SPONTANEOUS MAGNETIZATION OF QGP AT HIGH TEMPERATURE

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In quark-gluon plasma (QGP), at higher deconfinement temperatures $T \geq T_d$ the spontaneous generation of color magnetic fields, $b^3(T), b^8(T) \neq 0$ (3, 8 are color indexes), and usual magnetic field $b(T) \neq 0$ happens. Simultaneously, the Polyakov loop and/or algebraically related to it $A_0(T)$ condensate, which is solution to Yang-Mills imaginary time equations, are also created. Usually, in analytic quantum field theory these effects are investigated independently of each other within the effective potentials having different mathematical structures. The common generation of these condensates was detected in lattice Monte Carlo simulations.

Recently, with the new type two-loop effective potential, which generalizes the known integral representation for the Bernoulli polynomials and takes into consideration the magnetic background, this effect has been derived analytically. The corresponding effective potential $W(T, b^3, A_0)$ was investigated either in SU(2) gluodynamics or full QCD. The gauge fixing independence of it was proved within the Nielsen identity approach. The values of magnetic field strengths at different temperatures were calculated and the mechanism of stabilizing fields due to $A_0(T)$ condensate has been discovered. In the present review, we describe this important phenomenon in more details, as well as a number of specific effects happening due to vacuum polarization at this background. They could serve as the signals of the QGP creation in the heavy ion collision experiments.

Keywords: spontaneous magnetization, high temperature, asymptotic freedom, effective potential, A_0 condensate, effective charge, effective vertexes.

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1. Introduction

Deconfinement phase transition (DPT), as well as the properties of the quark-gluon plasma (QGP), are widely investigated for many years. Most results have been obtained in the lattice simulations because of the large coupling value $g \geq 1$ at the phase transition temperature T_c. But at high temperatures due to asymptotic freedom the analytic methods are also reliable. They give a possibility for investigating various phenomena in the plasma. Among them is the creation of gauge field condensates described by the classical solutions to field equations without sources. Only such type fields could appear spontaneously inside the QGP. The well known ones are the so-called A_0 condensate, which is algebraically related to the Polyakov loop (PL) and the chromomagnetic fields $b^3 = gH^3, b^8 = gH^8$ (3, 8 are color indexes of SU(3) group) which are the Savvidy vacuum states at high temperature. These condensates result in numerous proper new effects which could be the signals of the QGP. The condensation of A_0 alone is investigated by different methods. For recent works see, for instance, [1] and references therein.

All the mentioned condensates are the consequences of asymptotic freedom and follow from the important property that asymptotic freedom at high temperature inevitably results in an infrared instability at low one. The field condensation prevents such type instability that results in the formation of the physical vacuum state. In quantum field theory (QFT), the magnetic and the A_0 condensates are generated at different orders in coupling constant (or the number of loops) for the effective potential (EP) $W(T, b, b^3, b^8, A_0)$. So that they have different temperature dependencies and play different roles in the QGP dynamics. For example, A_0 is generated at g^4 order in coupling constant and determined by the ratio of two- and one-loop contributions to $W(A_0)$. So it has the g^2 order. The fields $b(T), b^3(T), b^8(T)$ are generated in tree - plus one-loop - plus daisy approximation and also have the order g^2 in coupling constant. But they have other temperature dependence due the contribution to the EP of the tree-level term coming from the classical equation solutions. On the other hand, the contribution of A_0 at tree level equals zero because it is a constant electrostatic potential. This difference is important at high temperature. All mentioned features require special comprehensive considerations.

The fields investigated below are an important topic towards a theory of confinement. The A_0-background is relevant because at finite temperature such field cannot be gauged away and is intensively investigated beginning with [2]. In the early 90-ies, two-loop contributions were calculated in QCD and with these, the EP has non-trivial minimums and related condensate fields (see, for instance, [3], [4]). They form a hexagonal structure in the plane of the color components A_0^3 and A_0^8 of the background field.

The other kind background is the chromomagnetic one. More details about this field and the ways of its stabilization at finite temperature can be found, in particular, in [5], [6], [7] . The magnetization is also resulted from the minimum of the EP, which is stable in the consistent approximation of one loop plus daisy diagram contributions. In the review [6] and book [7] the results of different approaches (analytic (in noted approximation) and numeric) are presented, in particular, on lattice calculations with A_0 and chromomagnetic field.

A common generation of both fields was studied analytically in [15]. Here, new representation generalizing the known integral representation for the Bernoulli polynomials, was worked out, which admits introducing either A_0 or any b fields up to two-loop order. Below we write b for each magnetic field, for brevity. Within this representation, in particular, the known results for separate generation of the fields have been reproduced. However, the spontaneous generation of chromomagnetic field up to two-loop order was not investigated in detail. So, the mechanism of the vacuum stabilization remained not clarified finally.

This problem was analytically investigated in [10]. It is of grate importance because in the lattice calculations accounting for both backgrounds [8] it has been observed that in the presence of the constant color magnetic field the PL acquires a non-trivial spatial structure along the direction of the field. More interesting, on the lattice also, a common spontaneous generation of both fields was detected [9], [7]. So, to clarify a mechanism of magnetic field stabilization in QFT taking into consideration both condensates, one has to turn to two-loop calculations. This is because the stabilization of magnetic field within the one-loop plus daisy diagrams does not work. Such approximation is insufficient in case of two fields. This is because the generation of the latter one is realized at two-loop level for the EP. In what follows, we discuss in details both these approximations and compare the obtained results. In fact, in the current literature the cases of the A_0 and b condensations are investigated separately. So, mutual relations of them remained not clarified and estimated qualitatively, only.

In what follows, first, we calculate the EP as the function of A_0, and $b = gH^3$, in $SU(2)$ gluodynamics. The extension to full QCD is trivial because it includes three such groups. The integral expressions for the EP of the A_0 are generalized to include the magnetic background. Also, we consider the limiting cases $A_0 = 0$ and $b \neq 0$ and find, for instance, the magnetic condensate in two-loop order, which was also considered in [5], [15] but using other approaches and in not wide temperature interval.

Note again that the spontaneous generation of a background field is meant in the sense that for the corresponding field the EP has a minimum below zero, which is energetically favorable. In QGP, the $A_0(T)$ (or/and PL) results in the color $Z(3)$ symmetry breaking and the Furry theorem violation. The magnetic fields considerably change the spectra of quarks and gluons as well.

So, new phenomena have to be realized. In particular, the induced color charges Q^3_{ind}, Q^8_{ind} and the physically unexpected new type vertexes joining photon and gluon states could be generated. Obviously that at low temperature this is impossible, the white states and the colored ones do not interact (unite) in one vertex. The PL as well as $A_0(T)$ are the order parameters for the DPT. At low temperature they equal zero. At high temperature they become nonzero. The same concerns the spontaneously created magnetic fields. Recently, on the principles of the Nielsen's identity method and new type integral-sum representation for the EP we derived the gauge invariant expression for the A_0 condensate in the magnetic fields in two-loop approximation, [15], [4]. This (in particular) opens a possibility for calculating the induced color charges and other effective vertexes for this general background of QGP.

