EFFECTIVE HAMILTONIAN FOR THE SYSTEM OF THE ELECTROMAGNETIC FIELD AND CHARGED PARTICLES

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A classical system of point charged particles and electromagnetic field is investigated in the general formulation, in which there is no direct interaction of the particles with each other. Particles are assumed to be non-relativistic for research simplicity. The point of view is defended that gauge choice determines the physical picture of processes in the system. The idea of extended gauge, in which scalar φ_k and longitudinal part A^l_{nk} of the vector potentials transform separately for large and small wave numbers, is proposed. Moreover, in the new gauge $\varphi_k \neq 0$ only when $k \geq k_0$ and $A^l_{nk} \neq 0$ only when $k \leq k_0$, where k_0 is a certain wave number. This greatly simplifies the study of the system dynamics, in which the transverse components A^t_{nk} of the vector potential are taken into account and do not change at the gauge transformation. Basing on the extended gauge idea, it is established that the system has a massive oscillatory mode described by a transverse potential A^t_{nk} and an oscillatory mod described by a longitudinal potential A^l_{nk} with the laws of dispersion $(\omega_k^2+\omega_0^2)^{1/2}$ i ω_0, respectively ($\omega_k = ck$ is the dispersion law for electromagnetic waves in vacuum, and ω_0 is the plasma frequency). These modes can be called electromagnetic and plasma ones. At the same time, effective interaction between charged particles is introduced, which describes the screening effect. A connection of our work with Bohm and Pines investigations related to the presence of plasma modes in a Coulomb system, in which they do not use idea of gauge transformations, is analyzed.

Keywords: gauge transformation, effective interaction, massive electromagnetic mode, plasma mode.

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1. Introduction

Classical system consisting of electromagnetic field and non-relativistic charged particles is investigated. Its Lagrangian in standard notations has the form

$$L = \sum_a \frac{m_a \upsilon_a^2}{2} + \frac{1}{c}\sum_a e_a \upsilon_{an} A_n(x_a) - \sum_a e_a \varphi(x_a) + \frac{1}{8\pi}\int_V d^3x \{E(x)^2 - B(x)^2\} \qquad (1)$$

(V is a volume of the system). Electric and magnetic fields are expressed through potentials $A_n(x)$, $\varphi(x)$ by usual relations

$$E_n = -\dot{A}_n / c - \mathrm{grad}_n\, \varphi, \qquad B_n = \mathrm{rot}_n\, A\,. \qquad (2)$$

Quantities $A_n(x)$, $\varphi(x)$, x_a are generalized coordinates of the system and $\dot{A}_n(x)$, $\dot{\varphi}(x)$, $\upsilon_a \equiv \dot{x}_a$ are the corresponding generalized velocities.

It is known that field potentials are not unique and can be changed by the gauge transformation

$$A_n(x,t) = A_{1n}(x,t) + \partial_n \lambda(x,t)\,, \qquad \varphi(x,t) = \varphi_1(x,t) - \partial_t \lambda(x,t)/c\,, \qquad (3)$$

despite the fact that they are generalized field coordinates (here $\lambda(x,t)$ is an arbitrary function, old potentials are supplied here with sub 1). The replacement of potentials as generalized coordinates, as well as gauge transformations, are used in the literature to introduce an effective interaction of charges, which allows describing the system dynamics without considering the degrees of freedom of the field. This was realized when Darwin [1] derived the Hamiltonian for a system of charged particles in a weakly relativistic approximation only with accuracy up to $(\upsilon_a / c)^2$ since the contributions of higher orders ultimately coincide with ones of electromagnetic waves (see also [2]).

