DISPERSION EQUATION FOR ELECTROMAGNETIC WAVES IN A DICKE SYSTEM

S.F. Lyagushyn*, A.I. Sokolovsky

Oles Honchar Dnipro National University, Dnipro, Ukraine
e-mail: 1yagush.new@gmail.com

The self-organization process in a Dicke model is studied with taking into account electromagnetic field states. The interaction between two-level emitters provided by the field results in generating field waves of increasing amplitudes. The picture of superradiance development in a prolonged system can be described in terms of electromagnetic field in continuous medium formed by emitters. The generalized induction in the Dicke system is introduced on the basis of the material equation for charge current obtained using the Bogolyubov reduced description method. The dispersion equation for waves and its solutions are derived and analyzed. Non-singular expressions for material coefficients are proposed and used for calculating the wave characteristics.

Keywords: Dicke model, reduced description, polarization current, material equations, Lorentz broadening, dispersion law, complex frequency.

Received 10.09.2022; Received in revised form 14.10.2022; Accepted 15.11.2022

1. Introduction

The inexhaustible interest in the Dicke model is due to its place in the development of the methods of non-equilibrium statistical physics and the prospects of practical use. The possibility of obtaining coherent generation in exotic wavelength ranges and applying it for creating non-classic field states is very attractive. In this connection new aspects of the superfluorescence analysis such as process peculiarities in a prolonged system [1], resonator influence [2], and correlation parameters [3] cause attention of researchers. The similarity of processes in quantum computers [4] and Dicke two-level systems is a powerful stimulating factor for superradiance research. It should be noted that qubit behavior is described with the Bloch sphere like the quasispin evolution of the Dicke model [5]. Our investigations of the Dicke phenomenon based on the Bogolyubov reduced description method (RDM) allows not only to calculate the pulse delay time, but also to analyze field states. We use the ideas and notions of the electrodynamics of continuous media (EDCM), such as electric polarization and material equations. In our previous works [6-8] the set of differential equations for the macrostate evolution of a system of two-level emitters interacting via field was constructed. The equations for field variables including binary correlation functions make this set rather difficult since the spatial dispersion plays the key role in the medium under discussion. Thus, we come to the expedience of studying the wave processes in the medium of two-level emitters. This program was begun in our paper of 2013 [9]. Now a more consistent way of applying EDCM to the Dicke system is proposed with using the found expressions for material coefficients, wave propagation theory, and dispersion relations.

The most general formulation of EDCM (the term “macroscopic electrodynamics” is also used), which is applicable for fields changing quickly in space and time, is based on introducing the generalized induction [10]. This vector physical value $\boldsymbol{\mathcal{D}}$ containing information both about electric and magnetic polarization of the medium is defined by the relation

$$\boldsymbol{\mathcal{D}} = \mathbf{E} + 4\pi \int_{-\infty}^{t} \mathbf{I}(t')dt' \qquad (1)$$

where $\boldsymbol{E}$ denotes average microscopic electric field strength, and $\mathbf{I}$ is an average electric current density formed by both free and bounded charge carriers of the matter (external charges must be taken into account separately). Hereafter vectors are in bold. It supposed that at $t \to -\infty$ matter and field are in equilibrium and external electromagnetic field was absent [11]. (1) leads to the following expression including the current density of matter charge carriers at a time moment *t*:

$$\frac{\partial \mathcal{D}}{\partial t} = \frac{\partial \mathbf{E}}{\partial t} + 4\pi \mathbf{I} \tag{2}$$

In terms of $\mathcal{D}$, Maxwell equations in a medium without external charges take the form:

$$\begin{aligned} &\nabla \cdot \mathcal{D} = 0, \quad \nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathcal{D}}{\partial t} \\ &\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}. \end{aligned} \tag{3}$$

Next, the dielectric permittivity tensor connecting $\mathcal{D}$ and **E** of the medium is traditionally introduced and harmonic plane waves are considered, i.e., all fields conceivably depend on time and coordinates through the factor $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$ [10]. Thus, the set of algebraic equations for field Fourier components providing the condition for wave mode existence in the medium is obtained, it is called the dispersion equation. This convenient way of constructing the wave theory in media can be modified by using the connection between $\mathcal{D}$ and **E** based on Eq. 2 including current density.