To realize that, we have to calculate the contribution of diagrams depicted in Fig. 1 and Fig. 2. There in, the solid line presents the quark propagator in the A_0 and magnetic fields and the wavy line presents the zero component of gluon fields G_0^3 or G_0^8. At finite temperature T, in the Matsubara formalism, one has to calculate the temperature sum over discrete energy values $p_4 = 2\pi T(l + 1/2), l = 0, \pm 1, ...$, integrate over momentum component p_3 oriented along the space field direction, calculate the sum over spin variable $\sigma = \pm 1$ and sum up over n = 0, 1, 2,..., in correspondence to the fermion spectrum in magnetic field b: $(p_4 + gA_0)^2 = m^2 + p_3^2 + (2n+1)b - \sigma b$. Here we write b as a general expression corresponding to each of the fields. This is eH for usual magnetic field, gH^3, gH^8 - for color fields.

In actual calculations of investigated effects, we apply the low level approximation, $n = 0, \sigma = +1$ giving a leading contribution for strong external fields. We obtain that the induced color charge Q^3_{ind} is nonzero. The presence of the magnetic fields changes the values of it compared to the zero field case. As a result, we derive that QGP has to be magnetized and color charged.

The way of presenting the results is as follows. First, in sect. 2 we introduce and discuss the general two-loop EP of both field calculated in the background $R_\xi^{ext.}$ gauge. This EP will be investigated for various cases of interest. So, one is able to proceed further taking into consideration only this one. However, for more detailed considerations, in sect. 3 we present the results on the Nielsen identity method for proving the gauge fixing parameter (ξ) independence of the EP. In sect. 4 we find the relation for the initial ξ-dependent EP and the EP of order parameter PL [4]. Then in sect. 5 we consider the case of b field generation. The case of A_0 only is additionally investigated in sect. 6. This section closes describing the creation of the stable QGP background at high temperature. In next sections we consider new type vertexes and related with them effects which have to happen and manifest the creation of QGP. Namely, in sect. 7 we calculate the induced color charge $Q^3_{ind.}$ followed from diagram in Fig. 1. In sect. 8 we calculate and investigate the effective vertex in Fig. 2 which joins one gluon and two photon states. A number of effects related to this vertex is discussed. In final section we summarize the results reported and describe prospects for the future.

2. Effective potential of fields

In the case of SU(2), the effective potential in the background R_ξ gauge reads [15]:

$$\begin{aligned} W_{gl}^{SU(2)} &= B_4(0,0) + 2B_4\,(a,b) \\ &+ 2g^2\left[B_2\,(a,b)^2 + 2B_2\,(0,b)\,B_2\,(a,b)\right] - 4g^2(1-\xi)B_3\,(a,b)\,B_1\,(a,b) \end{aligned} \tag{1}$$

with the notation

$$a = \frac{x}{2} = \frac{gA_0}{2\pi T}, \quad b = gH_3^3. \tag{2}$$

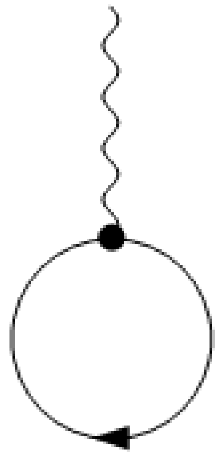

Fig. 1. Tadpole diagram

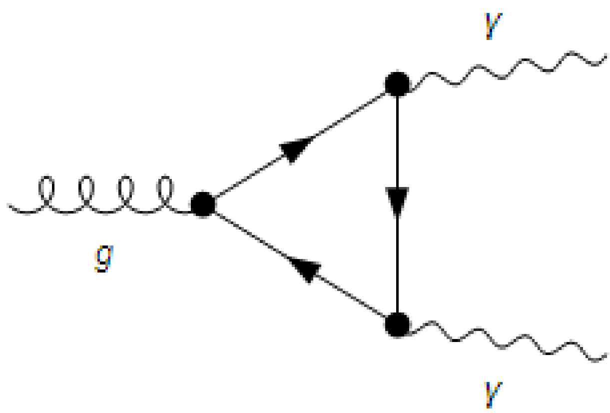

Fig. 2. Effective $\gamma - \gamma - G^3(g)$ diagram

The chomomagnetic field is directed along third directions in coordinate and color spaces. Since we work at finite temperature, W_{gl} is equivalent to the free energy.

The functions $B_n(a,b)$ are defined by

$$B_4(a,b)=T\sum_\ell\int\frac{dk_3}{2\pi}\frac{b}{4\pi}\sum_{n,\sigma}\ln\Big((2\pi T(\ell+a))^2+k_3^2+b(2n+1+\sigma-i0)\Big), \qquad (3)$$
$$B_3(a,b)=T\sum_\ell\int\frac{dk_3}{2\pi}\frac{b}{4\pi}\sum_{n,\sigma}\frac{\ell+a}{(2\pi T(\ell+a))^2+k_3^2+b(2n+1+\sigma-i0)}$$
$$B_2(a,b)=T\sum_\ell\int\frac{dk_3}{2\pi}\frac{b}{4\pi}\sum_{n,\sigma}\frac{1}{(2\pi T(\ell+a))^2+k_3^2+b(2n+1+\sigma-i0)},$$
$$B_1(a,b)=T\sum_\ell\int\frac{dk_3}{2\pi}\frac{b}{4\pi}\sum_{n,\sigma}\frac{\ell+a}{\Big((2\pi T(\ell+a))^2+k_3^2+b(2n+1+\sigma-i0)\Big)^2}.$$

In eq.(1), ξ is gauge fixing parameter, the summations run $n=0,1,\dots,\sigma=\pm2$ and ℓ runs over all integers. The $'-i0'$-prescription defines the sign of the imaginary part for the tachyon mode. These formulas and eq.(1) are the generalization of the corresponding two-loop expressions in [21], eqs.(3.8) and (A.2)-(A.5), [23], eq.(14), [24], eq.(4), and also [4], eq.(4), to the inclusion of the magnetic field. Note the sign "-" in eq.(5). Below we use also the relations

$$B_3(a,b)=\frac{1}{4\pi T}\partial_a B_4(a,b), \qquad B_1(a,b)=\frac{-1}{4\pi T}\partial_a B_2(a,b). \qquad (4)$$

For $b=0$ we have to replace $\frac{b}{4\pi}\sum_{n,\sigma}\to\int\frac{d^2k}{(2\pi)^2}$ and get

$$B_4(a,0)=\frac{2\pi^2T^4}{3}B_4(a),\;\; B_3(a,0)=\frac{2\pi T^3}{3}B_3(a), \qquad (5)$$
$$B_2(a,0)=\frac{T^2}{2}B_2(a),\qquad B_1(a,0)=-\frac{T}{4\pi B_1(a)},$$

where $B_n(a)$ are the Bernoulli polynomials, periodically continued. The special values for, in addition, $a=0$ are

$$B_4(0,0)=-\frac{\pi^2T^4}{45},\quad B_3(0,0)=0,\quad B_2(0,0)=\frac{T^2}{12},\quad B_1(0,0)=\frac{T}{8\pi}. \qquad (6)$$

We note that these formulas hold for $T>0$. The motivation for the above choice of the notations is that the functions $B_n(a,b)$, (3), are the corresponding mode sums without additional factors. More details about this representation as well as the renormalization and the case of $T=0$ are given in [15].

