ORDER PARAMETERS IN DICKE MODEL

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Dicke model provides describing the superradiance effect when the spontaneous emission of a great number of excited two-level atoms is running in a correlated way due to their interaction via field. The process is identical to a giant dipole emission. Since the ordering takes place, it is interesting to study the possibility for using the terms of phase transition theory in such physical situation. Equilibrium properties of Dicke type models were studied for a long time because of their relevance for explaining some phenomena in matter-field interaction. The existence of an ordered phase in the emitter subsystem together with macroscopic photon mode filling is established, wherein the correct solution requires the Bogolyubov concept of quasiaverages to exclude the phase uncertainty of quasispin component structures $\langle R^+ \rangle$ and $\langle R^- \rangle$ (for the concentrated model). In this paper, the behavior of similar order parameters for the non-equilibrium process description is analyzed and using the order parameter $\langle R^+ R^- \rangle$ associated with the squared electric dipole moment of the emitter complex is substantiated. Such fact justifies $\langle R_3 \rangle$ application in the literature devoted to the superradiance theory. The situation with quasiaverages is compared with the problem of uniformity breaking in the numerical modeling.

Keywords: ordering, phase transition, Jaynes–Cummings model, quasispin, Dicke superradiance, quasiaverages, macroscopic dipole moment.

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1. Introduction

In the fundamental Dicke`s paper [1] the superradiance effect was predicted through considering the selection rule for the interaction operator of a concentrated (small compared with a wavelength) system of two-level emitters. Such operator included collective operators of quasispin nature, its matrix elements squared determined the dipole radiation intensity. Some years later, laser invention encouraged attempts to use Dicke ideas in laser generation theory. Though the last required another approach because of the stimulated emission role [2], such attempts support the interest to superradiance problem. Moreover, just the laser technology allowed the first experimental observation of this phenomenon [3]. At the same time, the Dicke model was studied from the point of view of equilibrium statistical theory and showed the possibility to move into an ordered state at low enough temperatures. This "superradiant" phase transition proved to be very topical both for the problems of light and matter interaction in a cavity and for the general theory of systems including interaction with boson fields. The analysis of ordering in Dicke system showed the necessity of using quasiaverages for the removal of degeneracy of physical parameter values with respect to phases similarly to the Bogolyubov theory of superfluidity. For the purpose of obtaining physical results, we propose to use the phase invariant combination of parameters leading us to usually applied physical quantity (averaged 3rd component of quasispin). In numerical modeling of a prolonged system, we met with analogous difficulties that were overcome in a natural way. Thus, the paper connects the results concerning the phase transition in Dicke model and considering non-equilibrium processes in it.

The paper is structured in the following way. Section 2 discusses the quantum effect observed in a system with Jaynes– Cummings Hamiltonian and the free energy calculation for Dicke system. Section 3 explains introducing quasiaverages in Dicke model and the nature of equilibrium state degeneracy. Section 4 presents the main result concerning the expedience of using a parameter taking a zero value at the maximum ordering in the system and some aspects of comparing the superradiance effect with phase transitions.

2. Jaynes–Cummings model and superradiant phase transition

Dicke process does not require an optical cavity and takes place in an open system. A real experimental situation supposes the presence of a cavity always. In superradiance investigations the cavity influence is considered as an additional factor. But in quantum optics a prominent place belongs to the Jaynes–Cummings model [4]:

$$\hat{H}_{\mathrm{JC}} = \hat{H}_f + \hat{H}_m + \hat{H}_{\mathrm{int}} \tag{1}$$

where Hamiltonian components correspond to a single field mode $\hat{H}_f = \hbar\omega a^+ a$, a two-level atom $\hat{H}_m = \hbar\omega_0 r_3$, and their interaction $\hat{H}_{\mathrm{int}} = g r_1 \left(a^+ + a\right)$. Note, that r_1, r_2, r_3 are quasispin operators expressed via Pauli matrixes $r_i = \sigma_i / 2$, a, a^+ are boson operators. The combinations of quasispin operators $r^{\pm} = r_1 \pm i r_2$ are raising and lowering operators of the atom: $r^+ = |+\rangle\langle-|$, $r^- = |-\rangle\langle+|$. One can write down the interaction constant as