In [7], just the required expression for current density has been derived in the RDM framework. The material equation for quasiequilibrium medium of the Dicke model is

$$\mathbf{I}(\mathbf{x}) = \int d\mathbf{x}'[\sigma(\mathbf{x}-\mathbf{x}', \varepsilon(\mathbf{x}))\mathbf{E}(\mathbf{x}') + c\xi(\mathbf{x}-\mathbf{x}', \varepsilon(\mathbf{x}))\mathbf{Z}(\mathbf{x}')], \quad (\mathbf{Z} \equiv \nabla \times \mathbf{B}). \tag{4}$$

The view of (4) evidences the spatial dispersion in the considered medium. The main conditions for obtaining such expression are using the Gibbs distribution for averaging, that is a local temperature existing, and substance properties depending on the local energy density. Integration is performed over the whole system volume, the influence of boarders is negligible, the expression for **I** is valid in the 2nd order of the perturbation theory in terms of a small parameter *d* (atom dipole moment). For such reasons $\mathbf{E}^t$ (transversal field) is replaced by $\mathbf{E}$ (complete field) in the equation. The spatial form of the material equation is necessary for our attempts of the numerical investigation of the superradiance process with considering field fluctuations [12]. But the problem of wave propagation requires the primary form of this equation in terms of Fourier components

$$\mathbf{I}(\mathbf{x}) = \frac{1}{V}\sum_{\mathbf{k}} e^{i\mathbf{k}\mathbf{x}}[\sigma(\mathbf{k}, \varepsilon(\mathbf{x}))\mathbf{E}(\mathbf{k}) + c\xi(\mathbf{k}, \varepsilon(\mathbf{x}))\mathbf{Z}(\mathbf{k})]. \tag{5}$$

We denote the Fourier transforms by the same letters but indicate the argument. The Fourier images of material coefficients in the assumption of the random orientation of emitting atoms were calculated in [7] in the assumption of the same frequency ω of all emitters:

$$\sigma(\mathbf{k}, \varepsilon) = -s\,\delta(\omega - \omega_k), \quad \xi(\mathbf{k}, \varepsilon) = -\frac{2s}{\pi}\mathrm{P}\frac{1}{\omega^2 - \omega_k^2}, \quad \omega_k = ck, \quad s \equiv \frac{2\pi d^2 \varepsilon}{3\hbar^2}. \tag{6}$$

The fact that the coefficients are proportional to the energy density of the emitter subsystem allows considering the relation

$$\mathbf{I}(\mathbf{k}) = \sigma(\mathbf{k}, \varepsilon)\mathbf{E}(\mathbf{k}) + c\xi(\mathbf{k}, \varepsilon)\mathbf{Z}(\mathbf{k}) \tag{7}$$

as current density Fourier image in the case of a uniform medium. In another case a more complicated connection must be applied:

$$\mathbf{I}(\mathbf{k}) = \frac{1}{V}\sum_{\mathbf{k}'}\left[\sigma(\mathbf{k},\varepsilon_{\mathbf{k}\text{-}\mathbf{k}'})\mathbf{E}(\mathbf{k}') + c\xi(\mathbf{k}',\varepsilon_{\mathbf{k}\text{-}\mathbf{k}'})\mathbf{Z}(\mathbf{k}')\right] \tag{8}$$

where $\varepsilon_{\mathbf{k}\text{-}\mathbf{k}'}$ is the Fourier transform of $\varepsilon(\mathbf{x})$. Our consideration will be restricted with a homogeneous medium, which is an adequate model for the early stages of Dicke system evolution at an appropriate way of initial state preparation.