An important example of the study of effective interactions is the Bohm–Pines [3] theory of plasmons as collective motions in a system with a Coulomb interaction of charges (a non-relativistic system of charged particles). The equivalence of such a system to a system with some shortrange part of the Coulomb interaction in the presence of a longitudinal long-wave vector potential field was asserted. However, it was noted the need to use some additional conditions that compensate the increase in the number of degrees of freedom of the system. This shortcoming was overcome in our report at the MMET-2012 conference [4], where the implementation of the Bohm–Pines approach within the framework of the complete theory of the electromagnetic field and charges system by using a gauge transformation of the field potentials was proposed.

The current work develops and concretizes our approach [4] to studying the system of the electromagnetic field with charged particles. The task is to obtain an effective Hamiltonian of the system with subsystems of charges with a shortrange effective interaction between them, with electromagnetic and plasma modes.

The structure of the paper is as it follows. The Introduction describes the system under consideration. The second section introduces an idea of the extended gauge of the potentials of electromagnetic field. The third section constructs an effective interaction of charged particles with each other. The fourth part discusses properties of the Hamiltonian of the system. In the Conclusions the obtained results are summarized.

2. Extended gauge of electromagnetic field potentials

Below we use periodic boundary conditions and usual Fourier expansion of the potentials

$$A_n(x) = \frac{1}{V}\sum_k A_{nk} e^{ikx}, \quad A_{nk} = \int_V A_n(x) e^{-ikx} d^3x;$$
$$\varphi(x) = \frac{1}{V}\sum_k \varphi_k e^{ikx}, \quad \varphi_k = \int_V d^3x \varphi(x) e^{-ikx}, \tag{4}$$

introducing transversal and longitudinal parts A^l_{nk}, A^t_{nk} of the vector potential

$$A_{nk} = A^l_{nk} + A^t_{nk}, \quad A^l_{nk} \parallel k_n, \quad A^t_{nk} \perp k_n. \tag{5}$$

In these terms the Lagrangian (1) takes the form

$$L = \sum_a \frac{m_a \upsilon_a^2}{2} + \frac{1}{8\pi c^2 V}\sum_k (\dot{A}^{t*}_{nk}\dot{A}^t_{nk} - \omega_k^2 A^{t*}_{nk} A^t_{nk}) + \frac{1}{8\pi c^2 V}\sum_k \dot{A}^{l*}_{nk}\dot{A}^l_{nk} + \frac{1}{8\pi V}\sum_k k^2 \varphi^*_k \varphi_k +$$
$$+ \frac{i}{8\pi c}\sum_k k_n (\varphi_k \dot{A}^{l*}_{nk} - \varphi^*_k \dot{A}^l_{nk}) + \frac{1}{cV}\sum_k J^*_{nk}(A^t_{nk} + A^l_{nk}) - \frac{1}{V}\sum_k \rho^*_k \varphi_k \tag{6}$$

where Fourier transforms of the electric current $J_n(x)$ and charge $\rho(x)$ densities are introduced

$$J_n(x) = \sum_a e_a \upsilon_{an} \delta(x - x_a), \quad \rho(x) = \sum_a e_a \delta(x - x_a) \tag{7}$$

like notation (4) ($\omega_k \equiv ck$).

The potentials of the field are not unique and can be changed by a gauge transformation (3). These relations after Fourier transformation takes the form

$$A^l_{nk}(t) = A^l_{1nk}(t) + ik_n\lambda_k(t), \qquad A^t_{nk}(t) = A^t_{1nk}(t), \qquad \varphi_k(t) = \varphi_{1k}(t) - \partial_t\lambda_k(t)/c. \tag{8}$$

Each set of potentials describes the system from a certain physical point of view. Choosing this function as it follows

$$\lambda_k(t) = c\int_0^t dt'\varphi_{1k}(t') \quad \text{при } k \le k_0, \qquad \lambda_k(t) = i\frac{k_n}{k^2}A^l_{1nk}(t) \quad \text{при } k \ge k_0, \tag{9}$$

we obtain

$$\varphi_k = 0 \quad \text{at} \quad k \le k_0, \qquad A^l_{nk} = 0 \quad \text{at} \quad k \ge k_0. \tag{10}$$

Quantity k_0 (cut off) defines considered scale of the wave vectors (then the long waves correspond to $k \le k_0$). The concrete value of k_0 should be discussed in applications.