3. Effective potential and Nielsen's identity

Now, let us consider SU(2) gluodynamics in Euclidean space time, for the case of zero magnetic field and nonzero background field $\bar{A}^a_\mu=A_0\delta_{\mu0}\delta^{a3}=const$, described by the Lagrangian

$$L=\frac{1}{4}(G^a_{\mu\nu})^2+\frac{1}{2\xi}[(\bar{D}_\mu A_\mu)^a]^2-\bar{C}\bar{D}_\mu D_\mu C. \qquad (7)$$

The gauge field potential $A^a_\mu=Q^a_\mu+\bar{A}^a_\mu$ is decomposed in quantum and classical parts. The covariant derivative in eq.(7) is $(\bar{D}_\mu A_\mu)^{ab}=\partial_\mu\delta^{ab}-g\epsilon^{abc}\bar{A}^c_\mu$, $G^a_{\mu\nu}=(\bar{D}_\mu Q_\nu)^a-(\bar{D}_\nu Q_\mu)^a-$

$g\epsilon^{abc}Q^b_\mu Q^c_\nu, g$ is gauge coupling constant, internal index a = 1,2,3. The Lagrangian of ghost fields $\bar{C}, C$ is determined by the background covariant derivative $\bar{D}_\mu(\bar{A})$ and the total one $D_\mu(\bar{A}+Q)$. As in [22], [23] we introduce the вЂќcharged basisвЂќ of fields:

$$A^0_\mu = A^3_\mu, \;\; A^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm iA^2_\mu), \qquad (8)$$
$$C^0 = C^3, \;\; C^\pm = \frac{1}{\sqrt{2}}(C^1 \pm iC^2).$$

In this basis a scalar product is $x^a y^a = x^+y^- + x^-y^+ + x^0y^0$, and the structure antisymmetric factors are: $\epsilon^{abc} = 1$ for a = вЂќ+вЂќ, b = вЂќ-вЂќ, c = вЂќ0вЂќ. Feynman's rules are the usual ones for the theory at finite temperature with modification: in the background field the sum over frequencies should be replaced by $\sum_{k_0}, k_0 = (\frac{2\pi l}{\beta} \pm g\bar{A}_0)$ in all loops of the fields $Q^\pm_\mu, C^\pm$. Here, $l = 0, \pm1, \pm2, ...$ This frequency shift must be done not only in propagators but also in three particle vertexes.

Carrying out standard calculations we obtain the two-loop EP [4]

$$\begin{aligned} W(x) &= W^{(1)}(x) + W^{(2)}(x), \\ \beta^4 W^{(1)}(x) &= \frac{2}{3}\pi^2[B_4(0) + 2B_4(\frac{x}{2})], \\ \beta^4 W^{(2)}(x) &= \frac{1}{2}g^2[B^2_2(\frac{x}{2}) + 2B_2(0))B_2(\frac{x}{2})] + \frac{2}{3}g^2(1-\xi)B_3(\frac{x}{2})B_1(\frac{x}{2}), \end{aligned} \qquad (9)$$

where $B_i(x)$ are Bernoulli's polynomials defined $modulo$ 1 adduced in Appendix of the paper and $x = \frac{g A_0 \beta}{\pi}, \beta = 1/T$. This expression coincides with calculated already in [23], [24]. In what follows we consider the interval $0 \le x \le 2$.

Let us investigate the minima of it. We apply an expansion in powers of g and get

$$\begin{aligned} \beta^4 W_{min} &= \beta^4 W(0) - \frac{1}{192\pi^2}(3-\xi)^2 g^4, \\ x &= g^2\frac{(3-\xi)}{8\pi^2}, \end{aligned} \qquad (10)$$

where the first term is the value at zero field. Actually, an expansion parameter determined from the ratio of two- and one-loop contributions equals to $\frac{g^2}{8\pi^2}$, and therefore sufficiently large coupling values g are permissible. As we see, both the minimum position and the minimum energy value are gauge-fixing dependent. Hence the gauge invariance of the A_0 condensation phenomenon is questionable.

This problem was solved within Nielsen's identity method in [24], [25] for $SU(2)$ and $SU(3)$ gluodynamics and in [26], [3] for QCD with quarks. Since this approach is important for what follows, we describe it in short here.

In [27] Nielsen's identity for general type EP has been derived:

$$\delta' W(\phi) = W_{,i}\delta\chi^i(\bar{\phi}), \qquad (11)$$

which describes a variation of $W(\phi)$ due to variation of the gauge fixing term $F^\alpha(\phi)$. In eq.(11) ϕ^i is gauge field, $\bar{\phi}^i$ denotes a vacuum value of ϕ^i, comma after W means variation derivative with respect to corresponding variable. Variation $\delta\chi^i$ describes changing of field $(\bar{\phi})$ due to special gauge transformation which compensates variation of a classical action appearing after variation of gauge-fixing function $F^\alpha(\phi) \to F^\alpha(\phi) + \delta F^\alpha(\phi)$.

In field theory $\delta\chi^i$ is calculated from equation [27]:

$$\delta\chi^i = -\Big\langle D^i_\alpha(\phi)\Delta^\alpha_\beta(\phi)\delta^{'} F^\beta(\phi)\Big\rangle, \tag{12}$$

where $\big\langle O(\phi)\big\rangle$ denotes functional average of $O(\phi)$. In this expression $D^i_\alpha(\phi)$ is generator of gauge group, $\Delta^\alpha_\beta(\phi)$ is propagator of ghost fields, $\delta^{'} F^\beta(\phi)$ is variation of gauge fixing term.