3. Time dispersion in material coefficients. Problem of singularity

Pay attention that Eq. 4 does not show an explicit form of the time dispersion. The RDM provides the evolution equations with simultaneous values of physical quantities, and this is one of its advantages. Nevertheless, the influence of the previous time moments is taken into account in (4) due to the way of obtaining the coefficients $\sigma(\mathbf{k},\varepsilon)$ and $\xi(\mathbf{k},\varepsilon)$. Indeed, expressions for calculating physical values at an arbitrary time (usually $t=0$ is taken) contain integrals over time from $-\infty$ to 0 [7]. The integrals exist after the thermodynamic limit in the class of generalized functions, therefore singular formulas (6) are obtained. In some sense, the coefficient $\xi(\mathbf{k},\varepsilon)$ describes the time dispersion because of the relation $\mathbf{Z} = c^{-1}\partial_t\mathbf{E}^t$ confirming the influence of the time derivative of electrical field strength.

The simple form of (6) is complicated by the singular functions. Obviously, due to the presence of factors d^2 in expressions (6) we could speak about weak medium response to both electric and magnetic external influence. But response singularity at $\omega_k \to \omega$ shows the great particularity of Dicke model and expedience of special measures for obtaining physical results since emitter-field interaction does not suppose the mathematical equality of external signal and ω.The known way of overcoming the problem is non-uniform Lorentz broadening accounting [5]. A regular description of this technique is given in [8].

The emitter frequencies are supposed to be distributed near the fixed value ω_0 (that is a table value for the operation transition of Dicke emitters) with probability density $w_\alpha(\omega)$. Calculations are performed for an arbitrary ω, then the result is averaged with the weight function $w_\alpha(\omega)$. For many problems, it is sufficient to consider the limit at $\alpha \to 0$ and use the relation $\lim_{\alpha\to 0} w_\alpha(\omega-\omega_0) = \delta(\omega-\omega_0)$. Presumably, integrals with singular functions can be calculated, maybe, in Cauchy sense. But dealing with the wave nature, one must work with the function $w_\alpha(\omega)$ itself. Really α is a value determined by the atom-field interaction character. In our model the non-uniform broadening is absent, but the natural emitter line width plays the same role. The parameter α can be calculated proceeding from the operation level lifetime. So, the material coefficients (6) should be replaced by expressions

$$\sigma(\mathbf{k},\varepsilon) = -s\int_0^{+\infty} d\omega\, w_\alpha(\omega)\delta(\omega-\omega_k)\,, \qquad \xi(\mathbf{k},\varepsilon) = -\frac{2s}{\pi}\int_0^{+\infty} d\omega\, w_\alpha(\omega)\mathrm{P}\frac{1}{\omega^2-\omega_k^2} \tag{9}$$

where in accordance with [5]

$$w_\alpha(\omega) = c_\alpha\frac{\alpha}{(\omega-\omega_0)^2+\alpha^2}\,, \qquad \int_0^{+\infty} d\omega\, w_\alpha(\omega) \equiv 1\,. \tag{10}$$

Note, that $c_\alpha^{-1} = \pi/2 + \mathrm{arctg}(\omega_0/\alpha) \approx \pi$ for small α.

It can be proven that $\mathrm{P}\int_0^{+\infty} \frac{d\omega}{\omega^2 - \omega_k^2} = 0$ and therefore the second integral in (9) exists in the usual sense

$$\int_0^{+\infty} d\omega\, w_\alpha(\omega) \mathrm{P} \frac{1}{\omega^2 - \omega_k^2} = \mathrm{P}\int_0^{+\infty} d\omega \frac{w_\alpha(\omega)}{\omega^2 - \omega_k^2} = \int_0^{+\infty} d\omega \frac{w_\alpha(\omega) - w_\alpha(\omega_k)}{\omega^2 - \omega_k^2} \equiv S_\alpha(\omega_k).$$

So, further the material coefficients should be used in the form

$$\sigma(\mathbf{k},\varepsilon) = -s\, w_\alpha(\omega_k), \qquad \xi(\mathbf{k},\varepsilon) = -\frac{2s}{\pi} S_\alpha(\omega_k) \tag{11}$$

and from physical point of view for $\omega_k = ck \approx \omega_0$.

