APPLICATION OF FRACTIONAL-RATIONAL APPROXIMATION IN MODELING ELEMENTS OF MICROWAVE ENERGY TRANSMISSION SYSTEMS

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The article is devoted to the application of the Cauchy method for the analysis of pole-zero structures in the form of a matched one-dimensional array and a matched-reflective layered structure in a waveguide. This approach confirms the possibility of a very accurate approximation of frequency characteristics, which simplifies calculations when using ultra-wideband signals. The optimized layered structure allows the useful signal to pass and the second harmonic to be reflected for subsequent conversion, which increases the efficiency of energy transmission by microwave means.

Keywords: Cauchy method, reflection coefficient, pole-zero structure, layered matching-reflecting dielectric structure, matched antenna array.

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1. Introduction

The microwave energy transmission system contains a radiation system, a reception system, which includes both a receiving antenna system and detection devices [1]. To provide a narrow directional beam, it is advisable to use antenna arrays [2].In such arrays, matching elements should be used to ensure a high degree of radiation efficiency [3]. Considering the transmission of high-power signals, it is advisable to use ultra-wideband signals [4], for example, chirp signals, as is common in radar. The detection process is carried out by a nonlinear element, so a second harmonic is necessarily formed. If appropriate measures are not taken, radiation will be generated through the receiving antenna system, which will go into free space. This problem is similar to the one that occurs when using mixers. To solve the latter problem, it is possible to use a layered structure, which has a minimum reflection coefficient in the signal frequency band and a reflection coefficient close to unity in the second harmonic frequency band [5]. Calculation of the necessary characteristics based on the first-principles-model computations is a computational cost procedure, so it is tempting to carry out calculations for a small number of frequency samples, and obtain other necessary data by interpolation. The use of fractional rational approximation is quite universal. It means that the Cauchy method of fractional rational approximation [6] can be used to speed up the numerical computations of parameters including the input impedance, currents, and the scattering data of any linear time-invariant (LTI) electromagnetic system. The algorithm presented [6] showed promise in solving several problems.

The purpose of this article is to determine the limits of applicability of the presented approach for describing the frequency characteristics of specific designs of antenna arrays and layered structures that can be used in solving problems of contactless energy transmission by microwave means.

2. Cauchy method of fractional-rational approximation

According to [6] the transfer function $H(f)$ for an arbitraryLTI system can be described with fractional rational expression

$$H(f_i) \approx \frac{A(f_i)}{B(f_i)} = \frac{\sum_{k=0}^{P} a_k f_i^k}{\sum_{k=0}^{Q} b_k f_i^k}, \quad (1)$$

where $A(f)$ and $B(f)$arethe corresponding polynomials of the numerator and the denominator, respectively, P and Q are the orders of the numerator and the denominator .

polynomials correspondingly.

For simplification of the procedure of a solution searching, it is obvious enough to present the problem in the form of the system of linear algebraic equations [1]

$$[C]_{N\times(P+Q+2)}\begin{bmatrix}a\\b\end{bmatrix}_{(P+Q+2)\times 1}=0, \tag{2}$$

where $[a]=[a_0,a_1,a_2,\cdots,a_P]^T$ and $[b]=\left[b_0,b_1,b_2,\cdots,b_Q\right]^T$
and

$$[C]=\begin{bmatrix}1 & f_1 & \cdots & f_1^P & -\widetilde{H}(f_1) & -\widetilde{H}(f_1)f_1 & \cdots & -\widetilde{H}(f_1)f_1^Q\\ 1 & f_2 & \cdots & f_2^P & -\widetilde{H}(f_2) & -\widetilde{H}(f_2)f_2 & \cdots & -\widetilde{H}(f_2)f_2^Q\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & f_N & \cdots & f_N^P & -\widetilde{H}(f_N) & -\widetilde{H}(f_N)f_N & \cdots & -\widetilde{H}(f_N)f_N^Q\end{bmatrix}. \tag{3}$$

The symbol T denotes the transpose of a vector. The size of the matrix $[C]$ is $N\times(P+Q+2)$, so the values of $[a]$ and $[b]$ can be obtained only if the total number N of frequency points are greater than or equal to the sum of numbers of unknown coefficients of the numerator and the denominator polynomials $P+Q+2$. The last system of linear algebraic equations can be solved using the least square method.

The use of direct frequency values of microwave range in an algebraic system leads to a huge dynamic range of coefficients at large values of P and Q. This drawback is eliminated by normalization according to $(f_i-f_r)/f_r$, with a certain reference value f_r, in the simplest case we can limit ourselves to the relation f_i/f_r.