In our case according eq.(7) $\delta^{'} F^\beta(\phi) = -\frac{1}{2}(\bar{D}_\mu(\bar{A})Q_\mu)^\beta \frac{\delta\xi}{\xi}$, D^i_α is covariant derivative. In [24], eq.(26), the expression was derived (more details on calculations and discussions for $SU(3)$ case see in [25], [3]):

$$\delta\chi^0 = \frac{g}{4\pi\beta} B_1(\frac{x}{2})\delta\xi. \tag{13}$$

Nielsen's identity for two-loop EP reads

$$\frac{dW}{d\xi} = \frac{\partial W^{(2)}}{\partial\xi} + \frac{\partial W^{(1)}}{\partial x}\frac{\partial x}{\partial\xi} = 0, \tag{14}$$

where in the order $\sim g^2$ the derivative $\frac{\partial x}{\partial\xi}$ equals to $\frac{\delta\chi^0}{\delta\xi} \times (\frac{g\beta}{\pi})$ in eq.(13). The latter factor comes from the definition of $x = \frac{gA_0\beta}{\pi}$. Since $W^{(2)}$ has the order g^2, and $W^{(1)}$- g^0, the eq.(14) states that $W(x,\xi)$ does not change along the characteristic curve

$$x = x' + \frac{g^2}{4\pi^2} B_1(\frac{x'}{2})(\xi - \zeta) \tag{15}$$

in the plain of variables (x,ξ), ζ is an arbitrary integration constant. Thus, there is the set of orbits where $W(x')$ is gauge-fixing independent. Along them a variation in ξ is compensated by the special variation of x'.

4. Effective potential of order parameter

In this section we, following [23], express the EP eq.(9) in terms of $\langle L\rangle$. We call it "effective potential of order parameter" $W_L(x_{cl})$. In SU(2) group, in tree approximation, the PL is expressed in terms of x as follows: $\langle L\rangle = \cos(\frac{\pi x}{2})$. This formula can be used to relate a given value of PL (or A_0) and classical (observable) condensate value with accounting for radiation corrections: $\langle L\rangle = \cos(\frac{\pi x_{cl}}{2}) = \cos(\frac{\pi x}{2}) + \Delta\langle L\rangle$. The quantum correction was calculated in one-loop order (eq. (10) in [23]),

$$\Delta\langle L\rangle = -\frac{g^2\beta\sin(\frac{\pi x}{2})}{4\pi}\int\frac{dk}{k_0^+}\Big[\frac{1}{(k_0^+)^2 + \vec{k}^2} + \frac{(\xi-1)(k_0^+)^2}{((k_0^+)^2 + \vec{k}^2)^2}\Big], \tag{16}$$

where the notations are introduced:

$$\int dk = \int\frac{d^3k}{(2\pi)^3}\Big(\frac{1}{\beta}\sum_{l=-\infty}^{\infty}\Big), \;\; k_0^+ = k_0 + gA_0, \;\; k_0 = \frac{2\pi l}{\beta}. \tag{17}$$

Eq.(16) is crucial for what follows.

Obviously that the first term in eq.(16) and the term at $(\xi - 1)$ are positively defined functions and should have the same signs after integrations. The second integral in eq.(16) is well known, it is expressed in terms of Bernoulli's polynomials [22], [3],

$$I_2 = -\frac{(\xi - 1)}{4\pi\beta} B_1(\frac{x}{2}). \tag{18}$$

Now, we return to the initial expression in eq.(16) and calculate the first term by using a standard procedure. This is presented in Appendix of [4] and also in [7]. The result is

$$I_1 = -\frac{1}{2\pi\beta} B_1(\frac{x}{2}). \tag{19}$$

Substituting I_1 and I_2 in eq.(16), we obtain finally

$$\Delta \langle L \rangle = \frac{g^2 \sin(\frac{\pi x}{2})}{16\pi^2} B_1(\frac{x}{2})(\xi + 1). \tag{20}$$

Just this formula should be used in order to express the field x in terms of "classical observable one", x_{cl}.

In particular, the relation between x and x_{cl} looks as follows

$$x = x_{cl} + \frac{g^2}{4\pi^2} B_1(\frac{x_{cl}}{2})(\xi + 1). \tag{21}$$

Within Nielsen's identity approach, this formula corresponds to the choice in eq.(15) $x' = x_{cl}$ and $\zeta = -1$. Along this orbit the EP is gauge-fixing independent and expressed in terms of $\langle L \rangle$. In such a way these two methods are related.

Inserting eq.(21) in eq.(9) and expanding $B_4(\frac{x}{2})$ in powers of g^2, we obtain $W_L(x_{cl}) = W_L^{(1)}(x_{cl}) + W_L^{(2)}(x_{cl})$, where the first term is obtained from $W^{(1)}(x)$ by means of substitution $x \to x_{cl}$ and the second is

$$\beta^4 W_L^{(2)}(x_{cl}) = \frac{g^2}{2} \left[B_2^2(\frac{x_{cl}}{2}) + 2B_2(0)B_2(\frac{x_{cl}}{2}) + \frac{8}{3} B_3(\frac{x_{cl}}{2}) B_1(\frac{x_{cl}}{2}) \right]. \tag{22}$$

In the $W_L(x_{cl})$ the ξ-dependent terms are mutually cancelled, as it should be and demonstrate gauge-fixing independence.

We also note that the final expression for $W_L(x_{cl})$ can be obtained from $W(x)$ eq.(9) formally (omitting described consequent steps) by means of the next substitutions: $x \to x_{cl}$ and $\xi \to \zeta = -1$. As a result, according eq.(10) we get for the minimum values

$$\begin{aligned} \beta^4 W_L(x_{cl})|_{min} &= \beta^4 W_L(0) - \frac{1}{48\pi^2} g^4, \\ x_{cl}|_{min} &= \frac{g^2}{2\pi^2}. \end{aligned} \tag{23}$$

Thus, the EP $W_L(x_{cl})$ has a nonzero minimum position and does not depend on ξ. The condensation happens at the two-loop level. The minimum value of PL (corresponding to the physical states) equals to: $\langle L \rangle = \cos(\frac{g^2}{4\pi})$. In contrast, in [23] the value $\langle L \rangle = \pm 1$ was obtained.

The expression $-W_L(x_{cl})|_{min} = p$ eq.(23) describes thermodynamical pressure in the plasma. The first term is $\beta^4 W_L(0) = -0.657974 + \frac{g^2}{24}$. The function $W_L(x_{cl})$ can be used for calculating Debye's mass of neutral gluons defined as

$$m_D^2 = \frac{d^2 W_L(x_{cl})}{dA_0^2}|_{A_0=0}, \tag{24}$$

remind, $x_{cl} = \frac{gA_0^{cl}}{\pi T}$. We get

$$m_D^2 = \frac{2}{3}g^2T^2 + \frac{5}{4}\frac{g^4}{\pi^2}T^2. \tag{25}$$

Here, first term is well known one-loop contribution and the second one is two-loop correction.

To complete we note that the A_0 condensation is derived within the correlation of the one- and two-loop effective potentials. Whereas asymptotic freedom at high temperature is realized due to the correlation of the tree-level and one-loop contributions to the EP. Formally (as it is often doing in the literature), the latter can be summarized by the replacement of coupling constant $g^2 \to \bar{g}^2 \sim \frac{g^2}{\log(T/T_0)}$, T_0 is a reference temperature. In both cases, the ratio of the relevant terms is $\sim \frac{g^2}{4\pi^2}$. Hence at high temperature we can substitute $g^2 \to \bar{g}^2$ in above formulas, in particular, in eq. (23).