4. Dispersion relation and its solutions

Since the second of equations (3) includes the time derivative of $\boldsymbol{\mathcal{D}}$, (2) provides the necessary transition to vectors **E** and **I**. Assuming the dependence of the type $\mathbf{E}(\mathbf{x},t) = \mathbf{E}e^{-i(\omega t - \mathbf{kx})}$ for normal waves [10] leads the set of equations for Fourier transforms

$$\begin{gathered} \mathbf{k}\cdot\boldsymbol{\mathcal{D}} = 0, \qquad \mathbf{k}\times\mathbf{B} = -\frac{\omega}{c}\boldsymbol{\mathcal{D}}, \\ \mathbf{k}\cdot\mathbf{B} = 0, \qquad \mathbf{k}\times\mathbf{E} = \frac{\omega}{c}\mathbf{B}. \end{gathered} \tag{12}$$

Pay attention that in further we must differ the cyclic frequency ω of the mode we are looking for and the value ω_k connected with the wave vector **k** used in (11) and (12). If we put in the relation (7) $\mathbf{E}(\mathbf{k}) = \mathbf{E}e^{-i(\omega t - \mathbf{kx})}$ and analogous for **Z**, both the necessary form of coordinate-time dependence for field variables and the right view of $\mathbf{I}(\mathbf{x},t)$ will be secured. Thus, the condition $\varepsilon(\boldsymbol{x}) = \varepsilon = \mathrm{const}$ should be accepted. Obviously, dealing with non-equilibrium processes contradicts the assumption of spatial and time homogeneity, but in Dicke systems during the process of correlation development the emitter subsystem state (in terms of energy density) remains nearly unchanged for a long time. This circumstance greatly simplifies the numerical simulation of the early stages of superradiance process [12]. So, such assumption can be used in the study of wave processes in a Dicke model at the superradiant process beginning.

Substituting the described current representation into the Fourier transformed formula (2) gives the relation

$$\omega\boldsymbol{\mathcal{D}} = \left(\omega + 4\pi i\sigma(\mathbf{k},\varepsilon)\right)\mathbf{E} - 4\pi c\xi(\mathbf{k},\varepsilon)\left(\mathbf{k}\times\mathbf{B}\right) \tag{13}$$

where material coefficients are taken in the form (11). Next, as usually, we reduce the set (12) to the equation for electrical field $\boldsymbol{E}$. According to (12), the relation

$$\boldsymbol{\mathcal{D}} = -\frac{\omega}{c}\mathbf{k}\times\mathbf{B}$$

is true and so formula (13) leads to the connection

$$\mathbf{k}\times\mathbf{B} = -\frac{\omega}{c}u(\mathbf{k},\omega)\mathbf{E} \tag{14}$$

Where

$$u(\mathbf{k},\omega) \equiv \frac{1+4\pi i\sigma(\mathbf{k},\varepsilon)\omega^{-1}}{1-4\pi\xi(\mathbf{k},\varepsilon)}. \tag{15}$$

The last equation of (12) gives

$$\mathbf{k}\times\mathbf{B} = \frac{c}{\omega}\mathbf{k}\times(\mathbf{k}\times\mathbf{E}) = \frac{c}{\omega}[\mathbf{k}(\mathbf{k}\cdot\mathbf{E}) - \mathbf{E}k^2].$$

Obviously, the final view of the equation for $\boldsymbol{E}$ is

$$\mathbf{k}(\mathbf{k}\cdot\mathbf{E}) - k^2\mathbf{E} + \frac{\omega^2}{c^2}u(\mathbf{k},\varepsilon)\mathbf{E} = 0. \tag{16}$$

