

# INDUCED COLOR CHARGE AND QUARK PROPAGATION AT POLYAKOV'S LOOP BACKGROUND

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In quark-gluon plasma, the presence of the  $A_0$  (Polyakov loop (PL)) condensate results in the color  $Z(3)$  symmetry breaking and the Furry theorem violation. Due to these peculiarities, new phenomena - the induced color charges and other even-number diagram effects - are realized. Using the  $\bar{R}_\xi$  gauge, we obtain the  $\xi$ -independent two-loop effective potential  $W(A_0^{classical})$  expressed in terms of the PL. Its minimum position detects the value of the condensate. We calculate the one-loop tadpole diagram with one gluon line and the induced color charges  $Q_{ind}$ . Having these, we investigate the influence of the color charges on quark propagation and derive and partially investigate the contribution of them into the Schwinger-Dyson equation for quarks. It is found that the presence of  $Q_{ind}$  effectively increases the strength of the  $A_0$  potential acting on colored particles in the plasma.

**Keywords:** quark-gluon plasma, Polyakov loop, color symmetry, induced color charges, Schwinger-Dyson equation for quarks.

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## 1. Introduction

Quark-gluon plasma (QGP) is a new state of matter consisting of quarks and gluons liberated from nuclei at high temperature due to asymptotic freedom of non-Abelian gauge fields. QGP is one of the main object for researching nowadays. Despite high interest, many phenomena are not investigated. The order parameter for the phase transition is the Polyakov loop (PL) - an integral variable - or related with it so-called  $A_0$  condensate, which is a constant solution to Yang-Mills field equation at finite temperature in the Matsubara imaginary time formalism. At low temperature the PL and  $A_0$  equal zero. At high temperature they become nonzero signaling a deconfinement phase transition (DPT).

There is a number of processes suppressed in vacuum by the Furry theorem. This theorem states that due to C-parity conservation the Feynman diagrams with odd number of external photon (gluon) lines mutually cancel (and thus give zero total contribution to any amplitude). However in the plasma, C-parity is violated and the theorem ceases to be satisfied.

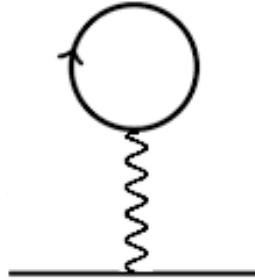


Fig. 1. Tadpole diagram

The generally recognized  $SU(3)$  global symmetry violation mechanism at high temperatures is the spontaneous generation of the  $A_0$  condensate. This leads to the appearance of the gluon magnetic mass and color charges.

The goal of the present paper is two-fold. First we calculate the value for the  $A_0$  condensate

by using gauge invariant effective potential of order parameter derived recently [1]. Second, we compute the "tadpole" diagram (Fig. 1), and the induced color charge generated in plasma, and also obtain the corrections to quark mass operator described by the Schwinger-Dyson equation.

## 2. $A_0$ condensate

$A_0$  condensate is a solution to field equations giving minimum to the effective potential for the gluon fields at high temperatures. It belongs to the center of the gauge group  $A_0 = \frac{2\pi n}{\beta g N}$ ,  $n \in Z_N$ ,  $n = 0, 1, \dots, N-1$ .

Actually, the value of the  $A_0$  has to be calculated from the full effective action with quantum corrections taken into account. Difference between  $\langle A_0 \rangle$  and  $\frac{2\pi n}{\beta g N}$  points is due to a spontaneous gauge symmetry violation [2]. The QCD Lagrangian in the relativistic background  $\bar{R}_\xi$  gauge reads

$$\begin{aligned} \mathcal{L} = & \frac{1}{4}(G_{\mu\nu}^a)^2 + \frac{1}{2\xi}(D_\mu^B Q_\mu^a)^2 + \bar{\chi} D_\mu^B D_\mu \chi \\ & + \bar{\psi}^a (\gamma_\mu + im) \psi^a + ig \bar{\psi}^a \gamma_\mu (A_\mu^c + Q_\mu^c) (t^c)^a_b \psi^b, \end{aligned} \quad (1)$$