Thus, the value of the order parameter PL in the minimum of the EP is

$$\langle L \rangle = \cos(\frac{\bar{g}^2}{4\pi}). \tag{26}$$

It gives a possibility for detecting the deconfinement phase transition and its type. Accounting for the explicit expression for the one-loop effective coupling $\alpha_s = \frac{\bar{g}^2}{4\pi}$ in the $SU(2)$ case

$$\bar{\alpha}_s = \frac{\alpha_s}{1 + \frac{11}{3\pi}\alpha_s \log(T/T_0)} \tag{27}$$

we see that the PL is continuously decreasing with temperature lowering and becomes zero at $\frac{\bar{g}^2}{4\pi} = \frac{\pi}{2}$. This signals confinement. If we set $T_d = T_0$ the value of the ratio $W^{(2)}/W^{(1)}$ is $\sim 1/2$, that is in the range of applicability of perturbation theory. The phase transition is second order, as it is well known for SU(2) gauge group. We note once again that due to the smallness of the expansion parameter $\frac{g^2}{8\pi}$ our perturbation EP of order parameter is suitable function for investigating the confinement-deconfinement phase transition.

An important observation, as we have seen, in order to obtain a ξ independent EP expressed in terms of PL it is sufficient to set $\xi = -1$ in expressions of interest. This systematically will be used in what follows.

5. The magnetic field at high temperature

Let us consider the case of chromomagnetic field at high temperature [10] and use eq.(59) of [15]. The EP reads

$$W_{gl}^{SU(2)} = \frac{b^2}{2g^2} - \frac{\pi^2T^4}{15} - \frac{a_1 b^{3/2}T}{2\pi} + \frac{11b^2\log(4\pi T/\mu)}{24\pi^2} + g^2\left(\frac{T^4}{24} - \frac{a_2\sqrt{b}T^3}{12\pi} + \frac{a_2{}^2 bT^2}{32\pi^2}\right). \tag{28}$$

The first term is energy of classical field. The terms proportional to T^4 constitute the gluon black body radiation. The contribution from the second loop is in the parenthesis. It has T^3

behaviour. The numbers $a_1 = 0.828, a_2 = 1.856$ are calculated in [15], eq.(22). Note that the one-loop part is $\xi-$indepedent and we set $\xi = -1$ in the two-loop part in order to get gauge invariant EP.

In one-loop order, the energy eq.(28) has a non-trivial minimum resulting from the term proportional to $b^{3/2}T$. The condensate and the minimum EP are

$$b^{one}_{min} = \frac{9{a_1}^2\alpha_s^2 T^2}{16\pi^2}, \qquad W^{SU(2),\,one}_{min} = -\frac{\pi^2 T^4}{15} - \frac{27a_1^4\alpha_s^3 T^4}{512\pi^4}, \tag{29}$$

where $\alpha_s = g^2/(1 + \frac{11}{12}\frac{g^2}{\pi^2}\log(4\pi T/\mu))$ is running coupling constant, μ is a normalization point for temperature. The first term of the energy is the gluon black body radiation. In this approximation, the condensate is always present, and the energy in the minimum is always negative. That means the spontaneous vacuum magnetization and SU(2) symmetry breaking. Here also an imaginary term presents, but we consider the real part. The standard way to remove the imaginary term of one-loop effective potential is adding the daisy diagram contributions (see [5] for details). From eq.(29) we see that the presence of α_s weakens the field strength at high temperature.

We turn to the two loop case. We consider the high temperature limit and take into consideration the $\sim T^3$ term in eq.(28). Denoting as $b^{1/2} = x$, we obtain the third-order polynomial equation for determining the condensate value:

$$x^3 - \frac{3}{4\pi}a_1 T\alpha_s x^2 - \frac{g^2}{24\pi}a_2 T^3\alpha_s = 0. \tag{30}$$

The real root of it can be found using formulas from the standard handbook [29], Chapter 3.8. The result is

$$x_0 = b^{1/2}_{min} = \frac{1}{4}\frac{(2a_2\alpha_s)^{1/3}}{\pi^{1/3}}T + \frac{1}{4\pi}a_1\alpha_s T. \tag{31}$$

If we compare this with eq.(29), we find that the second term is three times less than the one in eq.(29). The most interesting is the change of the temperature dependence coming from $\alpha_s^{1/3}$. Hence, the first term is dominant for this case. For the field strength we get in this limit

$$b_{min} = \frac{1}{16}\frac{(2a_2\alpha_s)^{2/3}}{\pi^{2/3}}T^2. \tag{32}$$

Note also, the value a_2 is larger than a_1. As a result, the role of the second loop is important. Formula eq.(31) was derived first in the literature in [10].

6. The minimum of the effective potential for pure A_0

In this section, we remind the known results for the case of a pure A_0-background discussed in details in sects. 3, 4. For the case $b = 0$, the general effective action eq.(1) with eq.(5) is expressed in terms of Bernoulli's polynomials. We restrict ourselves to the main topological sector and there to $0 \leq a \leq 1/2$. Here, the EP has a minimum at $a = a_{min}$ (see also eq.(6) in [24]) and takes in this minimum the value $W_{|a=a_{min}} = W_{min}$ with

$$(gA_0)_{min} = \frac{3-\xi}{16\pi}g^2 T, \qquad W_{min} = -\frac{\pi^2 T^4}{15} - \frac{(3-\xi)^2 T^4}{192\pi^2}g^4. \tag{33}$$

As mentioned in [4], [32], eq.(33) coincides with the gauge-invariant result for $\xi = -1$, what we assume in the following.

Let us compare eq.(33) with the minimal effective potential eq.(29) in the pure magnetic case. We see in the latter case, the extra temperature dependent factor $(1+\frac{11}{12}\frac{g^2}{\pi^2}[\log(4\pi T)/\mu)^{-1}$ is present and decreases the value of the magnetic condensate at high temperature. For the two loop result eq.(32) the strength of the field is larger. But again at sufficiently high temperature the $\alpha_s^{1/3}$ factor makes the value of $b_{min}(T)$ smaller compared to the value of $(gA_0)_{min}$ eq.(33). As a result, since both condensates have negative energies they should be generated. This decreases the total free energy of the system. The same takes place for SU(3) gluodynamics and full QCD.