For the cartesian components of $\boldsymbol{E}$, this equation leads to the set of homogeneous linear equations

$$\left[\left(k^2 - \frac{\omega^2}{c^2}u(\mathbf{k},\omega)\right)\delta_{nl} - k_n k_l\right]E_l = 0 \tag{17}$$

(here $n, l = 1, 2, 3$ and Einstein's rule is used). The necessary condition of the own electromagnetic wave existence is zero value of its determinant:

$$\left|\left(k^2 - \frac{\omega^2}{c^2}u(\mathbf{k},\omega)\right)\delta_{nl} - k_n k_l\right| = 0. \tag{18}$$

The obtained relation (18) is a dispersion equation for electromagnetic waves in Dicke system that allows establishing the connection between wave vector and frequency, i.e., the dispersion law $\omega(\mathbf{k})$. In our isotropic case $u(\mathbf{k},\omega) = u(k,\omega)$ and the vector $\mathbf{E}$ structure is defined only by the vector $\boldsymbol{k}$ and it has the longitudinal $\mathbf{E}^l$ and transverse $\mathbf{E}^t$ components $\mathbf{E} = \mathbf{E}^l + \mathbf{E}^t$. Substituting $\mathbf{E} = \mathbf{E}^l + \mathbf{E}^t$ in (17) gives

$$-\frac{\omega^2}{c^2}u(k,\omega)\mathbf{E}^l + \left(k^2 - \frac{\omega^2}{c^2}u(k,\omega)\right)\mathbf{E}^t = 0 \tag{19}$$

and therefore

$$-\frac{\omega^2}{c^2}u(k,\omega)\mathbf{E}^l = 0, \qquad \left(k^2 - \frac{\omega^2}{c^2}u(k,\omega)\right)\mathbf{E}^t = 0 \tag{20}$$

because vectors $\mathbf{E}^l$, $\mathbf{E}^t$ are linear independent. For longitudinal field $\mathbf{E}^l \neq 0$, hence the relation

$$u(k,\omega) = 0 \tag{21}$$

must be satisfied. For transverse field $\boldsymbol{E}^t \neq 0$ and the equality

$$k^2 - \frac{\omega^2}{c^2}u(k,\omega) = 0 \tag{22}$$

is valid. The last two equations define dispersion laws $\omega(k) \equiv \omega' + i\omega''$ if the corresponding waves are possible.

In view of small parameter squared d^2 presence in material coefficients (11), $\sigma(k,\varepsilon)$, $\xi(k,\varepsilon)$ are small, then $u(k,\omega) \approx 1$ and the medium of emitters almost does not differ from vacuum. At the same time in the k range of interest, where $k \approx \omega_0 / c$, material coefficients take large values due to the $w_\alpha(\omega_k) \equiv w_\alpha(ck)$ behavior (see (11), (28)).

For longitudinal field, according to (15), (21),

$$\omega' = 0, \qquad \omega'' = -4\pi\sigma(k,\varepsilon) = 4\pi s\, w_\alpha(ck), \tag{23}$$

i.e., for all k the imaginary part $\omega'' > 0$. Thus, longitudinal waves propagation is impossible, but the positive sign of ω'' shows that field energy grows rapidly when ck is close to ω_0. The considered material equations were derived with perturbation theory; hence they are valid for small fields. Nevertheless, our analysis is applicable at least at the beginning of the Dicke process when state changes touch on field correlations at the nearly stable energy of the emitter subsystem.