$$\begin{aligned} G_{\mu\nu}^a &= (D_\mu^B)^{ab} Q_\nu^b - (D_\nu^B)^{ab} Q_\mu^b - gf^{abc} Q_\mu^b Q_\nu^c, \\ (D_\mu^B)^{ab} &= \delta^{ab} \partial_\mu + gf^{abc} A_\mu^c, \\ (D_\mu)^{ab} &= \delta^{ab} \partial_\mu + gf^{abc} (Q_\mu^c + A_\mu^c), \\ A_\mu^c &= \delta_{\mu 0} (\delta^{c3} A_0^3 + \delta^{c8} A_0^8), \end{aligned} \quad (2)$$

where  $(t^c)^a_b$  are  $SU(3)$  generators,  $Q_\mu^a$  is a quantized field,  $A_\mu^a$  is classical background field,  $f^{abc}$  are structure constants,  $\chi$ ,  $\bar{\chi}$  are ghost fields,  $\xi$  is gauge-fixing parameter,  $g$  is coupling constant.

For further consideration it is convenient to introduce the charged basis of the gluon field

$$\begin{aligned} \pi_\mu^\pm &= \frac{1}{\sqrt{2}}(A_\mu^1 \pm iA_\mu^2), & \pi_\mu^0 &= A_\mu^3, & \eta_\mu &= A_\mu^8, \\ K_\mu^\pm &= \frac{1}{\sqrt{2}}(A_\mu^4 \pm iA_\mu^5), & \bar{K}_\mu^\pm &= \frac{1}{\sqrt{2}}(A_\mu^6 \pm iA_\mu^7). \end{aligned} \quad (3)$$

The basis splits in three subgroups  $\pi_\mu^\pm, \pi_\mu^0$ ,  $K_\mu^\pm, \eta_\mu$  and  $\bar{K}_\mu^\pm, \eta_\mu$  which are important for what follows. In it, in the momentum space, the background fields  $A_0^3$  and  $A_0^8$  are included as constant shifts. Actual calculations of the two-loop effective potential at the  $A_0$ ,  $W(A_0, \xi)$ , have been carried out by many authors and resulted in the  $\xi$ -dependent expression. Its minimum position is also gauge-fixing dependent. So, the condensation phenomenon could be gauge non-invariant and illusory. This point was discussed for many years in the literature. The gauge-fixing independent effective potential, expressed in terms of the PL, was derived recently in [1], [3] on the base of the Nielsen identity approach to the  $\xi$ -dependence of generating functionals and particle spectra.

As it was discovered, the minimum value of the condensate  $(A_0)_{min}$  is the same for the gluon two-loop effective potential and the quark one both calculated separately. Moreover, to obtain the effective potential expressed in terms of PL it is formally sufficient to substitute in  $W(A_0, \xi)$  the parameters  $A_0 \rightarrow A_0^{PL} = A_0^{classical}$ ,  $\xi \rightarrow -1$ . Such way derived new expression results in the classical effective potential  $W(A_0^{PL})$  for observable value of the condensate. More information see in noted papers.

For completeness we adduce the final expression of the gluon contribution obtained after these replacements [1]:

$$\begin{aligned} \beta^4 W_g = & \frac{4\pi^2}{3} \left[ -\frac{1}{30} + \sum_{i=1}^3 B_4(a_i) \right] + \frac{g^2}{2} \left\{ \sum_{i=1}^3 [B_2^2(a_i) + 2B_2(0)B_2(a_i)] \right. \\ & \left. + B_2(a_1)B_2(a_2) + B_2(a_2)B_2(a_3) + B_2(a_3)B_2(a_1) \right\} \\ & + \frac{2g^2}{3} \left\{ B_3(a_1)[2B_1(a_1) + B_1(a_2) - B_1(a_3)] + \right. \\ & B_3(a_2)[2B_1(a_2) + B_1(a_1) + B_1(a_3)] + \\ & \left. B_3(a_3)[2B_1(a_3) - B_1(a_1) + B_1(a_2)] \right\}, \end{aligned} \quad (4)$$

where the dimensionless parameters are introduced:  $x = \frac{\beta}{\pi} g A_0^3$ ,  $y = \frac{\beta}{\pi} g A_0^8$  and

$$a_1 = \frac{x}{2}, \quad a_2 = \frac{1}{4}(x + \sqrt{3}y), \quad a_3 = \frac{1}{4}(-x + \sqrt{3}y). \quad (5)$$