7. Induced Color charge

In this section, for QCD, we calculate the induced color charge generated by the tadpole diagram of Fig. 1. In charged basis, we have two components of the induced charge for the shifts A_0^3 and A_0^8. But accounting for the result [14] $A_0^8 = 0$, we have to calculate the contribution for the case $(A_0)^a_\mu = A_0\delta_{\mu 4}\delta^{a3}$. The explicit form in the Euclidean space-time is $Q_4^3 Q^3_{ind}$, and we have

$$Q^3_{ind} = \frac{g}{\beta}\sum_{p_4}\int\frac{d^3p}{(2\pi)^3}Tr\left[\frac{\lambda^3}{2}\gamma_4\frac{\hat{p}_\sigma\gamma_\sigma + m}{\hat{p}^2+m^2}\right], \tag{34}$$

where $\hat{p} = (p_4 = p_4 \pm A_0, \mathbf{p})$, $p_4 = 2\pi T(l + 1/2)$, $l = 0, \pm1, \ldots$. The trace is calculated over either space-time or color variables, λ^3 is Gell-Mann matrix. Here also we denoted as A_0 the value $A_0 = \frac{gA_0}{2}$. In what follows we use the Matsubara imaginary time formalism at finite temperature.

Calculating the traces over the space and the internal indexes we get,

$$Q^3_{ind} = \frac{4g}{\beta}\int\frac{d^3p}{(2\pi)^3}\sum_{p_4}\frac{(p_4 + A_0)}{(p_4 + A_0)^2 + \epsilon_p^2}, \tag{35}$$

where $\epsilon_p^2 = \vec{p}^2 + m^2$. In case of nonzero field, $\epsilon_p^2 = p_3^2 + m^2 + (2n+1)gH - gH\sigma$. Here, gH stands for any kind of magnetic field gH^3, gH^8, eH or even some combinations of chromomagnetic fields generated in the plasma at high temperature (see for details [13]). We have two coupling constants and therefore returned to the H-notations. It is important also that all the magnetic fields are oriented in one direction in coordinate space. In this case the EP of the fields has minimal energy and so such type ones are generated spontaneously.

To calculate the temperature sum we use the following representation ($\beta = 1/T$ is inverse temperature)

$$Q^3_{ind} = 4g\int\frac{d^3p}{(2\pi)^3}\frac{\beta}{\pi}\oint_C \tan\frac{\beta\omega}{2}\frac{(\omega + A_0)}{(\omega + A_0)^2 + \epsilon_p^2}d\omega. \tag{36}$$

The contour C encloses clockwise the poles of tangent in ω-plane. This is in the case of zero field. If the field is nonzero, we have to replace $\frac{d^3p}{(2\pi)^3} - > \frac{dp_3}{2\pi}\frac{gH}{(2\pi)^2}$ in correspondence to the particle spectrum. The integrand function has two complex poles of first order in the ω-plane. We deform and move the contour to infinity and calculate the residues of the integrand to find the charge value.

The result, after transformation into spherical coordinates and angular integration, is

$$Q_{ind}^3 = \frac{g \sin(A_0 \beta)}{\pi^2} \int_0^\infty p^2 dp \frac{1}{\cos \beta A_0 + \cosh(\beta \epsilon_p)}. \tag{37}$$

In what follows, we calculate the integral in the high-temperature limit $T \to \infty$. In this case we use

$$\epsilon_p = \sqrt{\mathbf{p}^2 + m^2} \approx |\mathbf{p}| + \frac{1}{2} \frac{m^2}{|\mathbf{p}|} \tag{38}$$

because large values of momentum give dominant contribution.

After integration over momentum we obtain at zero field [14]

$$Q_{ind}^3 = g A_0^3 [\frac{T^2}{3} - \frac{m^2}{2\pi^2}]. \tag{39}$$

As we see, the first term depends on temperature as $\sim T^2$. The second one depends on mass, only. At high-temperature, the first term is dominant and plasma acquires the spontaneous induced charge in the case $m = 0$, also.

Now, we consider nonzero H. Using the low Landau level approximation, $\sigma = +1, n = 0$, we get after integration over p_3 momentum

$$Q_{ind}^3(H, T) = g \frac{gH}{2\pi^3} \frac{\sin(A_0^3 \beta)}{\beta} (1 + 7\beta^2 m^2 Zeta'(-2)). \tag{40}$$

Note, numerically $Zeta'(-2) = -0.03044485$. Thus, one of the consequences of the A_0 condensate presence is the $Z(3)$ symmetry and the C-parity violation, which leads to the induction of color charge in the plasma.

Let us compare the values of induced color charges given by formulas eq.(39) and eq.(40). The first leading in temperature terms are of interest. Both expressions have the factor gA_0^3 but a different temperature dependence. In former case, the factor $\sim T^2$ stands and determines high temperature behavior. In latter case, it is determined by the temperature dependence of the magnetic field. This behavior has been investigated in sect. 5 in two-loop approximation eq.(32). Because of the factor $\alpha_s = g^2/(1 + \frac{11}{12}\frac{g^2}{\pi^2}\log(T/\mu))$, the value of the field strength is always smaller compared to T^2. As a result, the induced color charge eq.(40) in the magnetic field is also smaller compered to eq.(39).

On the other hand, during carried out calculations we have taken the field strength as a given number which is arbitrary. So that it can be the field produced by some external current. In this case the induced color charge will be completely determined by external field. Such a situation is expected and discussed for heavy ion collision experiments.

As we noted in Introduction, the factor gH marks different magnetic fields - usual magnetic field eH, color magnetic fields gH^3, gH^8 or even some combination of them. For instance, in [13] it was shown that in QGP at the LHC energies the combinations of fields $H_f^1 = q_f H + g(\frac{H^3}{2} + \frac{H^8}{2\sqrt{3}}), H_f^2 = q_f H + g(-\frac{H^3}{2} + \frac{H^8}{2\sqrt{3}}), H_f^3 = q_f H - g\frac{H^8}{2\sqrt{3}}$, have to be spontaneously produced, where q_f is electric charge of quark species. The strengths of the fields at various temperatures have been estimated. In particular, it was shown that the strength of color fields is two order stronger compared to the usual magnetic one. The spectra of all charged particles become discrete that influences and specifies the manifestations of QGP. It is also important that (as we noted above) all the generated fields are collinear each other in space.

It is also interesting that the field presence decreases the phase transition temperature. This also has been obtained in analytic [13] and lattice computations [9], [11], [12]. As a general conclusion of above consideration, the most important consequence of the induced charge $Q^3_{ind.}$ is the generation of classical static corol potential $q^3_{stat.}$ that opens a possibility for new scattering processes in the QGP. The magnetic fields modify them in an essential way. So, numerous manifestations of the plasma creation could be observed in experiments. For example, these influence the number of direct photons radiated from QGP, modify scattering of photons on it, etc. These phenomena will be discussed elsewhere.