For transverse waves, (22) results in the relations (see (11) and (15))

$$\omega^2 = c^2k^2 \frac{1+8sS_\alpha(ck)}{1-4\pi i s w_\alpha(ck)\omega^{-1}} = c^2k^2 \frac{\omega\left(1+8sS_\alpha(ck)\right)}{\omega - 4\pi i s w_\alpha(ck)},$$

$$\omega\left[\omega - 4\pi i s w_\alpha(ck)\right] = c^2k^2\left[1+8sS_\alpha(ck)\right],$$

$$\omega^2 - 4\pi i s w_\alpha(ck)\omega - 8sc^2k^2S_\alpha(ck) - c^2k^2 = 0,$$

$$\omega'^2 + 2i\omega'\omega'' - \omega''^2 - 4\pi i s w_\alpha(ck)\omega' + 4\pi s w_\alpha(ck)\omega'' - 8sc^2k^2S_\alpha(ck) - c^2k^2 = 0$$

The set of two equations for ω' and ω'' is obtained:

$$\omega'^2 - \omega''^2 + 4\pi s w_\alpha(ck)\omega'' - 8sc^2k^2S_\alpha(ck) - c^2k^2 = 0,$$

$$\omega'\omega'' - 2\pi s w_\alpha(ck)\omega' = 0.$$

From the 2nd equation at once

$$\omega'' = 2\pi s w_\alpha(ck). \tag{24}$$

Inequality $\omega'' > 0$ evidences energy growth for transverse modes, especially fast for the indicated area of interest. Next, from the 1st equation

$$\omega'^2 = c^2k^2\left(1+8sS_\alpha(ck)\right) + 4\pi^2 s^2 w_\alpha^2(ck). \tag{25}$$

It is necessary to have an estimate for $S_\alpha(ck)$. The explicit view of the weight function (10) should be used. Thus,

$$S_\alpha(ck) = \int_0^{+\infty} d\tilde{\omega}\, \frac{c_\alpha \alpha}{\tilde{\omega}^2 - (ck)^2}\left[\frac{1}{(\tilde{\omega}-\omega_0)^2+\alpha^2} - \frac{1}{(ck-\omega_0)^2+\alpha^2}\right] =$$
$$= \frac{w_\alpha(ck)}{c_\alpha \alpha}\int_0^{+\infty} d\tilde{\omega}\, w_\alpha(\tilde{\omega}) \frac{\left[ck+\tilde{\omega}-2\omega_0\right]}{-(ck+\tilde{\omega})} = \frac{w_\alpha(ck)}{c_\alpha \alpha}\int_0^{+\infty} d\tilde{\omega}\, w_\alpha(\tilde{\omega})\left[\frac{2\omega_0}{ck+\tilde{\omega}} - 1\right]. \tag{26}$$

The last integral can be calculated exactly using formulas for rational function integration. The final expression is based on the formula 2.18 (4) from [13]:

$$\int_0^{+\infty} d\tilde{\omega} w_\alpha(\tilde{\omega})\left[\frac{2\omega_0}{(ck+\tilde{\omega})}-1\right]=\int_0^{+\infty} d\tilde{\omega}\frac{c_\alpha\alpha}{(\tilde{\omega}-\omega_0)^2+\alpha^2}\cdot\frac{2\omega_0}{ck+\tilde{\omega}}-1=2\omega_0 c_\alpha\alpha\int_0^{+\infty}\frac{d\tilde{\omega}}{\left[(\tilde{\omega}-\omega_0)^2+\alpha^2\right](ck+\tilde{\omega})}-1=$$

$$=2\omega_0 c_\alpha\alpha\frac{1}{2\left[(\omega_0+ck)^2+\alpha^2\right]}\ln\frac{(ck+\tilde{\omega})^2}{(\omega_0-\tilde{\omega})^2+\alpha^2}\Bigg|_0^{\infty}-2\omega_0 c_\alpha\alpha\frac{(-2)(\omega_0+ck)}{2\left[(\omega_0+ck)^2+\alpha^2\right]}\times \quad (27)$$