The first term of (4) in rectangular brackets describes one-loop contribution and others present the two-loop one. The part standing with factor  $\frac{2g^2}{3}$  comes from the  $\xi$ -dependent terms (see [1]). Explicit expressions for Bernoulli's polynomials  $B_i(x)$  defined *modulo* 1 are

$$\begin{aligned} B_1(x) &= x - \frac{x}{2|x|}, \quad B_2(x) = x^2 - |x| + \frac{1}{6}, \\ B_3(x) &= x^3 - \frac{3}{2} \frac{x^3}{|x|} + \frac{1}{2}x, \\ B_4(x) &= x^4 - 2|x|^3 + x^2 - \frac{1}{30}. \end{aligned} \quad (6)$$

That is, for  $x = C\beta/(2\pi)$ ,  $|x| \leq 1$ , and therefore  $C, x$  are periodic. At  $x = 0$  the  $B_1(x)$  is defined to be 0. Here and in what follows we denote  $x = x_{classical} = x^{PL}$ ,  $y = y_{classical} = y^{PL}$  are variables expressed in terms of Polyakov's loops for  $A_0^3$  and  $A_0^8$ .

The minimum position of  $W_g$  gives the condensate values in the plasma:

$$x = \frac{g^2}{\pi^2}, \quad y = 0. \quad (7)$$

The minimum value of the potential is

$$\beta^4 W_g|_{min} = \beta^4 W_g(0) - \frac{g^4}{2\pi^2}, \quad (8)$$

where the first term is the value at zero condensate. Hence we see that the condensate decreases the potential and is energetically favorable. It is important that the  $A_0^8$  is absent at two-loop level. In above formulas  $x$  and  $y$  are defined in the main topological sector:

$$0 \leq a_1 \leq 1, \quad 0 \leq a_2 \leq 1, \quad -1 \leq a_3 \leq 0. \quad (9)$$

Other values can be obtained by means rotation on the angle  $\frac{\pi}{3}$  in the  $(x, y)$  plain. In what follows we shall consider the main sector, only.

As a final conclusion, we see that in *QGP* the gauge invariant condensate values of classical fields are  $(gA_0^3)_{min} = \frac{g^2}{\pi}T, y = 0$ . They are temperature dependent and will be assumed in what follows at place of  $gA_0$ .

### 3. Induced color charge

In this section, we calculate the induced color charge generated by the tadpole diagram of Fig. 1. In charged basis, we have two components of the induced charge for the shifts  $A_0^3$  and  $A_0^8$ . But accounting for the result (7) we calculate the contribution for the case  $(A_0)_\mu^a = A_0 \delta_{\mu 4} \delta^{a3}$ . The explicit form in the Euclid space-time is  $Q_4^3 Q_{ind}^3$ , and we have

$$Q_{ind}^3 = \frac{g}{\beta} \sum_{p_4} \int \frac{d^3 p}{(2\pi)^3} Tr \left[ \frac{\lambda^3}{2} \gamma^4 \frac{\hat{p}_\sigma \gamma_\sigma + m}{\hat{p}^2 + m^2} \right], \quad (10)$$

where  $\hat{p} = (p_4 = p_4 \pm A_0, \mathbf{p})$ ,  $p_4 = 2\pi T(l + 1/2)$ ,  $l = 0, \pm 1, \dots$ . The trace is calculated over either space-time or color variables. Here also we noted as  $A_0$  the value  $A_0 = \frac{gA_0}{2}$ .

Calculating the traces over the space and the internal indexes we get,

$$Q_{ind}^3 = \frac{4g}{\beta} \int \frac{d^3 p}{(2\pi)^3} \sum_{p_4} \frac{(p_4 + A_0)}{(p_4 + A_0)^2 + \varepsilon_{\mathbf{p}}^2}. \quad (11)$$

To calculate the sum we use the following representation [7]

$$Q_{ind}^3 = 4g \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\pi i} \oint_C \tan \frac{\beta \omega}{2} \frac{(\omega + A_0)}{(\omega + A_0)^2 + \varepsilon_{\mathbf{p}}^2} d\omega. \quad (12)$$

The integrand function has two imaginary poles of the first order. We use residues to find its value.