$$g = \sqrt{\frac{2\pi\hbar}{\omega V}} (\mathbf{e}\cdot\mathbf{d})\omega_0 \tag{2}$$

where **e** stands for the mode polarization vector, and **d** defines the non-diagonal matrix element of the atom dipole moment: $\langle\pm|\hat{\mathbf{d}}|\mp\rangle = \pm i\mathbf{d}$. The field is quantized in the cavity volume and the fixed mode is considered. The view of $\hat{H}_{\mathrm{int}}$ may be reduced to the form $-\hat{\mathbf{E}}\cdot\hat{\mathbf{d}}$ where $\hat{\mathbf{E}}$ is the electric field operator in the point of atom localization [5, 6]. The two-level atom interacts with a single mode of photon field, and they form an isolated system. Its behavior studied on the consistently quantum basis proved the fruitfulness of the attention to the quantum description of electromagnetic field. Such approach allowed the substantiation of collapse and revival of probabilities to find the atom in the excited state due to the discrete structure of energy levels of the atom. The indicated phenomenon was observed 20 years later [7], but it also stimulated the interest to properties of a complex of two-level atoms interacting with photon field. The Jaynes–Cummings problem has a quantum-mechanical character, not statistic. The ordering in it consists in return to the pure quantum state.

The next question is the possibility of arrangement in a system of many atoms emitting photons. It is just a Dicke system. Based on the above, we consider it in the quantum terms and use the quasispin operators for describing emitter states. Dicke Hamiltonian can be written down in the same form as (1):

$$\hat{H}_{\mathrm{D}} = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} a^+_{\mathbf{k}} a_{\mathbf{k}} + \sum_{n} \hbar\omega_0 r_{3n} + \sum_{\mathbf{k},n} g_{\mathbf{k},n} \left(r^+_n a_{\mathbf{k}} + r^-_n a^+_{\mathbf{k}}\right) \tag{3}$$

For simplicity, we restrict ourselves with Dicke Hamiltonian in rotating wave approximation [2]. Its thermodynamic properties were studied intensively since the famous paper by Hepp and Lieb [8]. It turned out that in the one-mode case ($\omega_{\mathbf{k}} = \omega_0$) this system undergoes a phase transition to the state with spontaneous polarization of the emitter subsystem $\left|\left\langle R^{\pm}\right\rangle\right| \neq 0$ at temperatures less than θ_c to be found from the equation

$$\theta_c = \frac{\hbar\omega_0}{2}\left\{\mathrm{Arth}\left(\frac{\hbar\omega_0}{\bar{g}}\right)^2\right\}^{-1}. \tag{4}$$

Here the notation for collective quasispin operators $R^{\pm}=\sum_n r_n^{\pm}$ is used, temperatures are measured in power units $\theta = kT$, k is Boltzmann constant. The phase transition is possible if strong coupling condition is observed:

$$\bar{g} \geq \hbar\omega_0 \tag{5}$$

where $\bar{g}=g\sqrt{V}$ is a dipole-photon coupling constant of zero thermodynamic order (g is given by expression (2)). The order parameter $\xi=\left|\frac{1}{N}\left\langle R^{\pm}\right\rangle\right|$ is determined from the equation

$$\sqrt{(\hbar\omega_0)^4+4\bar{g}^4\xi^2}=\bar{g}^2 \mathrm{th}\left\{\sqrt{(\hbar\omega_0)^4+4\bar{g}^4\xi^2}\Big/2\hbar\omega_0\theta\right\}. \tag{6}$$

Note that even quantum-mechanical averages of $R^{\pm}$ expressed through $r_n^{\pm}$ possessing zero diagonal matrix elements should be zero. Thus, in Dicke model we deal with spontaneous symmetry breaking.

