends to increase with the lateral distance. Such effect is expected due to the complicated space spectrum coming from the antenna but not plain wave radiation.

8. Conclusions

It was shown that segmented approach to the Prony's method is applicable to the analysis of signals with "frequency" modulation and that it can yield results of the same or better quality then the Skljar's method. It was shown that the segmented Prony's method is applicable to estimation of distances on radioimages obtained with the help of a horn antenna. The optimal parameters of the method were found with the help of a mathematical model of the antenna and the reflector. The optimal antenna for the calculations was chosen by varying the surface area of the aperture. The results of scanning for different distances were obtained with the help of the Prony's method. It was shown that the Prony's method can be applied to obtain the distances on a radioimage for improving the calibration procedure in aperture synthesis implementation with inverse filtering approach.

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