The result, after transformation into spherical coordinates and angular integration, is

$$Q_{ind}^3 = \frac{g \sin(A_0 \beta)}{\pi^2} \int_0^\infty p^2 dp \frac{1}{\cos \beta A_0 + \cosh \beta \varepsilon_{\mathbf{p}}}. \quad (13)$$

In what follows, we calculate the integral in the high-temperature limit  $T \rightarrow \infty$  ( $\beta \rightarrow 0$ ). In this case  $|\mathbf{p}| \gg m$ , so we use

$$\varepsilon_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2} \approx |\mathbf{p}| + \frac{1}{2} \frac{m^2}{|\mathbf{p}|}. \quad (14)$$

After integration over momentum we obtain

$$Q_{ind}^3 = gA_0 \left[ \frac{T^2}{3} - \frac{m^2}{2\pi^2} \right]. \quad (15)$$

As we see, the first term does not depend on mass and depends on temperature as  $\sim T^2$ . The second term depends on mass, only. At high-temperature, the first term is dominant and the plasma acquires the spontaneous induced charge in the case  $m = 0$ , also. Remind also that here  $A_0 = \frac{gA_0}{2}$ .

Thus, one of the consequences of the  $A_0$  condensate presence is the  $Z(3)$  symmetry and the  $C$ -parity violation, which leads to the induction of color charge in the plasma.

#### 4. Quark self-energy

Now, let us calculate a quark self-energy in the presence of the  $A_0$  condensate. As it is known, the particle spectra are determined by the poles of the Schwinger-Dyson equation

$$G^{-1} = \hat{p} + m - \Sigma, \quad (16)$$

where  $\Sigma$  is a polarization operator.

Let us calculate it in one-loop approximation. For this, in addition to the tadpole diagram, we also need to account for the one shown in Fig. 2. That has been done in [5] for a chemical

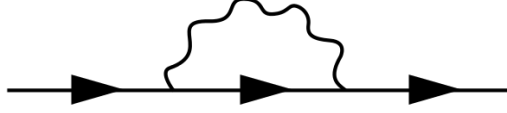


Fig. 2. One-loop quark self-energy.

potential  $\mu$ . We use that results in case  $N = 3$  for QCD. Using the Feynman rules (in the Feynman gauge ( $\xi = 1$ )) [7] we get

$$\Sigma(q) = \frac{4g^2}{3\beta} \sum_{p_4} \int \frac{d^3p}{(2\pi)^3} \mathcal{D}_{\mu\nu}(p-q) \gamma_\mu G(p) \Gamma_\nu(p, q|p-q), \quad (17)$$

where the bare functions have the form

$$\Gamma_\mu^0(p, q|p-q) \frac{\lambda}{2} = \gamma_\mu \frac{\lambda}{2}, \quad G^0(p) = \frac{-i\gamma_\mu \hat{p}_\mu + m}{\hat{p}^2 + m^2}, \quad \mathcal{D}_{\mu\nu}^0(p) = \frac{\delta_{\mu\nu}}{p^2}, \quad (18)$$

where we omitted internal indexes.

We sum over the spinor indices and omit again internal ones to obtain

$$\Sigma(q) = \frac{8g^2}{3\beta} \sum_{p_4} \int \frac{d^3p}{(2\pi)^3} \frac{i\gamma_\mu \hat{p}_\mu + 2m}{(\hat{p}^2 + m^2)(p-q)^2}. \quad (19)$$

After that we calculate the temperature sum

$$\begin{aligned} \Sigma(q) &= -\frac{2g^2}{3\pi i} \int \frac{d^3p}{(2\pi)^3} \oint_C \tan \frac{\beta\omega}{2} \\ &\quad \times \frac{i(\omega\gamma_4 + \mathbf{p}\mathbf{q}) + 2m}{[(\omega - A_0)^2 + \varepsilon_{\mathbf{p}}^2][(\omega - q_4)^2 + |\mathbf{p} - \mathbf{q}|^2]}. \end{aligned} \quad (20)$$

