EFFECTIVE VERTEXES IN MAGNETIZED QUARK-GLUON PLASMA

V. Skalozub

Oles Honchar Dnipro National University, 49010 Dnipro, Ukraine
e-mail: Skalozub@dnu.edu.ua

In quark-gluon plasma (QGP), at high temperatures T the spontaneous generation of color magnetic fields, $b^3(T), b^8(T) \neq 0$ (3, 8 are color indexes), and usual magnetic field $b(T) \neq 0$ happens. Also, the Polyakov loop and related to it the $A_0(T)$ condensate, which is solution to Yang-Mills imaginary time equations, create.

Recently, with the new type two-loop effective potential, which generalizes the known integral representation for the Bernoulli polynomials and takes into consideration the magnetic background, these effects were derived. The corresponding effective potential $W(T, b^3, b^8, b, A_0)$ was calculated either in SU(2) gluodynamics or full quantum chromodynamics (QCD). The values of magnetic field strengths at different temperatures were calculated and the mechanism for stabilizing the background due to $A_0(T)$ was also discovered.

In present paper, we concentrate on the one-loop quark contributions. In particular, we derive the effective vertexes, which couple magnetic fields and A_0. The vertexes result in new specific effects signalling the creation of QGP in heavy ion collision experiments.

Keywords: spontaneous magnetization, high temperature, asymptotic freedom, effective potential, A_0 condensate, effective vertexes.

Received 07.12.2025; Received in revised form 22.12.2025; Accepted 23.12.2025

1. Introduction

Deconfinement phase transition (DPT), as well as the properties of QGP, are widely investigated for many years. Most results have been obtained in the lattice simulations because of the large coupling value $g \geq 1$ at the lower the phase transition temperature T_d. But at high temperatures due to asymptotic freedom the analytic methods are also reliable. They give a possibility for investigating various phenomena in the plasma. Among them is the creation of gauge field condensates described by the classical solutions to field equations without sources. Only such type fields could appear spontaneously inside the QGP. The well known ones are the so-called A_0 condensate, which is algebraically related to the Polyakov loop (PL) and the chromomagnetic fields $b^3 = gH^3, b^8 = gH^8$ (3, 8 are color indexes of SU(3) group) which are the Savvidy vacuum states at high temperature. These condensates result in numerous proper new effects which could be the signals of the QGP. The condensation of A_0 alone is investigated by different methods. For recent works see, for instance, [1] and references therein.

All the mentioned condensates are the consequences of asymptotic freedom and follow from the important property that asymptotic freedom at high temperature inevitably results in an infrared instability at low one. The field condensation prevents such type instability that results in the formation of the physical vacuum state. In quantum field theory (QFT), the magnetic and the A_0 condensates are generated at different orders in coupling constant (or the number of loops) for the effective potential (EP) $W(T, b, b^3, b^8, A_0)$. So that they have different temperature dependencies and play different roles in the QGP dynamics. For example, A_0 is generated at g^4 order in coupling constant and determined by the ratio of two- and one-loop contributions to $W(A_0)$. The fields $b(T), b^3(T), b^8(T)$ are generated in tree - plus one-loop - plus daisy approximation and also have the order g^2 in coupling constant. On the other hand, the contribution of A_0 at tree level equals zero because it is a constant electrostatic potential.

The fields investigated below are an important topic towards a theory of confinement. The A_0-background is relevant because at finite temperature such field cannot be gauged away and is intensively investigated beginning with [2]. In the early 90-ies, two-loop contributions were calculated in QCD and with these, the EP has non-trivial minimums and related condensate fields (see, for instance, [3], [4]). They form a hexagonal structure in the plane of the color components A_0^3 and A_0^8 of the background field.

A common generation of both fields was studied analytically in [15]. Here, new representation generalizing the known integral representation for the Bernoulli polynomials, was worked out, which admits introducing either A_0 or any b fields up to two-loop order. The magnetic fields considerably change the spectra of quarks and gluons as well. So, new phenomena have to be realized. The PL as well as $A_0(T)$ are the order parameters for the deconfinement phase transition. At low temperature they equal zero. At high temperature they become nonzero. The same concerns the spontaneously created magnetic fields.