3. Quasiaverages in Dicke model

Dicke model is attractive for equilibrium statistical study. Considering the system of two-level emitters in boson thermostat at a fixed temperature θ, we can easily calculate the statistical sum Z. Then the free energy can be obtained as $F=-\theta \ln Z$ and all thermodynamic properties become known. The rigorous investigation requires separating boson and quasispin variables and thermodynamic limit procedure. Such approach was implemented on the basis of approximating Hamiltonian method developed by Bogolyubov (Jr.). Initially, in the one-mode model, the shift transformation, which is canonical in thermodynamic limit, is applied to photon operators:

$$\alpha^{+}=a^{+}+\frac{\bar{g}}{\hbar\omega_0\sqrt{N}}R^{+}, \qquad \alpha=a+\frac{\bar{g}}{\hbar\omega_0\sqrt{N}}R^{-} \tag{7}$$

and the Hamiltonian comes down to the form

$$\hat{H}_{\mathrm{D}}=\hbar\omega_0\alpha^{+}\alpha+\hbar\omega_0 R_3-\frac{\bar{g}^2}{\hbar\omega_0}\frac{1}{N}R^{+}R^{-}, \tag{8}$$

then its quasispin part equivalence at $N\to\infty$ with the operator structure

$$\hbar\omega_0 R_3-\frac{\bar{g}^2}{\hbar\omega_0}\frac{1}{N}\left(R^{+}+R^{-}-2\bar{\xi}\right)\bar{\xi} \tag{9}$$

is established under the condition that real positive $\bar{\xi}$ minimizes the free energy of (9). The corresponding equation and its solution [9] reproduce results of [8]. The obtained value of parameter $\bar{\xi}=N\xi$ where ξ is expressed from (6).

Further steps are based on Bogolyubov ideas put forward in studies of ideal Bose-gas. If a system Hamiltonian commutes with a physical quantity operator, the corresponding conservation law is valid. The boson part of (8) commutes with particle number operator. Hence selection rules for thermodynamic averages calculated for the system (8) are observed:

$$\langle\alpha\rangle=\langle\alpha^{+}\rangle=0\,.$$

All results are formulated in the thermodynamic limit. Taking into account (7), we have the relations

$$\langle a \rangle = \langle a^+ \rangle = -\frac{\bar{g}}{\hbar\omega_0\sqrt{N}}\langle R^\pm \rangle = -\frac{\bar{g}}{\hbar\omega_0\sqrt{N}}\bar{\xi}\,, \quad \left\langle \frac{a^+a}{N} \right\rangle = \left(\frac{\bar{g}}{\hbar\omega_0}\right)^2 \xi^2 \tag{10}$$

that mean the macroscopic filling of the resonance mode of electromagnetic field.

Note that operators R^+ and R^- are hermitian conjugate, and their averages must be complex conjugate. Fixing the real value of $\bar{\xi}$ in the approximating Hamiltonian (9), we bypass the problem of indefinite phase factor in the indicated averages and the necessity of using formalism of quasiaverages.

The regular way of introducing quasiaverages for the models including interaction with boson field was developed in [10] (see also [11]). A more general Hamiltonian (with many-mode boson field, but the number of modes *s* is finite) is considered:

$$\hat{H} = \sum_{\kappa=1}^{s}\left\{\hbar\omega_\kappa a_\kappa^+ a_\kappa + \sqrt{N}\left(\lambda_\kappa a_\kappa^+ L_\kappa + \lambda_\kappa^* a_\kappa L_\kappa^+\right) - N\mu_\kappa L_\kappa L_\kappa^+\right\} + T\,. \tag{11}$$

κ numerates boson modes and some collective operators L. Norms of operators L_κ and T as well as their certain commutators satisfy the necessary restrictions from above. Approximating Hamiltonian has the view

$$\hat{H}_0(C) = T - N\sum_{\kappa=1}^{s} g_\kappa\left(C_\kappa^* L_\kappa + C_\kappa L_\kappa^+ - C_\kappa C_\kappa^*\right) \tag{12}$$

where $g_\kappa = \mu_\kappa + |\lambda_\kappa|^2/\omega_\kappa$, C_κ and C_κ^* are complex variational parameters. It is also convenient to use the auxiliary Hamiltonian

$$\hat{H}' = T - N\sum_{\kappa=1}^{s}\mu_\kappa L_\kappa L_\kappa^+\,. \tag{13}$$

