CALCULATION OF DIELECTRIC CONSTANT CALIBRATION CHARACTERISTICS OF SENSORS USING THE METHOD OF MOMENTS

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This research is devoted to the issues of further development of methods of experimental measurement of dielectric characteristics of solid materials, liquids and liquid–gas media, which are foam structures. In recent decades, foam structures have gradually filled and supplanted traditional dielectric materials. There are still no scientific and metrology-based methods and devices for correct measurements of foam. In the paper, the method of moments (MoM) is employed for resonator measurements. The obtained results for a wide range of dielectric constants from 1 to 80 are interesting for determining the sensitivity of measurements, the influence of frequency measurement errors, and determining the number of measurement points.

Keywords: dielectric constant, calculation, microwave measurements, materials, media, foam structure, liquid foam, method of moments, microstrip antenna, patch antenna, wavegide, resonator.

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1. Introduction

Dielectric permittivity is a measure of how a substance can respond to a change in the electrical state of the substance by an external static or alternating electric field relative to a vacuum. This is a very important material value. For scientists, understanding dielectric constant is key in the fields of physics, materials science, electrical engineering, and other related fields. In manufacturing, knowledge of dielectric constant is essential for the development of new materials and technologies.

Currently, there are many books and publications for the development of methods and means of measuring the dielectric constant in waveguides, resonators and in open space. In recent decades, many scientific sources have been devoted to the study of dielectric constant using antennas, especially small ones [1, 2].

While the study of the dielectric constants of homogeneous dielectrics is provided both methodically and with a base of laboratory devices, the study of foamed dielectric structures is still only at the stage of experiments, development, and production methods. Many scientists join the new research topics on foamed dielectrics. For example, publications [3, 4] demonstrate the method of measuring and calculating the dielectric parameters of foam structures ε and $\mathrm{tg}\,\delta$ in waveguides.

In papers [5-7] homogeneous and foamed structures in resonators of various systems are investigated. Foam coatings were studied in open space in the millimeter range, and it was shown that foam structures have a dielectric constant close to unity, which is very important for the development of low-reflective coatings [9-13].

The publications [14-17] employ the method of moments (MoM), a technique rooted in transverse electromagnetic (TEM) mode analysis, to estimate the complex permittivity of liquids. This approach addresses an inverse problem by integrating the coaxial probe method, which enables the measurement of liquid dielectric constants across a wide frequency spectrum. The process involves comparing S11 measurement values with calculated values derived from electromagnetic (EM) analysis during periods of contact with an unknown material.

Competing with the method of moments, the finite element method is used to calculate changes in the electromagnetic properties of a material in several subdomains. The proposed method is computationally efficient and extremely fast, despite the use of a large number of samples and variations in both dielectric constant and dielectric loss tangent parameters [18].

In publications [19-22], wideband line-line dielectrometric methods are used for liquids, soils, and planar substrates to measure the complex permittivity of materials, primarily in coaxial elements.

Microwave sensors are widely used for complex permittivity characterization of materials [23, 24]. Their ability to non-destructively monitor specific properties of a material undergoing physical or chemical changes has led to many applications in science, industry, medicine, and pharmaceuticals.

The purpose of this research is to evaluate numerical modeling methods for use in practical tasks of building experimental measuring devices for resonator measurements.

2. Construction of a theoretical calibration dependence for a resonant sensor of dielectric permittivity of liquids

In many industries, the control of material parameters is an important task. Many liquids can be considered as lossy dielectrics. Resonator full filling methods are among the most accurate, but in the presence of significant losses, the resonance becomes uncertain, which leads to the need for resonator partial filling methods.

A classic approach to a partially filled resonator is a model based on a system of transcendental equations obtained based on cross-linking of field components at the boundaries of the interface. This method is used only for dielectrics with small losses and for a limited number of filling methods. With large losses in the dielectric, the character of the connection between the field components, characteristic of a certain mode, for which the system of equations is being compiled, changes.

The use of numerous methods of solving electrodynamic problems becomes a universal method for constructing calibration dependencies that determine the relationship between the dielectric permittivity of the sample and the resonant frequency of the cell. Examples of such methods are finite difference methods, mesh methods, and the integral equation method.

The construction of the calibration characteristic is reduced to the construction of the geometric model of the cell, the setting of the excitation ports, the calculation of the frequency characteristic of the reflection or transmission coefficient between the excitation ports. A table of connections between the set of values of the dielectric permittivity of the sample (designated *eps*) and the parameters of the resonance curve is constructed. Interpolation and approximation methods are used to determine dependencies for intermediate permeability values.