8. Effective $\gamma\gamma G$ vertexes in QGP

Other interesting objects which have to be generated in the QGP with A_0 condensate are the effective three-line vertexes $\gamma\gamma G^3$, $\gamma\gamma G^8$. They should exist because of Furry's theorem violation and relate color and white states. These vertexes, in particular, have to result in observable processes of new type - inelastic scattering of photons, splitting (dissociation / conversion) of gluon $\bar{\phi}^3$, $\bar{\phi}^8$ classical potentials in two photons. As we have shown above, Q^8_{ind} is not generated in the considered approximation. But it is not excluded in higher-loop orders. So we have to take in mind the second type vertex also.

In this section, we calculate the vertex $\gamma\gamma G^3$ depicted in Fig. 2 and investigate some related processes in the plasma. The vertex $\Gamma^\nu_{\mu\lambda}$ consists of two such type diagrams. The second one is obtained by changing the direction of the quark line. We use the notations: all the momenta are ingoing, first photon $\gamma_1(k^1_\mu)$, second photon $\gamma_2(k^3_\lambda)$, color a=3 gluon $Q^3(k^2_\nu)$, and $k^1+k^2+k^3 = 0$. $k^{1,2,3}$ are momenta of external fields.

We consider the contributions coming from the traces of four γ-matrixes, which are proportional to the quark mass and dominant for small photon momenta $k^1, k^3 << m$. The analytic expression reads

$$\Gamma^\nu_{\mu\lambda}(k^1,k^3) = \Gamma^{\nu,(1)}_{\mu\lambda}(k^1,k^3) + \Gamma^{\nu,(2)}_{\mu\lambda}(k^1,k^3), \tag{41}$$

where

$$\Gamma^{\nu,(1)}_{\mu\lambda}(k^1,k^3) = \frac{1}{\beta}\sum_{p_4}\int \frac{d^3p}{(2\pi)^3}\frac{N_1}{D(\tilde{P})D(\tilde{P}-k^1)D(\tilde{P}+k^3)}. \tag{42}$$

Here $\beta = T^{-1}$, summation is over $p_4 = 2\pi T(l+1/2), l = 0, \pm1, \pm2, ...$, integration is over three dimensional momentum space p, N_1 denotes the numerator coming from the first diagram, $\tilde{P} = (\tilde{P}_4 = p_4 - A_0, \vec{p})$, $D(\tilde{P}) = (p_4 - A_0)^2 + \vec{p}^2 + m^2 = \tilde{P}_4^2 + \epsilon_p^2$ and $\epsilon_p^2 = \vec{p}^2 + m^2$ is energy of free quark squared. In case of nonzero field, $\epsilon_p^2 = p_3^2 + m^2 + (2n+1)gH - gH\sigma$ as in previous section. We also have to replace $\frac{d^3p}{(2\pi)^3} -> \frac{dp_3}{2\pi}\frac{gH}{(2\pi)^2}$.

First we consider the zero field case. The functions $D(\tilde{P}-k^1), D(\tilde{P}+k^3)$ assume a corresponding shift in momentum. The numerator N_1 is

$$(N_1)_{\mu\nu\lambda} = \delta_{\mu\nu}(\tilde{P}-k^2)_\lambda + \delta_{\lambda\nu}(\tilde{P}-k^2)_\mu + \delta_{\mu\lambda}(\tilde{P}-q)_\nu, \tag{43}$$

where $q = k^3 - k^1$ is photon momentum transferred.

The expression for the second term in eq.(41) is coming from the second diagram and obtained from eqs.(42), (43) by substitutions $k^1 \to -k^1, k^2 \to -k^2, q \to -q$. We denote the second numerator as N_2. In what follows we carry out actual calculations for the first term in eq.(41) and adduce the results for the second one.

Now, we take into consideration the fact that in the high temperature limit the large values of the integration momentum p give leading contributions. Therefore we can present the functions $D(\tilde{P}), D(\tilde{P}-k^1), D(\tilde{P}+k^3)$ in the form:

$$\begin{aligned} D(\tilde{P}) &= \tilde{P}_4^2 + \epsilon_p^2 = \tilde{P}^2, \\ D(\tilde{P}-k^1) &= \tilde{P}^2\Big(1 - \frac{2\tilde{P}\cdot k^1 - k_1^2}{\tilde{P}^2}\Big), \\ D(\tilde{P}+k^3) &= \tilde{P}^2\Big(1 + \frac{2\tilde{P}\cdot k^3 + k_3^2}{\tilde{P}^2}\Big). \end{aligned} \tag{44}$$

Here, $k_1^2 = (k_4^1)^2 + \vec{k}_1^2, k_3^2 = (k_4^3)^2 + \vec{k}_3^2$. At high temperature and $\tilde{P}^2 \to \infty$ the k-dependent terms are small. So, we can expand in these parameters and obtain for the integrand in eq.(42)

$$Intd. = \frac{N_1}{(\tilde{P}^2)^3}\Big[1 + \sum_{i=1}^{4} A_i\Big], \tag{45}$$

where

$$A_1 = -2\frac{(\tilde{P}\cdot q)}{\tilde{P}^2}, \;\; A_2 = -\frac{k_3^2 - k_1^2}{\tilde{P}^2} \tag{46}$$

$$A_3 = -4\frac{(\tilde{P}\cdot k^1)(\tilde{P}\cdot k^3)}{\tilde{P}^2}, \;\; A_4 = 4\frac{(\tilde{P}\cdot k^1)^2 + (\tilde{P}\cdot k^3)^2}{\tilde{P}^2} \tag{47}$$

and vector $q_\mu = (q_4, \vec{q})$.

For the second diagram we have to substitute $q \to -q$, other terms are even and do not change.

Further we concentrate on the scattering of photons on the potential Q_4^3 in the medium rest frame and set the thermostat velocity $u_\nu = (1, \vec{0}), \nu = 4$. The corresponding terms in the numerators are

$$N_1 -> \delta_{\mu\lambda}(\tilde{P}+q)_4, \;\; N_2 -> \delta_{\mu\lambda}(\tilde{P}-q)_4, \tag{48}$$

remind that $\tilde{P}_4 = p_4 - A_0$ and $\tilde{P}^2 = (p_4 - A_0)^2 + \epsilon_p^2$. In this case the numerators do not depend on space momentum and therefore also the magnetic field presence. So that we proceed further with the zero field case and take the field into consideration when it will be necessary.