In the thermodynamic limit the specific free energy calculated for the Hamiltonians under consideration coincides if the variational parameters values $\bar{C}_\kappa$ are chosen from the condition of the absolute minimum of this function for (12). Then in some cases $\langle L_\kappa \rangle_{\hat{H}_0(C)} = \bar{C}_\kappa \neq 0$, though for the initial Hamiltonian this value should equal zero because of the symmetry of (11). The contradiction is overcome through introducing infinitesimal terms breaking this symmetry in (11). In [10] such form of the necessary operator structure is substantiated:

$$\hat{H}_\nu = \hat{H} + 2N\sum_{\kappa=1}^{s}\nu_\kappa\omega_\kappa\left(\frac{a_\kappa^+}{\sqrt{N}} + \frac{\lambda_\kappa^*}{\omega_\kappa}\bar{C}_\kappa^*\right)\left(\frac{a_\kappa}{\sqrt{N}} + \frac{\lambda_\kappa}{\omega_\kappa}\bar{C}_\kappa\right). \tag{14}$$

Here ν_κ are real positive parameters. Leading them to zero after $N \to 0$, we come to physically correct result. In the thermodynamic limit

$$\langle L_\kappa \rangle_{\hat{H}_\nu} = -\frac{\omega_\kappa}{\lambda_\kappa}\left\langle \frac{a_\kappa}{\sqrt{N}} \right\rangle_{\hat{H}_\nu} = \bar{C}_\kappa, \qquad \langle L_\kappa^+ \rangle_{\hat{H}_\nu} = -\frac{\omega_\kappa}{\lambda_\kappa^*}\left\langle \frac{a_\kappa^+}{\sqrt{N}} \right\rangle_{\hat{H}_\nu} = \bar{C}_\kappa^*. \tag{15}$$

Since infinitely small terms in Hamiltonian cannot change the thermodynamic properties of a stable system, using quasiaverages (14) instead the usual averages ensures the symmetry breaking description. According to Bogolyubov [11], changes of thermodynamic parameters when introducing infinitely small terms is named the degeneracy of the thermodynamic equilibrium state, and quasiaverages remove such degeneracy.

The one-mode Dicke model corresponds to the structure of (11), but $\mu = 0$, so in (12) $g = |\lambda|^2 / \omega$. Matter operators L, L^+ are collective quasispin ones R^-, R^+. To our mind, in this case the conservation law of particle number is valid for "transformed" photons (7). Hence the zero values of averages $\langle \alpha \rangle, \langle \alpha^+ \rangle$ relate to additional averaging of random phase factor $e^{i\varphi}$ just like such operation concerning the bose condensate in an ideal gas. Indeed, the operators a, a^+ (as well as R^-, R^+) are non-Hermitian and not associated with measurable physical quantities, and we can choose the indicated factor arbitrarily. Another convincing argument for this choice possibility is the physical sense of these operators: a, a^+ describe the oscillating object, i.e. field, and R^-, R^+ are connected with non-diagonal matrix elements of the dipole moment operator, which values depend of the phase factors of wave functions of the two atom states involved in interaction process [5]. This is the reason of the sufficiency of the calculations (7) – (10) for polarization evidence in Dicke model.

4. Dipole moment in Dicke superradiance process

Any real superradiance process is a non-equilibrium phenomenon and must not be described in terms of equilibrium theory. Electromagnetic waves in the Dicke system are enhanced during passing through the medium of emitters [12] and leave the region of their localization. Nevertheless, studying the equilibrium state of an emitter system interacting with the generated field gives the key to the understanding of the ordering nature in the system under consideration. It is clear that macroscopic values of $\langle R^- \rangle, \langle R^+ \rangle$ arise when the dipole moments of emitters oscillating with the same frequency do it in phase. Such phasing appears due to their interaction via field. Since the phase factor in the indicated operators is arbitrary, considering their Hermitian product R^+R^- seems to be expedient, and this structure can be interpreted using the quasispin properties.

It is interesting to compare this phenomenon with the other one, in which the phasing is created by a radiation pulse. We mean photon echo predicted by Kopvillem and Nagibarov [13]. It is analogous to spin echo. A great number of emitters are transferred to an excited coherent state by a short pulse and start emitting in phase (signal of free induction). Then the induced macroscopic polarization of the medium gradually decreases because of a dephasing of the dipole oscillations. The second pulse change the sign of inhomogeneous dephasing and the initial phasing is almost restored, thus primary echo is formed. The phenomenon can be visually described in terms of quasispin behavior [14].