The matrix $[C]$ consists of two submatrices $[A]$, which contains only powers of frequency samples, and a submatrix $[B]$, which contains the products of the corresponding transfer function samples and powers of frequency samples. The submatrix $[B]$contains noise, but at the same time we can assume that the submatrix $[A]$is free from the noise component.

At a further step the algorithm [1] provides QR decomposition of the submatrix [A]

$$[A]=[Q][R], \tag{4}$$

$$[Q^T][A\quad -B]_{N\times(P+Q+2)}\begin{bmatrix}a\\b\end{bmatrix}_{(P+Q+2)\times 1}=$$

$$=[R\quad -Q^TB]\begin{bmatrix}a\\b\end{bmatrix}=\begin{bmatrix}R_{11} & R_{12}\\ 0 & R_{22}\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}=0, \tag{5}$$

where $[Q]$ is $N\times N$ orthogonal matrix and the matrix $[R]$ is an upper triangular matrix. $[R_{11}]$ is also an upper triangular matrix whose elements are not affected by noise. Elements of both $[R_{12}]$ and $[R_{22}]$ are corrupted with the noise in $\widetilde{H}(f_1)$. From (5), it is clear

$$[R_{22}][b]=0, \tag{6}$$

$$[R_{11}][a]=-[R_{12}][b]\Rightarrow[a]=-[R_{11}]^{-1}[R_{12}][b]. \tag{7}$$

Application of SVD to $[R_{22}]$ gives

$$[U][\Sigma][V]^H[b]=0. \tag{8}$$

The superscript H denotes the conjugate transpose of a matrix. If we assume that the eigenvalues on the diagonal of the matrix $[\Sigma]$ are ordered in descending order, then the zero eigenvalue with number $Q+1$ corresponds to an eigenvector $[V]_{Q+1}$, which is the solution of the system (proportional to the solution of the system)

$$[b] = [V]_{Q+1}. \tag{9}$$

This approach has shown high efficiency in solving the system of linear Prony equations [4, 23]. This is the optimal solution for the coefficients of the denominator polynomial under the given conditions. For comparison, the pseudo-inversion method based on singular value decomposition and a method of algebraic projections were applied to the system (6) for the data considered in Section 3 and the models from [6] added with additive noise in the latter case. These methods did not provide an advantage over the described method; at best, they allowed one to achieve results very close to the method of eigenvector.

Using (9) and (7), the coefficients of the numerator polynomial can be calculated. Finally, the transfer function $H(f)$ can be reconstructed.After this, the values of the poles and zeros can be found by standard numerical methods as the roots of the corresponding polynomials.

To analyze the frequency response of a linear waveguide array made of open-ended waveguides, the following transfer function was considered:

$$H(f) \approx \frac{A(f)}{B(f)} = \frac{\sum_{k=0}^{P} a_k f^k}{\sum_{k=0}^{Q} b_k f^k} \approx K \frac{\prod_{k=1}^{P}(f - f_{0,k})}{\prod_{m=1}^{Q}(f - f_{p,m})}, \tag{10}$$

where f_0 are the zeros of the structure under consideration, f_p are the poles of the corresponding structure.

3. Numerical simulation results

Matching the radiating system with free space, along with the formation of the required radiation pattern, is a key issue in creating an effective antenna system. An antenna array with a matching element in the form of a resonator coupling region was considered (Fig. 1). Open ends of waveguides are used as radiators in the antenna array. The operating frequency of the antenna array was 10 GHz. The number of radiators was 5. The distance between the radiators was $d = \lambda = 30$ mm, where λ is the wavelength in free space. The radiators were surrounded by an infinite conducting surface. The apertures of the radiators in the form of open end of the waveguides are holes in a metal screen of infinite transverse dimensions. The antenna array under consideration corresponds to the case of scanning in the *E*-plane. The phase of the radiators' action changes according to each other in accordance with a linear dependence.

The dimensions of the resonator coupling region of the antenna array (*Rz*1, *Rz*2 – *Rz*1, *c*) have been optimized (Fig. 1). The optimal distance from the resonator coupling region to the aperture was *Rz*1 = 0.275Λ = 11 mm, where Λ is the wavelength in the waveguide. The optimal width of the resonator coupling region was *Rz*2 – *Rz*1 = 0.025Λ = 1 mm. The optimal size of the compensation cutouts in the outer waveguides was *c* = 0.1Λ = 4 mm.

For the fractional-rational approximation by the Cauchy method [6], 13 points are used (ψ varies from 0° to 180° with the step of 15°). For each of the three cases ($P = 4$, $Q = 6$; $P = 5$, $Q = 6$; $P = 6$, $Q = 6$) the total program time operating was 1.8 seconds (approximation time).The total time spent on the direct calculation of 37 points of the frequency response of the antenna array (i.e. from 0° to 180° with a step of 5°) is approximately equal to 7 minutes.