We have to calculate in general the series of two types corresponding to these numerators:

$$S_1^{(n)} = \frac{1}{\beta}\sum_{p_4}\frac{p_4 - A_0}{(\tilde{P}^2)^n}, \;\; S_2^{(n)} = \frac{1}{\beta}\sum_{p_4}\frac{q_4}{(\tilde{P}^2)^n}, n = 3, 4, 5. \tag{49}$$

These functions can be calculated from the $S_1^{(1)}$ and $S_2^{(1)}$ by computing a number of derivatives with respect to ϵ_p^2. The latter series result in simple expressions. First is the one calculated already for the tadpole diagram eq.(37). But now we have to change the sing $A_0 \to -A_0$.

$$S_1^{(1)} = \frac{1}{\beta}\sum_{p_4}\frac{p_4 - A_0}{\tilde{P}^2} = -\frac{1}{2}\frac{\sin(A_0\beta)}{\cos(A_0\beta) + \cosh(\epsilon_p\beta)}. \tag{50}$$

The function $S_2^{(1)}$ is

$$S_2^{(1)} = \frac{1}{\beta}\sum_{p_4}\frac{q_4}{\tilde{P}^2} = -\frac{q_4}{2\epsilon_p}\frac{\sinh(\epsilon_p\beta)}{\cos(A_0\beta)+\cosh(\epsilon_p\beta)}. \tag{51}$$

Hence, the explicit analytic expressions can be obtained for the integrand and the integration over d^3p is carried out in terms of known functions and their derivatives.

Let us adduce the expressions for A_i obtained after some simplifying algebraic transformations:

$$A_1 = -2\frac{(p_4 - A_0)q_4}{\tilde{P}^2}, \tag{52}$$

$$A_3 = -\frac{4}{\tilde{P}^2}\Big[(1-\frac{\epsilon_p^2}{\tilde{P}^2})k_4^1 k_4^3 + \frac{(\vec{p}\cdot\vec{k}_1)(\vec{p}\cdot\vec{k}_3)}{\tilde{P}^2}\Big], \tag{53}$$

$$A_4 = \frac{4}{\tilde{P}^2}\Big[(1-\frac{\epsilon_p^2}{\tilde{P}^2})((k_4^1)^2+(k_4^3)^2) + \frac{(\vec{p}\cdot\vec{k}_1)^2+(\vec{p}\cdot\vec{k}_3)^2}{\tilde{P}^2}\Big], \tag{54}$$

Accounting for the structure of the numerators in eq.(48), we see that the terms without $\tilde{P}_4$ are canceled in the sum of two diagrams and the resulting amplitude consists of the expressions

$$M_1 = 2\delta_{\mu\lambda}\frac{p_4 - A_0}{(\tilde{P}^2)^3}(1 + A_1 + A_3 + A_4) \tag{55}$$

and

$$M_2 = -4\delta_{\mu\lambda}\frac{(p_4 - A_0)q_4^2}{(\tilde{P}^2)^4}. \tag{56}$$

Thus, all the contributions of the $S_2^{(n)}$ series are canceled in the total.

Now we tern to d^3p integration. The expressions in eqs.(55), (56) contain different powers of $\tilde{P}^2$, and hence different powers of β appear even in the leading $p \to \infty$ approximation, which corresponds to the first term in the expansion $\epsilon_p = p + \frac{1}{2}\frac{m^2}{p} + O(p^{-3})$. Below, we carry out integration in this leading in $T \to \infty$ approximation.

We present our procedure considering the first term in eq.(55) which is calculated as the second derivative of $S_1^{(1)}$ over ϵ_p^2 and equaled to

$$S_3 = -A_0\beta\,\frac{(Sech(\beta\epsilon_p/2))^4}{64p^3}(-2\beta\epsilon_p + \beta\epsilon_p Cosh(\beta\epsilon_p) + Sinh(\beta\epsilon_p)). \tag{57}$$

Then in spherical coordinates we calculate the integral

$$I_3 = \int_{-\infty}^{\infty} d^3p\, S_3 = 4\pi\int_0^{\infty} p^2 dp\, S_3(p). \tag{58}$$

In leading order $\epsilon_p\beta = p\beta$. Making the change of variables $p\beta = y$ we obtain for eq.(58),

$$I_3 = -\frac{A_0\pi\beta}{16}\int_0^{\infty}\frac{dy}{y}(Sech(y/2))^4(-2y + yCosh(y) + Sinh(y)). \tag{59}$$

Note that this integral is convergent. Numeric integration in eq.(59) gives

$$I_3 = -A_0 \pi \beta \, (0.3348). \tag{60}$$

In such a way all the other integrations in eqs.(55), (56) can be carried out.

Performing analogous calculations for other terms we obtain the expression for scattering amplitude in high temperature approximation. A not complicated problem is to find next-to-leading corrections having the order $(m\beta)^l, l = 1, 2, ...$ As a result, the explicit high temperature limits for the scattering amplitude can be calculated in terms of elementary functions.

Now, we consider the case of non zero magnetic fields. As mentioned before, the expression eq.(57) independs of the magnetic field presence and does not change. In the lower level approximation $n = 0, \sigma = 1$ we have $\epsilon_p^2 = p_3^2 + m^2$. So, for large momenta $\epsilon_p = p_3(1 + \frac{1}{2}\frac{p_3}{m})$. The expression eq.(58) now has the form

$$I_3 = \int\limits_{-\infty}^{\infty} \frac{d^3p}{(2\pi)^3} \, S_3 = \int\limits_{-\infty}^{\infty} \frac{dp_3}{2\pi} \frac{gH}{(2\pi)^2} \, S_3(p_3). \tag{61}$$

The integrand function is even with respect to the change $p_3 \to -p_3$, so we can calculate two integrals in the limits $(0, \infty)$. Performing numeric integrations we obtain,

$$I_3(H) = gHA_0\beta^3 \, (0.0283). \tag{62}$$

Here, for magnetic field gH different variants can be substituted, as it is noted in previous sections. The most important conclusion is that in magnetized QGP various processes generated by these effective three-linear photon-photon-gluon vertexes have to happen. They should serve as the signals of the deconfinement phase transition. Since magnetic fields depend on temperature in the described above way, corresponding numeric estimates (and numbers) could be obtained for investigated processes.

Among interesting processes is the conversion of classical static gluon fields $\bar{\phi}^3(k), \bar{\phi}^8(k)$ (generated in the plasma due to the color charges $Q^3_{ind.}, Q^8_{ind.}$), in photons. This conversion formally looks as a super radiance in condense matter physics (because of static initial states). Due to the effective vertex $\Gamma^{\nu}_{\mu\lambda}(k^1, k^3)$, in the rest frame of the plasma two photons moving in opposite directions and having specific energies, which correspond to quantum or classical states, have to be radiated. In fact, one has to detect a classical photon flow corresponding to the decaying classical state.

One of the consequences of this effect has to be an increase of calculated in the literature direct infrared photons radiated from the QGP in heavy ion collisions. The increase acts to remove the deficit of direct low frequency photons compared to the experimental data. Independently of the process resulting in direct photons, there exists a cut in photon frequency which can be radiated from the plasma. The A_0 condensate lowers this frequency and in this way increases in theory the yield of direct low frequency photons. More details see, for instance, in [33] and references therein. The magnetic fields modify the vertex $\Gamma^{\nu}_{\mu\lambda}(k^1, k^3)$ that also influences the output of direct photons as well.

These processes could serve as the signals of the QGP creation.

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