In this paper we call the gauge transformation (8), (9) as the extended one. At the same time, we assume that the field A^t_{nk} describes the transverse mode of the system, which is a modification of electromagnetic waves taking into account the interaction with charges, and the field A^l_{nk} describes the longitudinal mode of the system, which exists only in the presence of the interaction of the field with charges and which should be called plasma mode. Close to the result of these transformations is the Bohm–Pines [3] theory of Coulomb plasma with the Lagrangian

$$L = \sum_a \frac{m_a\upsilon_a^2}{2} - \frac{1}{2}\sum_{a,b}\frac{e_a e_b}{|x_a - x_b|}$$

without taking into account the degrees of freedom of the electromagnetic field. However, they transformed the long-range part

$$U^c_{\mathrm{lr}} \equiv \frac{1}{2V}\sum_{k\le k_0}\rho^*_k\varphi^c_k \tag{11}$$

of the Coulomb interaction

$$U^c \equiv \frac{1}{2}\sum_{a,b}\frac{e_a e_b}{|x_a - x_b|} = \frac{1}{2}\sum_a e_a\varphi^c(x_a) = \frac{1}{2}\int_V \rho(x)\varphi^c(x) = \frac{1}{2V}\sum_k \rho^*_k\varphi^c_k \tag{12}$$

into a long-wave part A^l_{nk} (with $k \le k_0$) of the vector potential of an electromagnetic field. Here $\varphi^c(x)$, φ^c_k are Coulomb potential of the system and its Fourier transform

$$\varphi^c(x) = \sum_a \frac{e_a}{|x - x_a|}, \qquad \varphi^c_k = \frac{4\pi}{k^2}\rho_k. \tag{13}$$

It was done with some difficulties related to number of freedom degrees.

3. Effective interaction of charged particles with each other

The Lagrange function of the system (6) in the extended gauge takes the form

$$L=\sum_a \frac{m_a\upsilon_a^2}{2}+\frac{1}{8\pi c^2 V}\sum_k(\dot{A}_{nk}^{t*}\dot{A}_{nk}^t-\omega_k^2 A_{nk}^{t*}A_{nk}^t)+\frac{1}{8\pi c^2 V}\sum_{k\le k_0}\dot{A}_{nk}^{l*}\dot{A}_{nk}^l+\frac{1}{8\pi V}\sum_{k\ge k_0}k^2\varphi_k^*\varphi_k+$$
$$+\frac{1}{cV}\sum_k J_{nk}^* A_{nk}^t+\frac{1}{cV}\sum_{k\le k_0}J_{nk}^* A_{nk}^l-\frac{1}{V}\sum_{k\ge k_0}\rho_k^*\varphi_k \tag{14}$$

and describes a system of three non-interacting fields A_{nk}^t, A_{nk}^l, φ_k (since now $A_{nk}^l\varphi_k=0$) and particles that interact with these fields. Let us introduce instead of the scalar potential φ_k a new scalar field ψ_k defined by the formula

$$\varphi_k=\varphi_k^c+\psi_k \tag{15}$$

where according (13) φ_k^c is the Coulomb field created by charge distribution ρ_k. Then the Lagrangian of the system (13) takes the form

$$L=\sum_a \frac{m_a\upsilon_a^2}{2}-U(k_0)+\frac{1}{8\pi c^2 V}\sum_k(\dot{A}_{nk}^{t*}\dot{A}_{nk}^t-\omega_k^2 A_{nk}^{t*}A_{nk}^t)+$$
$$+\frac{1}{8\pi c^2 V}\sum_{k\le k_0}\dot{A}_{nk}^{l*}\dot{A}_{nk}^l+\frac{1}{cV}\sum_k J_{nk}^* A_{nk}^t+\frac{1}{cV}\sum_{k\le k_0}J_{nk}^* A_{nk}^l+\frac{1}{8\pi V}\sum_{k\ge k_0}k^2\psi_k^*\psi_k \tag{16}$$