We introduce the notation for Bose and Fermi occupation numbers

$$n_{\mathbf{p}}^B = \frac{1}{e^{\beta|\mathbf{p}|} - 1}, \quad n_{\mathbf{p}}^\pm = \frac{1}{e^{\beta(\varepsilon_{\mathbf{p}} \mp iA_0)} + 1}. \quad (21)$$

The result is

$$\begin{aligned} \Sigma(q) = & -\frac{4g^2}{3} \int \frac{d^3p}{(2\pi)^3} \\ & \times \left\{ \frac{\pi n_{\mathbf{p}}^+}{\varepsilon_{\mathbf{p}}} \frac{\gamma_4 \varepsilon_{\mathbf{p}} + i\gamma \mathbf{p} + 2m}{(q_4 + i\varepsilon_{\mathbf{p}} - A_0)^2 + |\mathbf{q} - \mathbf{p}|^2} \right. \\ & + \frac{\pi n_{\mathbf{p}}^B}{|\mathbf{p}|} \frac{[|\mathbf{p}| + i(A_0 - q_4)]\gamma_4 + i\gamma(\mathbf{p} - \mathbf{q}) - 2m}{(q_4 - A_0 + i|\mathbf{p}|)^2 + \varepsilon_{\mathbf{p}-\mathbf{q}}^2} \\ & - \frac{\pi n_{\mathbf{p}}^-}{\varepsilon_{\mathbf{p}}} \frac{\gamma_4 \varepsilon_{\mathbf{p}} + i\gamma \mathbf{p} - 2m}{(q_4 + i\varepsilon_{\mathbf{p}} + A_0)^2 + |\mathbf{q} - \mathbf{p}|^2} \\ & \left. - \frac{\pi n_{\mathbf{p}}^B}{|\mathbf{p}|} \frac{[|\mathbf{p}| - i(A_0 + q_4)]\gamma_4 + i\gamma(\mathbf{p} - \mathbf{q}) - 2m}{(q_4 + A_0 + i|\mathbf{p}|)^2 + \varepsilon_{\mathbf{p}-\mathbf{q}}^2} \right\}. \end{aligned} \quad (22)$$

Then we introduce two new functions

$$\Sigma(q) = i\gamma_{\mu} K_{\mu}(q) + mZ(q), \quad (23)$$

and calculate them separately. Using  $Tr\Sigma(q)/4 = mZ(q)$  we find  $Z(q)$

$$\begin{aligned} Z(q) = & -\frac{4g^2}{3} \int \frac{d^3p}{\pi^3} \frac{n_{\mathbf{p}}^B}{|\mathbf{p}|} \left[ \frac{1}{(q_4 - A_0 + i|\mathbf{p}|)^2 + \varepsilon_{\mathbf{p}-\mathbf{q}}^2} \right. \\ & \left. - \frac{1}{(q_4 + A_0 + i|\mathbf{p}|)^2 + \varepsilon_{\mathbf{p}-\mathbf{q}}^2} \right]. \end{aligned} \quad (24)$$

Similarly, for  $Tr\gamma_4\Sigma(q)/4 = iK_4(q)$  we obtain the function  $K_4(q)$

$$\begin{aligned} K_4(q) = & -mZ(q) + \frac{2ig^2}{3} \int \frac{d^3p}{\pi^3} \left[ \frac{n_{\mathbf{p}}^+}{(q_4 - A_0 + i|\mathbf{p}|)^2 + |\mathbf{q} - \mathbf{p}|^2} \right. \\ & \left. - [h.c(A_0 \rightarrow -A_0)] \right]. \end{aligned} \quad (25)$$

And from  $Tr\gamma_n\Sigma(q)/4 = iK_n(q)$  we get the vector  $K_n(q)$  ( $n = 1, 2, 3$ )

$$\begin{aligned} K_n(q) = & \frac{2g^2}{3} \int \frac{d^3p}{\pi^3} \frac{p_n}{\varepsilon_{\mathbf{p}}} \left[ \frac{n_{\mathbf{p}}^+}{(q_4 - A_0 + i|\mathbf{p}|)^2 + |\mathbf{q} - \mathbf{p}|^2} \right. \\ & \left. - [h.c(A_0 \rightarrow -A_0)] \right]. \end{aligned} \quad (26)$$

Unlike the calculations with chemical potential  $\mu$ , the  $A_0$  is included as a real shift of the zero momentum component. Therefore, it also changes the frequencies of modes and affects the stability of spectra.