Since the analysis of the antenna array in the electromagnetic modeling program at all points takes a lot of time, it is time-efficient to construct a fractional-rational model based on a small number of points using the Cauchy method. The approximation accuracy of the fractional-rational model with the orders of the numerator and denominator polynomials equal to 4 and 6, 5 and 6, 6 and 6 were $5.2941 \cdot 10^{-4}$, $4.6479 \cdot 10^{-4}$, $1.0609 \cdot 10^{-4}$ respectively.

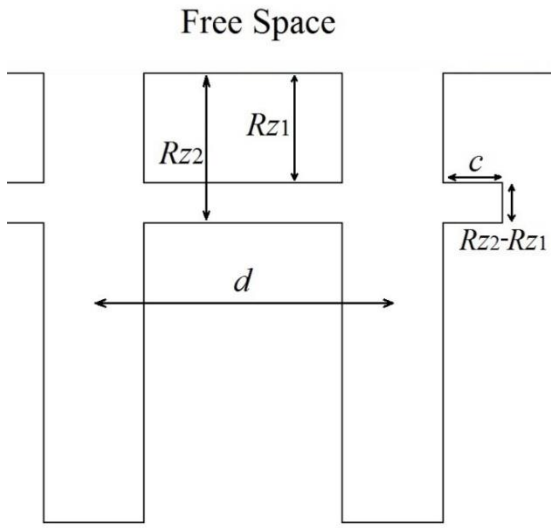

Fig. 1. Geometric dimensions of the antenna array with a resonator coupling region.

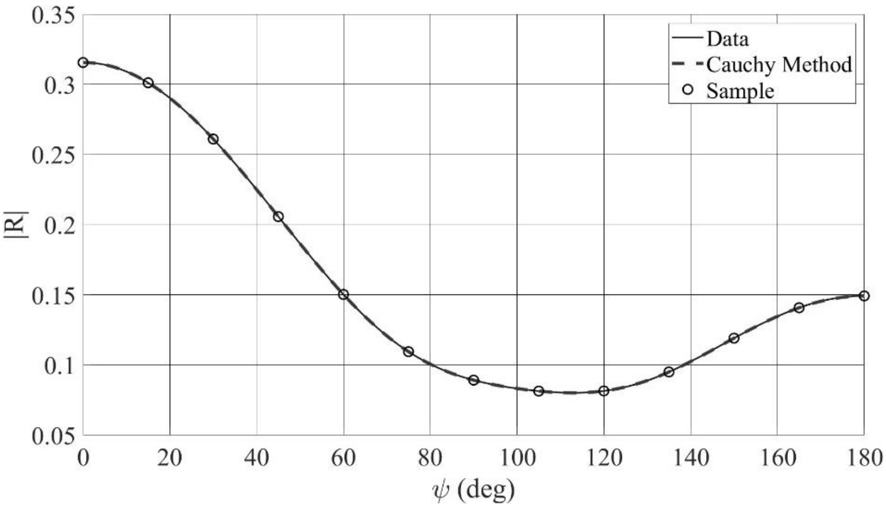

Fig. 2. Dependence of the modulus of the reflection coefficient of the central radiator against the phase difference between the fields of adjacent radiators.

The values of the poles and zeros of the frequency dependence are given in the table 1. It should be noted that the values of the zeros for all orders of the model are practically preserved, while the poles experience fixed changes upon reaching the order $P = 6$, $Q = 6$.

For the synthesized layered structure [5], the application of fractional rational approximation for the polynomial orders ($P = 12$, $Q = 12$) yields the root-mean-square

Table 1

Values of zeros and poles for different orders of the numerator and denominator for antenna array

N	*P* = 6, *Q* = 6		*P* = 5, *Q* = 6		*P* = 4, *Q* = 6	
	Zeros	Poles	Zeros	Poles	Zeros	Poles
1	121.0 + 36.0i	135.8 + 86.2i	121.9 + 40.9i	151.5 + 58.6i	120.1 + 41.0i	153.6 + 60.5i
2	121.0 – 36.0i	135.8 – 86.2i	121.9 – 40.9i	151.5 – 58.6i	120.1 – 41.0i	153.6 – 60.5i
3	83.0 + 35.9i	96.7 + 48.9i	82.0 + 31.1i	86.0 + 37.0i	83.7 + 30.0i	88.5 + 34.2i
4	83.0 – 35.9i	96.7 – 48.9i	82.0 – 31.1i	86.0 – 37.0i	83.7 – 30.0i	88.5 – 34.2i
5	–79.9	–0.8 + 128.8i	–1380.9	13.1 + 78.6i		15.7 + 77.7i
6	263.4	–0.8 – 128.8i		13.1 – 78.6i		15.7 – 77.7i