The classical model of superradiance outlined in [14] explained the role of phasing in the coherent spontaneous emission. The intensity of the mode **k** dipole emission of the complex of N identical atoms into a solid angle $d\Omega$ is expressed as

$$dI = \frac{\omega^4}{8\pi c^3} \sum_{\alpha=1,2} \left| \left(\mathbf{e}_{\alpha,\mathbf{k}} \cdot \mathbf{d} \right) \right|^2 \left\{ N + \sum_{i \neq j} \exp i \left[\left(\varphi_i - \varphi_j \right) + \mathbf{k} \cdot \left(\mathbf{r}_i - \mathbf{r}_j \right) \right] \right\} d\Omega \qquad (16)$$

where ω is emitter frequency, $\mathbf{e}_{\alpha,\mathbf{k}}$ is polarization vector, $\mathbf{d}$ is atom dipole moment, φ_i, φ_j are initial phases of emitters, $\mathbf{r}_i, \mathbf{r}_j$ describe their positions. If the condition of "spatial synchronism" $\left(\varphi_i - \varphi_j \right) + \mathbf{k} \cdot \left(\mathbf{r}_i - \mathbf{r}_j \right) = 0$ is satisfied, the second term in the braces in (16) is proportional to N^2.

In the quantum case we come to the well-known results obtained by Dicke [1]. Instead of phasing of the separate emitters, the collective states are the object of study. For the concentrated system the general view of the interaction term of the Hamiltonian can be written down as $H_{\text{int}} = -\mathbf{A}(0)\cdot(\mathbf{e}_1 R_1 + \mathbf{e}_2 R_2)$ where $\mathbf{A}(0)$ is the vector potential of the field, $\mathbf{e}_1, \mathbf{e}_2$ are constant real vectors the same for all emitters and R_1, R_2 are collective quasispin operators. We consider the emitter subsystem states defined with the eigenvalues of the operators $R^2 = R_1^2 + R_2^2 + R_3^2$ and R_3, respectively $R(R+1)$ and *M*. For the *N*-particle system these quantum numbers satisfy inequalities $|M| \le R \le N/2$. The symmetry properties of the Hamiltonian results in the selection rules for the matrix elements. Because the probability of spontaneous transition is determined by the corresponding matrix element square, only transitions with $\Delta M = 1$ are allowed in the dipole approximation. The matrix element

$$\langle R,M|\mathbf{e}_1 R_1 + \mathbf{e}_2 R_2|R,M-1\rangle = \frac{1}{2}(\mathbf{e}_1 \pm i\mathbf{e}_2 R_2)\left[(R\pm M)(R\mp M+1)\right]^{\frac{1}{2}}, \tag{17}$$

and setting $R = M = 1$, we have the radiation rate of one molecule $I_0 = \frac{1}{3}\frac{\omega^2}{c}\left(e_1^2 + e_2^2\right)$. At $R = M = N/2$ (all emitters are excited) $I = NI_0$, i.e., cooperative effect is absent. It is maximal when $R = N/2,\ M = 0$, and that means superradiation. More thorough analysis can be found in [14, 15].

Pay attention that if $P_{R,M}$ denotes the probability of a state $|R,M\rangle$, any superposition state radiates with intensity

$$I = I_0 \sum_{R,M} P_{R,M}(R+M)(R-M+1) = I_0\left\langle (R_1 + iR_2)(R_1 - iR_2)\right\rangle \equiv I_0 \mathrm{Sp}\left(\rho R^+ R^-\right). \tag{18}$$

Here ρ stands for the statistical operator of the system. We associate the total intensity of oscillating dipole radiation with the expression $\frac{|\mathbf{D}|^2\omega^4}{3c^3}$ where **D** is electric dipole moment of emitting system. Therefore (18) connects $\langle R^+R^-\rangle$ with the dipole moment squared. Now the place of the operator R_3 in ordering description should be elucidated. In (17) and (18) the relations

$$(R_1 + iR_2)(R_1 - iR_2) = R_1^2 + R_2^2 - i[R_1, R_2] = R^2 - R_3^2 + R_3 \tag{19}$$

are taken into account. Obviously, it is convenient to use the operator R_3 with a simple physical sense (the level population half-difference) instead of complex non-Hermitian operators. The operator R^2 commute with the accepted Hamiltonian and its value $\langle R^2\rangle$ determined by the pumping is fixed during the radiation process. This allows to construct the evolution equation for a superradiant system [15]. Neglecting stimulated emission processes and population difference fluctuations we come to the differential equation for $\langle R_3\rangle$:

$$\frac{d}{dt}\langle R_3\rangle = -\frac{1}{T}\left[\langle R^2\rangle - \langle R_3\rangle^2 + \langle R_3\rangle\right]. \tag{20}$$

Here the time of an excited atom decay $T = w_0^{-1}$ where the probability of such decay w_0 per time unit is proportional to $|\mathbf{d}|^2$. This equation coincides with Dicke's result if the initial state is fully inverted $|N/2, N/2\rangle$.