where one can consider

$$U(k_0)\equiv\frac{1}{2V}\sum_{k\ge k_0}\rho_k^*\varphi_k^c \tag{17}$$

as an effective interparticle interaction. According to (12), $U(k_0)$ can be called a short-range part of the Coulomb interaction. The Lagrangian (16) does not contain linear in ψ_k terms. The field ψ_k does not interact with the fields A_{nk}^t, A_{nk}^l and charged particles. Therefore, it does not affect the dynamics of the system and can be dropped.

Effective interaction U can be evaluated starting from transition to large volume

$$U(k_0)=\frac{1}{2(2\pi)^3}\int_{|k|\ge k_0}\rho_k^*\varphi_k^c d^3k=\frac{1}{(2\pi)^2}\sum_{a,b}e_a e_b\int_{|k|\ge k_0}k^{-2}e^{ik(x_a-x_b)}d^3k=\frac{1}{2}\sum_{a,b}\Phi(|x_a-x_b|,k_0) \tag{18}$$

where

$$\Phi(x,k_0)\equiv\frac{2}{\pi x}\int_{xk_0}^{+\infty}dt\frac{\sin t}{t};$$
$$\Phi(x,k_0)=\frac{1}{x}[1+O(xk_0)]\quad(xk_0\to 0), \tag{19}$$
$$\Phi(x,k_0)=\frac{2}{\pi x}[\frac{\cos xk_0}{xk_0}+O((xk_0)^{-2})]\quad(xk_0\to\infty).$$

At long distances, function $\Phi(x,k_0)$ decays faster than the Coulomb interaction and describes a screening effect. The same effective interaction between particles was introduced in [1] from other considerations. Namely, the authors start from the Coulomb interaction and use part of this interaction with $k \le k_0$ to introduce field of longitudinal vector potential A_{nk}^{l} with $k \le k_0$.

4. Hamiltonian of the system

The generalized energy of the system and its moments are defined by expressions

$$E = \sum_k \dot{A}_{nk}^{t} \frac{\partial L}{\partial \dot{A}_{nk}^{t}} + \sum_k \dot{A}_{nk}^{l} \frac{\partial L}{\partial \dot{A}_{nk}^{l}} + \sum_a \upsilon_{an} \frac{\partial L}{\partial \upsilon_{an}} - L ;$$

$$\pi_{nk}^{t} \equiv \frac{\partial L}{\partial \dot{A}_{nk}^{t}}, \qquad \pi_{nk}^{l} \equiv \frac{\partial L}{\partial \dot{A}_{nk}^{l}}, \qquad p_{an} \equiv \frac{\partial L}{\partial \upsilon_{an}}$$

that gives

$$E = \sum_a \frac{m_a \upsilon_a^2}{2} + U(k_0) + \frac{1}{8\pi c^2 V} \sum_k (\dot{A}_{nk}^{t*} \dot{A}_{nk}^{t} + \omega_k^2 A_{nk}^{t*} A_{nk}^{t}) + \frac{1}{8\pi c^2 V} \sum_{k \le k_0} \dot{A}_{nk}^{l*} \dot{A}_{nk}^{l} ,$$

$$p_{an} = m_a \upsilon_{an} + \frac{e_a}{cV} \sum_k A_{kn}^{t} e^{ikx_a} + \frac{e_a}{cV} \sum_{k \le k_0} A_{kn}^{l} e^{ikx_a} , \tag{20}$$

$$\pi_{kn}^{t} = \frac{1}{4\pi c^2 V} \dot{A}_{kn}^{t*}, \quad \pi_{kn}^{l} = \frac{1}{4\pi c^2 V} \dot{A}_{kn}^{l*} .$$