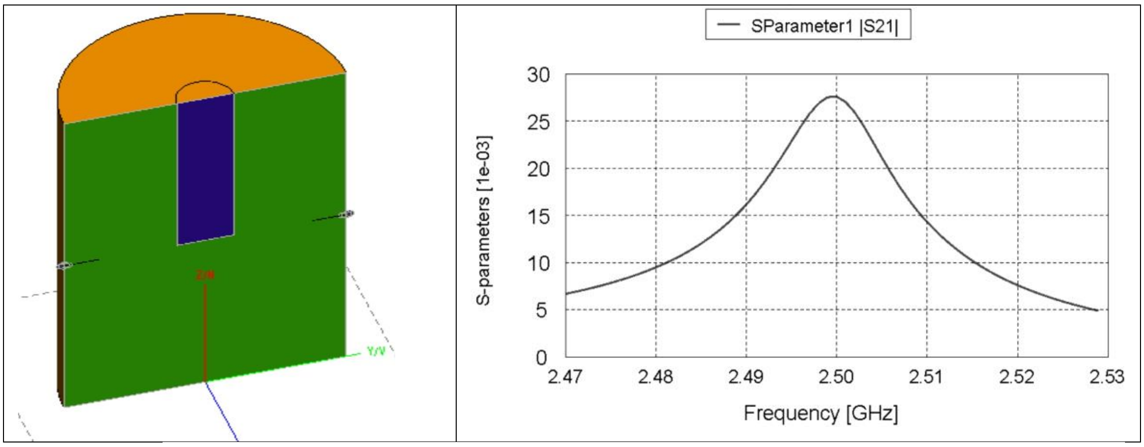
3. Cylindrical resonator filled with fluoroplastic with a cylindrical cavity filled with the liquid of test

Fig. 1a shows the cross-section of a cylindrical resonator filled with fluoroplastic, on the axis of which a cylindrical cavity is made, which is filled with the liquid under study. The H111 oscillation mode is used, which is excited by coaxial pins 6 mm long. A typical resonance curve of the transmission coefficient from the first pin to the second is shown in Fig. 1b, where S21 is the electromagnetic wave transmission coefficient.

For the main parameters of the resonance curve, we take the position of the maximum and the width of the resonance curve 2df at half the power level (–3 dB from the peak level).

The FEKO program environment was used to model the problem, in which the method of moments (MoM) was implemented. The determination of the resonant frequency of an empty and filled dielectric resonator is in good agreement with theory and practical measurements of reference liquids.

The following parameters were used: dimensions of the resonator: radius $R1 = 25$ mm, length $L1 = 80$ mm, full filling – fluoroplastic with $eps1 = 2.1$; dimensions of the cavity length $L2 = 40$mm, radius $R2 = 5$ mm, for all samples of the model, the tangent of the loss angle = 0.008. As a result of the simulation, the data presented in Table 1 were obtained.



a) **b)**

Fig. 1. a) cross-section of a cylindrical resonator, b) a typical resonance curve.

Table 1

Dependence between the dielectric permittivity (eps, which is related to experimental measurements) and the parameters of the resonance curve

	*L*2=30mm, *R*2=5 mm		*L*2=40 mm, *R*2=5 mm	
eps	f1	2df	f1	2df
1	2768	11	2784	11
10	2708	12	2677	13
20	2690	12	2652	13
30	2680	12	2638	11
40	2670	12	2627	12
50	2657	13	2612	14
60	2635	15	2592	14
70	2594	17	2558	15
80	2524	26	2499	22

To obtain the gauge dependence, we use interpolation polynomials, which are available in many mathematical packages. The calibration curve is shown below in Fig. 2.

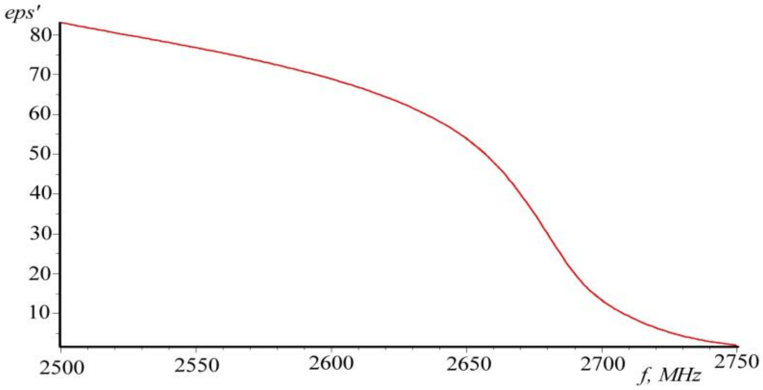


Fig. 2. Calibration dependence for finding dielectric permittivity *(eps'* **is obtained by calculations) when measuring the resonance frequency for *L*2=30mm, *R*2=5mm.**

4. Using of interpolation polynomials

Note that the modeling of the frequency distribution of the transmission coefficient operates at small discrete frequency points. Most of the modern measuring tools of the VNA (vector network analyzer) type also perform measurements on a certain frequency grid. Thus, to determine the exact position of the resonance, it is necessary to use interpolation polynomials or other models of the resonance curve (Fig. 3).