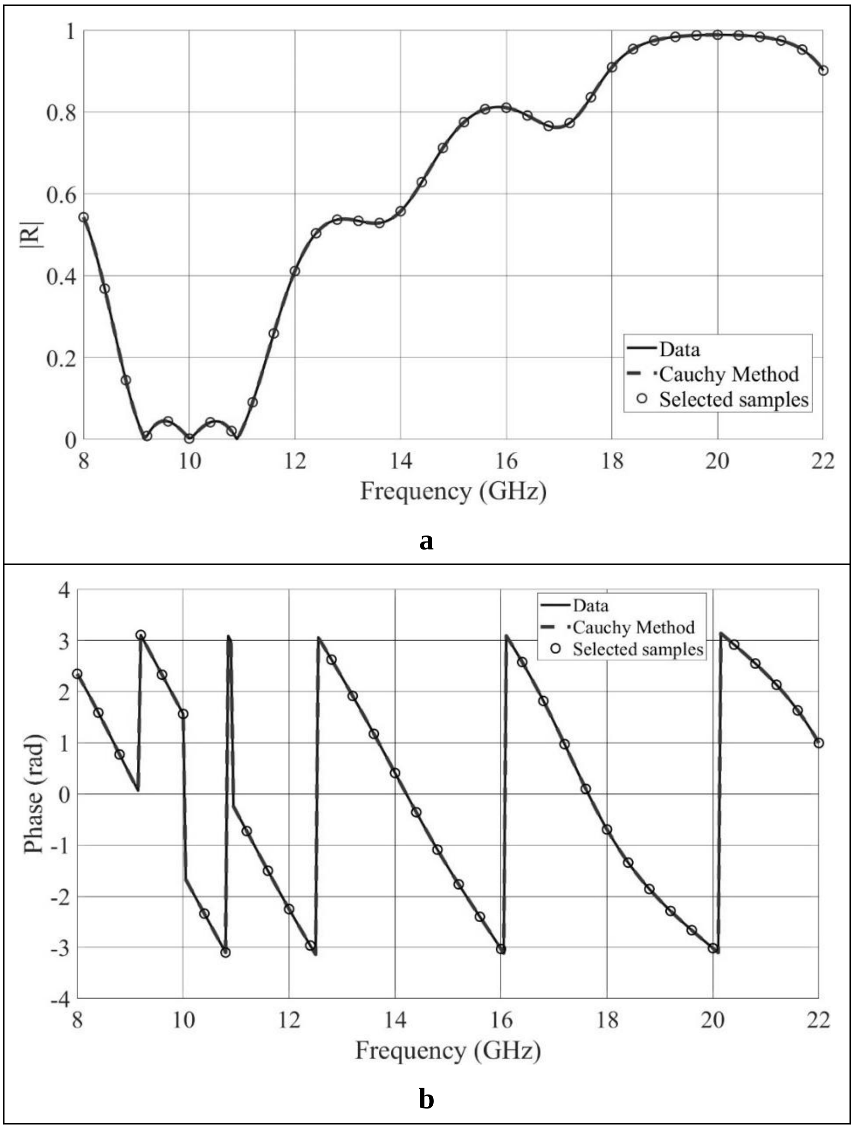

Fig. 3. Dependence of the modulus (a) and the phase (b) of the reflection coefficient of the synthesized matching-reflecting layered structure.

deviation $2.7549 \cdot 10^{-5}$. An interesting feature of the synthesized matching-reflecting layered dielectric structure is the presence of purely real zeros at 9.17, 10.01, 10.90 GHz in the matching band, which is typical for a layered structure in free space; in the reflection band, the zeros have a significant imaginary part.

When studying the layered structure, accurate approximation results (especially for the phase-frequency characteristic) were obtained for such orders of the system when 5-6 eigenvalues of the matrix R_{22}were practically equal to zero. In this case, summing up the specified group of eigenvectors gave results that were less accurate compared to the method of eigenvector corresponding to the smallest eigenvalue.

For the antenna array, accurate approximation results were achieved when the order of the matrix $R_{22}$was reached value, when only one eigenvalue practically equal to zero was formed.

4. Conclusions

The use of fractional-rational approximation by the Cauchy method [6] allows obtaining complex frequency characteristics with a high degree of accuracy about ten to the minus 5th power. This allows replacing calculations with direct methods by using fractional-rational approximation, which opens the possibility of saving calculation time when analyzing the application of broadband signals. The use of broadband signals, as in radar, allows achieving high indicators when transmitting powerful electromagnetic signals of the microwave range.

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