Kinetic energy of the particles can be written in the form

$$\begin{aligned} \sum_a \frac{m_a \upsilon_a^2}{2} = \sum_a \frac{p_a^2}{2m_a} - \frac{1}{cV} \sum_k j_{nk}^{*} A_{nk}^{t} - \frac{1}{cV} \sum_{k \le k_0} j_{nk}^{*} A_{nk}^{l} + \frac{1}{2c^2 V^2} \sum_{k,k'} \chi_{k-k'} A_{kn}^{t*} A_{k'n}^{t} + \\ + \frac{1}{c^2 V^2} \sum_{k' \le k_0, k} \chi_{k-k'} A_{kn}^{t*} A_{k'n}^{l} + \frac{1}{2c^2 V^2} \sum_{k,k' \le k_0} \chi_{k-k'} A_{kn}^{l*} A_{k'n}^{l} \end{aligned} \tag{21}$$

where gauge non-invariant current $j_n(x)$ and quantity $\chi(x)$ are introduced

$$j_n(x) = \sum_a \frac{p_a}{m_a} \delta(x - x_a) , \qquad \chi(x) = \sum_a \frac{e_a^2}{m_a} \delta(x - x_a) . \tag{22}$$

As a result, the Hamiltonian of the system is given by the expression

$$\begin{aligned} H = \sum_a \frac{p_a^2}{2m_a} + U(k_0) + \frac{1}{8\pi c^2 V} \sum_k [\dot{A}_{nk}^{t*} \dot{A}_{nk}^{t} + \omega_t^o(k)^2 A_{nk}^{t*} A_{nk}^{t}] + \frac{1}{2c^2 V^2} \sum_{\substack{k,k' \\ k' \ne k}} \chi_{k-k'} A_{kn}^{t*} A_{k'n}^{t} + \\ + \frac{1}{8\pi c^2 V} \sum_k [\dot{A}_{nk}^{l*} \dot{A}_{nk}^{l} + \omega_l^o(k)^2 A_{nk}^{l*} A_{nk}^{l}] + \frac{1}{2c^2 V^2} \sum_{\substack{k,k' \le k_0 \\ k' \ne k}} \chi_{k-k'} A_{kn}^{l*} A_{k'n}^{l} + \\ + \frac{1}{2c^2 V^2} \sum_{k' \le k_0, k} \chi_{k-k'} A_{nk}^{t*} A_{nk'}^{l} - \frac{1}{cV} \sum_k j_{nk}^{*} A_{nk}^{t} - \frac{1}{cV} \sum_{k \le k_0} j_{nk}^{*} A_{nk}^{l} \end{aligned} \tag{23}$$

Here new notations are introduced

$$\omega_t^o(k) = \sqrt{\omega_p^2 + \omega_k^2} , \qquad \omega_l^o(k) = \omega_0 , \qquad \omega_0 \equiv \sqrt{\frac{4\pi}{V} \sum_a \frac{e_a^2}{m_a}} = \sqrt{4\pi \sum_i \frac{n_i e_i^2}{m_i}} \tag{24}$$

(sub i is a number of the component of particles in the system, n_i is density of the i-th component). Frequency ω_0 is called as the plasma one and the fifth summand in (23) describes the plasma oscillations in the system which are oscillations of the longitudinal part of the vector potential of the electromagnetic field. The third summand in (23) describes oscillations of the transversal part of the vector potential with frequency $\omega_t^o(k)$. The first two summands in (23) describe charged particles of the system which interact with effective interaction $U(k_0)$. So, we completely reproduced results of the paper [1] on a new basis considering transversal oscillations of the electromagnetic field in the medium.