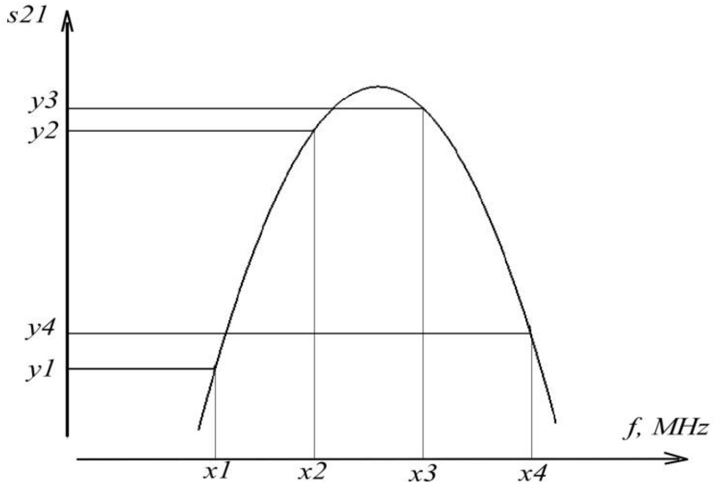


Fig. 3. Determination of the resonance position using discrete frequency and gain reference points.

Let's consider an example of searching for polynomial coefficients to determine the position of the resonance based on the results of measurements at four frequency points. For a given set of points, we use the interpolation polynomial

$$y = a_3 x^3 + a_2 x^2 + a_1 x + a_0 . \tag{1}$$

Then we compose a set of equations to determine its coefficients:

$$\left.\begin{aligned} a_3 x_1^3 + a_2 x_1^2 + a_1 x_1 + a_0 &= y_1 \\ a_3 x_2^3 + a_2 x_2^2 + a_1 x_2 + a_0 &= y_2 \\ a_3 x_3^3 + a_2 x_3^2 + a_1 x_3 + a_0 &= y_3 \\ a_3 x_4^3 + a_2 x_4^2 + a_1 x_4 + a_0 &= y_4 \end{aligned}\right\} \tag{2}$$

Here the unknowns are the coefficients of the polynomial. This set is transformed to the following form

$$\left.\begin{aligned} a_3\left(x_4^3 - x_3^3\right) + a_2\left(x_4^2 - x_3^2\right) + a_1\left(x_4 - x_3\right) = \left(y_4 - y_3\right) \\ a_3\left(x_3^3 - x_2^3\right) + a_2\left(x_3^2 - x_2^2\right) + a_1\left(x_3 - x_2\right) = \left(y_3 - y_2\right) \\ a_3\left(x_2^3 - x_1^3\right) + a_2\left(x_2^2 - x_1^2\right) + a_1\left(x_2 - x_1\right) = \left(y_2 - y_1\right) \end{aligned}\right\} \tag{3}$$

The system of equations (3) is solved by Cramer’s method, because of which we find

$$a_3 = \frac{\Delta_1}{\Delta},\ a_2 = \frac{\Delta_2}{\Delta},\ a_1 = \frac{\Delta_3}{\Delta},\ \Delta \neq 0, \tag{4}$$

$$a_0 = y_4 - a_3 x_4^3 - a_2 x_4^2 - a_1 x_4 . \tag{5}$$

Here Δ, Δ_i are the main and auxiliary determinants of the system (3).

The maximum of the curve will correspond to zero derivative of the polynomial (1):

$$3a_3 x^2 + 2a_2 x + a_1 = 0 . \tag{6}$$

The roots of Eq. (6) are defined as the roots of the quadratic equation

$$x_{\max,1} = \frac{-2a_2 + \sqrt{4a_2^2 - 12a_3 a_1}}{6a_3}, \qquad x_{\max,2} = \frac{-2a_2 - \sqrt{4a_2^2 - 12a_3 a_1}}{6a_3} \tag{7}$$

From these roots, the root located between the values of frequency x_2 and x_3 is selected.

$$x_2 < x_{\max} < x_3 .$$

These procedures are not demanding on computing power and can be implemented to build devices for measuring dielectric constant based on microcontrollers such as PIC, Arduino, Raspberry.

5. Conclusions

From the analysis of the results obtained because of numerical modeling, it is possible to determine the sensitivity of dimming, the addition of distortions in the dimming frequency, which can affect the generator, the transient attenuation of the signal, the number of points of dimming, and verify the value of the signal with theoretical models.

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