$$\times\int_0^{+\infty}\frac{d\tilde{\omega}}{(\tilde{\omega}-\omega_0)^2+\alpha^2}-1=\frac{\omega_0 c_\alpha\alpha}{\left[(\omega_0+ck)^2+\alpha^2\right]}\ln\left(\frac{\omega_0^2+\alpha^2}{c^2k^2}\right)+\frac{2\omega_0(\omega_0+ck)}{\left[(\omega_0+ck)^2+\alpha^2\right]}-1,$$

$$S_\alpha(ck)=w_\alpha(ck)\left\{\frac{\omega_0}{\left[(\omega_0+ck)^2+\alpha^2\right]}\left[\ln\left(\frac{\omega_0^2+\alpha^2}{c^2k^2}\right)+\frac{2(\omega_0+ck)}{c_\alpha\alpha}\right]-\frac{1}{c_\alpha\alpha}\right\}. \quad (28)$$

Thus, (25) gives the exact expression for wave frequency

$$\omega'=ck\sqrt{1+8sw_\alpha(ck)\left\{\frac{\omega_0}{c_\alpha\alpha}w_\alpha(-ck)\left[\ln\left(\frac{\omega_0^2+\alpha^2}{c^2k^2}\right)+\frac{2(\omega_0+ck)}{c_\alpha\alpha}\right]-\frac{1}{c_\alpha\alpha}+\frac{\pi^2 sw_\alpha(ck)}{2(ck)^2}\right\}}\approx$$

$$\approx ck\sqrt{1+8sw_\alpha(ck)\left\{\frac{2\omega_0}{(\omega_0+ck)^2}\ln\frac{\omega_0}{ck}+\frac{\pi(\omega_0-ck)}{\alpha(\omega_0+ck)}+\frac{\pi^2 sw_\alpha(ck)}{2(ck)^2}\right\}}\quad \text{at } \alpha\ll\omega_0 \quad (29)$$

For obtaining the group velocity of the considered waves the derivative $\frac{d\omega'}{dk}$ must be calculated. The derivative of the accepted weight function is

$$\frac{dw_\alpha(ck)}{dk}=c_\alpha\alpha\frac{d}{dk}\left[\frac{1}{(ck-\omega_0)^2+\alpha^2}\right]=-2c_\alpha\alpha c\frac{ck-\omega_0}{\left[(ck-\omega_0)^2+\alpha^2\right]^2}. \quad (30)$$

Basing on (30), the necessary derivative has been obtained. It proves to be too cumbersome. In the range of resonant waves when $ck\approx\omega_0$, the relations between different small parameters of the theory α, s see (6)) and $g=ck-\omega_0$ should be considered.

The medium influence on wave propagation is maximal near the maximum point of the weight function. For the group velocity $\frac{d\omega'}{dk}$ at $k=\frac{\omega_0}{c}$ calculations give the value $c\left(1-\frac{2s}{\alpha^2}\right)\left(1+\frac{4s^2}{\omega_0^2\alpha^2}\right)^{-1/2}$. The result obtained in the approximation $\alpha\ll\omega_0$ seems to be physically substantiated because the velocity decrease is proportional to the small parameter s and becomes large when a narrow distribution is accepted. The dimensionless quantity $\beta=\frac{s}{\omega_0\alpha}$ appears as a small parameter in a more detailed study that will be discussed in a subsequent paper. For ω'' the result is obvious from (23), (24): in any case $\omega''>0$, wave enhancement in the active medium takes place and this effect is proportional to $w_\alpha(ck)$ and *s*.

5. Conclusions

In the paper notions of the electrodynamics of continuous medium are applied to the Dicke system of two-level emitters interacting via field. The generalized induction is constructed using the material equation for current density. Maxwell equations for Fourier components are obtained basing on such material equation. They lead to the dispersion relations for transverse and longitudinal fields. The wave propagation problem requires considering the non-singular material coefficients for taking into account non-resonant interaction. Longitudinal waves are impossible in a superradiant system. For transverse waves, the dispersion law is established in the form of expressions for the real and imaginary parts of complex frequency corresponding to a certain wave vector. The medium influence is essential for modes, which are close to resonance with emitters. The increment value is proportional to the interaction intensity and emitter frequency distribution function. The group velocity is analyzed in brief.