Note that from the sums in (23) with $\chi_{k-k'}$ only two terms have been extracted. However, the mentioned sums are quadratic forms and can be diagonalized. Let us consider the corresponding eigen value problem. The matrix $4\pi\chi_{k-k'}/V$ is Hermitian one. For the mentioned purpose we need two kinds of the eigenfunctions

$$\sum_{k'\le k_0}\frac{4\pi}{V}\chi_{k-k'}\xi^l_{k''k'}=\lambda^l_{k''}\xi^l_{k''k} \qquad (k''\le k_0), \qquad \sum_{k'}[\omega_k^2\delta_{k,k'}+\frac{4\pi}{V}\chi_{k-k'}]\xi^t_{k''k'}=\lambda^t_{k''}\xi^t_{k''k} \tag{25}$$

because for longitudinal waves wave vectors are restricted by relation $k\le k_0$. Here and further sub k'' numerates eigen vectors and eigen values and takes the same values as an arbitrary wave vector of the corresponding waves. Eigen vectors $\xi^t_{k''k}$, $\xi^t_{k''k}$ of these matrices constitute complete orthonormalized system of vectors

$$\sum_k \xi^t_{k_1''k}\xi^{t*}_{k_2''k}=\delta_{k_1'',k_2''}\,, \qquad \sum_{k''}\xi^t_{k''k}\xi^{t*}_{k''k'}=\delta_{k,k'}\,; \qquad \sum_{k\le k_0}\xi^l_{k_1''k}\xi^{l*}_{k_2''k}=\delta_{k_1'',k_2''}\,, \qquad \sum_{k''\le k_0}\xi^l_{k''k}\xi^{l*}_{k''k'}=\delta_{k,k'}\,. \tag{26}$$

Eigenvalues $\lambda^t_{k''}$, $\lambda^l_{k''}$ are positive. This follows from the Fourier transform definition of the function $\chi(x)$ and formulas

$$\sum_{k,k'}[\omega_k^2\delta_{k,k'}+\frac{4\pi}{V}\chi_{k-k'}]\xi^t_{k''k'}\xi^{t*}_{k''k}=\lambda^t_{k''}\,|\,\xi^t_{k''k}\,|^2\,, \qquad \sum_{k,k'\le k_o}\frac{4\pi}{V}\chi_{k-k'}\xi^l_{k''k'}\xi^{l*}_{k''k}=\lambda^l_{k''}\,|\,\xi^l_{k''k}\,|^2\,, \tag{27}$$

which relations (19) give. Eigenvalues $\lambda^t_{k''}$, $\lambda^l_{k''}$ are roots of the equations

$$\det(4\pi\chi_{k-k'}/V-\lambda^i\delta_{k,k'})=0 \qquad (k,k'\le k_0); \qquad \det(\omega_k^2\delta_{k,k'}+4\pi\chi_{k-k'}/V-\lambda^t\delta_{k,k'})=0\,. \tag{28}$$

At large volume of the system wave vectors take quasi-continuous set of values. Therefore, equations (22) contain infinite matrices and calculation of $\lambda^t_{k''}$, $\lambda^l_{k''}$ is possible only on the basis of these matrices truncation.

The next transformations are conducted as it follows. Let us expand the fields A^t_{nk}, A^l_{nk} in the eigenvectors

$$A^t_{nk}=\sum_{\mu}a^t_{n\mu}\xi^t_{\mu k}\,, \qquad a^t_{n\mu}=\sum_k A^t_{nk}\xi^{t*}_{\mu k}\,; \qquad A^l_{nk}=\sum_{\mu\le k_0}a^l_{n\mu}\xi^l_{\mu k}\,, \qquad a^l_{n\mu}=\sum_{k\le k_0}A^l_{nk}\xi^{l*}_{\mu k}\,. \tag{29}$$