Next we consider the contribution of the induced color charge. For the tadpole diagram we

have

$$\begin{aligned} \Sigma^{tp}(q) = & \frac{4g^2}{3\beta} \sum_{k_4} \int \frac{d^3k}{(2\pi)^3} \sum_{p_4} \int \frac{d^3p}{(2\pi)^3} \\ & \times \mathcal{D}_{\mu\nu}(k + m_{gl}^3) \gamma_\mu \frac{\lambda^3}{2} \text{Tr} \left[ \frac{\lambda^3}{2} G(p) \Gamma_\nu(p, q|p - q) \right]. \end{aligned} \quad (27)$$

Here, we introduces temperature Debye's mass  $m_{gl}^3$  in gluon propagator. Due to equality of initial and final neutral gluon momenta  $q = q'$  the transfer momentum  $\vec{k} = 0$ . At finite temperature the  $k_4$  and space  $\vec{k}$  indexes are independent. The latter momentum is zero. In high temperature limit  $k_4$  is also zero. So, after k-integration we get only the temperature mass contribution from the gluon propagator. The second integral gives  $Q_{ind}^3$ , which was calculated above. The result is

$$\Sigma^{tp} = \frac{16g^2}{3(m_{gl}^3)^2} Q_{ind}^3, \quad (28)$$

where we omitted the color and  $\gamma_4$  matrixes and  $(m_{gl}^3)^2 = Cg^2T^2$ . Here, it enters the gluon propagator as a phenomenological parameter. It can be calculated, in particular, from the effective potential  $W_g$  (4). We present this calculation below. Using  $Q_{ind}^3$  expansion (15) we obtain finally

$$\Sigma^{tp} = \frac{gA_0}{C} \left[ \frac{16}{9} - \frac{8m^2}{3\pi^2T^2} \right] \gamma_4 \frac{\lambda^3}{2}. \quad (29)$$

As we see, the induced color charge gives correction to quark masses. The second term is next-to-leading at high-temperature. Accounting for equations (16), (29) and (1), we conclude that taking into consideration the induced charge results in the replacement the interaction potential  $gA_0^3$  by  $gA_0^3 + \frac{g^2A_0^3}{C} \left[ \frac{8}{9} - \frac{4m^2}{3\pi^2T^2} \right]$ . This increases effectively the influence of the  $A_0$  condensate on motion of quarks and gluons. In final expression we have substituted  $A_0 \rightarrow \frac{gA_0}{2}$  coming from  $Q_{ind}^3$ .

## 5. Debye's mass of neutral gluons

Now, let us calculate Debye's temperature mass of longitudinal neutral gluons (plasmons). For that we use the definition

$$m_D^2 = \frac{d^2W_g(x)}{d(A_0^3)^2} \Big|_{A_0^3=0}, \quad (30)$$

and take the one loop contribution of (4), only. The result is  $(m_{gl}^3)^2 = g^2T^2$ , so that the factor  $C = 1$ . This value coincides with the one calculated from the one-loop polarization operator of neutral gluons.

## 6. Discussion

In this paper, we have considered the quark-gluon plasma at  $A_0$  background. We used the charged basis, which admits to consider the  $A_0$  condensate as the constant shifts of the zero momentum component. Since  $A_0^8 = 0$  in adopted approximation, we calculated the induced color charge for the case  $(A_0)_\mu^a = (A_0)_\mu \delta^{a3}$ . The gauge-fixing independent expression for  $A_0$  condensate was obtained and used in followed computations.

Then we calculated the proper diagram Fig. 2 in the presence of the  $A_0$  condensate and the induced charge – the tadpole diagram. This diagram brings new corrections to quark self

energy. It can be included in consideration by shifting the initial  $A_0$  potential, as it was shown above. This effect increases the strength of the condensate and should be accounted for when different problems in quark-gluon plasma are studied.

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