If the emitter dipole moment operator $\hat{\mathbf{d}}$ has only non-diagonal matrix elements, in the space of two-level atom states it can be represented as $\hat{\mathbf{d}} = r^+\mathbf{d} + r^-\mathbf{d}^*$. Therefore, the total dipole moment acquires the form

$$\hat{\mathbf{D}} = R^+\mathbf{d} + R^-\mathbf{d}^* \equiv \mathbf{D}^+ + \mathbf{D}^- . \qquad (21)$$

In accordance with (17), the hermitian conjugate operators introduced in (21) have non-zero matrix elements only between neighboring in energy levels of Dicke system and possess all properties of the physical quantity operator in the quasiclassical limit [15]. The average value of $\mathbf{D}^+$ for a package $|R, M\rangle$ where M lies in a small interval ΔM near $\bar{M}$ equals to $\mathbf{d}\left[\left(R+\bar{M}\right)\left(R-\bar{M}+1\right)\right]^{\frac{1}{2}}$ that gives the dipole radiation intensity

$$I = \frac{4\omega^4}{3c^3}|\mathbf{d}|^2\left[R(R+1) - \overline{M^2} + \bar{M}\right]. \qquad (22)$$

Thus, we can speak that a macroscopic dipole is formed in Dicke system and for its dipole moment squared the operator $|\mathbf{d}|^2 R^+R^-$ gives adequate description. The essence of ordering is phased emission of a great number of microscopic emitters. In quantum theory terms, we deal with a collective state in the system of identical objects. Initially after pumping, the cooperation effect is absent, but gradually the ordering takes place and intensity grows rapidly. Some electric polarization arises in the system, but it doesn't have a static nature. The polarization is of oscillating character, and its amplitude reaches macroscopic values. The quasispins turns out to be in the plane *xy*, and it is the moment of the maximal cooperative effect. In authoritative monographs a graphic image is presented only for individual quasispin [5, 14]. So, the convenient parameter – level population difference R_3 – is zero when the peak of emission is reached. Very special nature of quasispin arrangement in Dicke system put forward a lot of questions for many years after the underlying theory creation [5]. This fact explains the term "dynamic phase transition" concerning the Dicke process.

The authors studied just the kinetics of the non-equilibrium Dicke process and the picture of correlations. This problem was solved for a concentrated system 35 years ago [16] for the concentrated system Trying to implement the numerical research of extended system, we faced the problem of fixing the phase of the wave field. This question has a lot in common with the choice of phases of non-Hermitian operators in the equilibrium problem. Infinitesimal field introducing is like the method of quasiaverages. This research led us to the establishing the order parameter in the non-equilibrium situation and using the results of works devoted to Dicke model thermodynamics.

5. Conclusions

Ordering nature has been analyzed both for equilibrium and nob-equilibrium Dicke model with comparing such properties and necessary approaches in Jaynes–Cummings model, photon echo studies, and general theory of matter and boson field interaction. Polyatomic system in Dicke model can transfer into the ordered state with macroscopic boson mode filling, the order parameters formally can be complex. The general way of solving such problem based on the method of quasiaverages is presented with underlying the role of phase factors. The leading role of emitters phasing in superradiance and photon echo is substantiated, though in Dicke process it arises spontaneously, and this phenomenon requires consistent quantum analysis. The paradoxical situation in its theory is commented: the generally used description parameter (the collective quasispin component R_3) proves to be zero at the ordering maximum. The application of the order parameter R^+R^- constructed from those of equilibrium model is reasoned. Such parameter takes a macroscopic value, but corresponds to an oscillating quantity, so thermodynamical ideas can hardly be applied, and dynamic phase transition seems to be a better version.

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