As a result, the Hamiltonian of the system takes the form

$$H=\sum_a \frac{p_a^2}{2m_a}+U(k_0)+\frac{1}{8\pi c^2 V}\sum_k[\dot{a}_{nk}^{t*}\dot{a}_{nk}^{t}+\omega_t(k)^2 a_{nk}^{t*}a_{nk}^{t}]+$$
$$+\frac{1}{8\pi c^2 V}\sum_k[\dot{a}_{nk}^{l*}\dot{a}_{nk}^{l}+\omega_l(k)^2 a_{nk}^{l*}a_{nk}^{l}]+\frac{1}{8\pi c^2 V}\sum_{k\le k_0}\omega_l(k)^2 a_{nk}^{t*}a_{nk}^{l}-\frac{1}{cV}\sum_k j_{nk}^{t*}a_{nk}^{t}-\frac{1}{cV}\sum_{k\le k_0} j_{nk}^{l*}a_{nk}^{l} \qquad (30)$$

where compared with (18) renormalized dispersion laws for transversal $\omega_t(k)$ and longitudinal $\omega_l(k)$ waves are given and new notations for currents are used

$$\omega_t(k)=\sqrt{\omega_k^2+\lambda_k^t}\,,\quad \omega_l(k)=\sqrt{\lambda_k^l}\,;\qquad j_{nk''}^t=\sum_k j_{nk}\xi_{k''k}^{t*}\,,\quad j_{nk''}^l=\sum_{k\le k_0} j_{nk}\xi_{k''k}^{l*}\,. \qquad (31)$$

Quantization of the system with the Hamiltonian (31) can be easily conducted.

5. Conclusions

The main development of this work is an idea of extended gauge transformation. It is introduced in terms of the Fourier components of the scalar φ_k and longitudinal vector potentials A_{nk}^l in different intervals of the wave number. In our case, this is the domain $k>k_0$, where $A_{nk}^l=0$, and the domain $k<k_0$, where $\varphi_k=0$. The wave number k_0 determines the characteristic length of the problem k_0^{-1}. This unequivocally determines the effective potential of the interaction of system charges with account for screening. At the same time, the interaction between particles decreases according to the law $\frac{\sin k_0 x}{x^2}$ at distances $x>>k_0^{-1}$ As a result, it was established that the scalar potential $\varphi_k=0$ does not depend on time in the extended gauge.

It is proved that contributions to the Hamiltonian quadratic in the field describe transverse oscillations with the dispersion law $\omega_t(k)\equiv(\omega_k^2+\lambda_k^t)^{1/2}$ where $\omega_k=ck$ and λ_k^t, is a positive function. These are electromagnetic waves modified by the influence of charged particles. In the quantum version of the theory $\omega_t(k)$ is the law of dispersion of photons that acquire mass. In the main approximation, $\lambda_k^t\approx\omega_0^2$, ω_0 is the plasma frequency given by the formula $\omega_0\equiv\left(\frac{4\pi}{V}\sum_a\frac{e_a^2}{m_a}\right)^{1/2}$. It is also proved that the contributions to the Hamiltonian, which are quadratic in the field A_{nk}^l, describe longitudinal oscillations with the dispersion law $\omega_l(k)\equiv\lambda_k^{l\,1/2}$ where λ_k^l is a positive function. These oscillations are absent in a vacuum, they should be considered as plasma ones with $\lambda_k^l\approx\omega_0^2$. In the quantum variant of the theory $\omega_t(k)$ is plasmon dispersion law.

Our study of the system of charged particles and the electromagnetic field is influenced by the works of Bohm and Pines. However, these authors did not consider the degrees of freedom of the electromagnetic field and had a problem with the number of degrees of freedom of the system. They did not use gauge transformations but modified the Hamiltonians of the system based on the idea of their unitary equivalence (in a quantum case). In another approach, these authors obtained an effective Hamiltonian of the interaction of charged particles of the system, which coincides with ours. Plasma oscillations in the Hamiltonians are also described by the longitudinal part of the vector potential A^{l}_{nk} with approximate frequency ω_0.

The leading idea of our study is that the choice of potential gauge determines the physics of the problem. We introduced the term “extended gauge” for the name of the gauge used in this work clarifying the role of Bohm and Pines in the study of the system of charged particles and the electromagnetic field.

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