References

1. **Liu, Qihang**. Photonic Fock state generation using superradiance / Qihang Liu, Jung-Tsung Shen // Optics Letters. – 2022. – Vol. 47, Issue 17. – P. 4576 – 4579.

2. **Liang, Zhengchen.** Coupling-Induced Tunable Broadband Superradiance in 2D Metal-Dielectric-Metal Nanocavity Arrays / Zhengchen Liang, Dong Wang, M. Ziegler, U. Hübner, Peng Xie, Qi Ding, Hong Zhang, Wei Wang // Laser & Photonics Reviews. – 2022, Early View 2200393 (17 September 2022). – doi.org/10.1002/lpor.202200393

3. **Lewalle, P.** Entanglement of a pair of quantum emitters via continuous fluorescence measurements: a tutorial / P. Lewalle, C. Elouard, S. K. Manikandan, Xiao- Feng Qian, J. H. Eberly, A. N. Jordan // Advances in Optics and Photonics. – 2021. – Vol. 13, Issue 3. – P. 517 – 583.

4. **Sebastiano, F.** Cryo-CMOS Electronic Control for Scalable Quantum Computing / F. Sebastiano, H. Homulle, B. Patra, R. Incandela, J. van Dijk, Lin Song, M. Babaie, A. Vladimirescu, E. Charbon // Proceedings of the 54th Annual Design Automation Conference (June 2017). – Article No. 13. – P. 1 – 6.

5. **Allen L.** Optical Resonance and Two-level Atoms / L. Allen, J.H. Eberly. North Chelmsdorf (Mass.) : Courier Corporation, 1987. – 233 p.

6. **Lyagushyn, S.F.** Kinetics of system of emitters and nonequilibrium electromagnetic field / S.F. Lyagushyn, A.I. Sokolovsky // Physics of Particles and Nuclei. – 2010. – Vol. 41, No. 7. – P. 1035 – 1038.

7. **Lyagushyn, S.F.** Material equations in electrodynamics of medium consisting of two-level emitters / S.F. Lyagushyn, A.I. Sokolovsky, S.A. Sokolovsky // Journal of Physics and Electronics. – 2019. – Vol. 27(1). – P. 9 – 18.

8. **Lyagushyn, S.F.** Kinetics of electromagnetic field in nonequilibrium medium / S.F. Lyagushyn, A.I. Sokolovsky, S.A. Sokolovsky // Journal of Physics and Electronics. – 2019. – Vol. 27(2). – P. 17 – 28.

9. **Lyagushyn, S.F.** Dynamics of electromagnetic field and its correlations in a medium consisting of two-level emitters / S.F. Lyagushyn, Yu.M. Salyuk, A.I. Sokolovsky // Visnyk Dnipropetrovskoho universytetu. Seriya «Fizyka. Radioelektronika». – 2013. – Vol. 21 (Issue 20). – P. 47 – 55.

10. **Skalozub, V.V.** Klasychna makroskopichna elektrodynamika / V. V. Skalozub, O. V. Gulov. – Dnipropetrovsk : DNU Publishing House, 2010. – 172 p.

11. **Akhiezer, A.I.** Methods of Statistical Physics / A.I. Akhiezer, S.V. Peletminsky. – Oxford : Pergamon Press. – 1981.

12. **Lyagushyn, S.** To the numerical simulation of the correlation picture in Dicke superradiance / S. Lyagushyn // Low Temperature Physics/Fizika Nizkikh Temperatur. – 2022. – Vol. 48, No. 2. – P. 114 – 118.

13. **Gradshtein, I.S.** Tabitsy integralov, summ, ryadov i proizvedeniy / I.S. Gradshtein, I.M. Ryzhik. – Moscow : Gos. izd-vo fiz.-mat. literatury, 1963. – 1108 p.