The SU(3) gauge group can be presented as three SU(2) groups. So, the most results are relevant. In SU(2), the EP in the background R_ξ gauge reads [15]:

$$\begin{aligned} W_{gl}^{SU(2)} &= B_4(0,0) + 2B_4\left(a,b\right) \\ &+ 2g^2\left[B_2\left(a,b\right)^2 + 2B_2\left(0,b\right)B_2\left(a,b\right)\right] - 4g^2(1-\xi)B_3\left(a,b\right)B_1\left(a,b\right) \end{aligned} \tag{1}$$

with the notation

$$a = \frac{x}{2} = \frac{gA_0}{2\pi T}, \quad b = gH_3^3. \tag{2}$$

The chromomagnetic field is directed along third directions in coordinate and color spaces. Since we work at finite temperature, W_{gl} is equivalent to the free energy.

The functions $B_n(a,b)$ are defined by

$$\begin{aligned} B_4(a,b) &= T\sum_\ell \int \frac{dk_3}{2\pi}\frac{b}{4\pi^2}\sum_{n,\sigma} \ln\Big((2\pi T(\ell+a))^2 + k_3^2 + b(2n+1+\sigma-i0)\Big), \qquad (3) \\ B_3(a,b) &= T\sum_\ell \int \frac{dk_3}{2\pi}\frac{b}{4\pi^2}\sum_{n,\sigma} \frac{\ell+a}{(2\pi T(\ell+a))^2 + k_3^2 + b(2n+1+\sigma-i0)} \\ B_2(a,b) &= T\sum_\ell \int \frac{dk_3}{2\pi}\frac{b}{4\pi^2}\sum_{n,\sigma} \frac{1}{(2\pi T(\ell+a))^2 + k_3^2 + b(2n+1+\sigma-i0)}, \\ B_1(a,b) &= T\sum_\ell \int \frac{dk_3}{2\pi}\frac{b}{4\pi^2}\sum_{n,\sigma} \frac{\ell+a}{\Big((2\pi T(\ell+a))^2 + k_3^2 + b(2n+1+\sigma-i0)\Big)^2}. \end{aligned}$$

In eq.(1), ξ is gauge fixing parameter, the summations run $n = 0, 1, \ldots$, $\sigma = \pm 2$ and ℓ runs over all integers. The $' - i0'$-prescription defines the sign of the imaginary part for the tachyon mode. These formulas and eq.(1) are the generalization of corresponding two-loop expressions in [21], [23], [24] and also [4] for including a magnetic field. We note the relations

$$B_3(a,b) = \frac{1}{4\pi T}\partial_a B_4(a,b), \qquad B_1(a,b) = \frac{-1}{4\pi T}\partial_a B_2(a,b) \tag{4}$$

proper to the Bernoulli polynomials.

For $b = 0$ we have to replace $\frac{b}{4\pi^2}\sum_{n,\sigma} \to \int \frac{d^2k}{(2\pi)^2}$ and get

$$B_4(a,0) = \frac{2\pi^2 T^4}{3} B_4(a), \quad B_3(a,0) = \frac{2\pi T^3}{3} B_3(a), \tag{5}$$
$$B_2(a,0) = \frac{T^2}{2} B_2(a), \qquad B_1(a,0) = -\frac{T}{4\pi B_1(a)},$$

where $B_n(a)$ are the Bernoulli polynomials, periodically continued. The special values for $a = 0$ are

$$B_4(0,0) = -\frac{\pi^2 T^4}{45}, \quad B_3(0,0) = 0, \quad B_2(0,0) = \frac{T^2}{12}, \quad B_1(0,0) = \frac{T}{8\pi}. \tag{6}$$

These formulas hold for $T > 0$.

Note also that the function $B_4(a,b)$ describes the one loop contribution and the others give the two loop part of the EP. The motivation for the above choice of notations is that the functions $B_n(a,b)$, (3) are the corresponding mode sums without additional factors. More details about this representation as well as the renormalization and the case of $T = 0$ are given in [15]. Above expressions with whole $l = \pm1, \pm2, ...$ correspond to boson contributions. For fermions $l = \pm(1 + 1/2), \pm(2 + 1/2), ...$

2. One loop approximation for EP

As mentioned in the previous section, the spontaneous creation of A_0 field in the Matsubara imaginary time formalism happens in two-loop approximation presented in eq.(1). In one-loop order, only magnetic fields are generated. The latter part at finite temperature is described by the expression $B_4(a,b)$ which will be the main object below. In case of a number of fields it produces numerous specific interactions between them (due to vacuum fluctuations).

To explain the origin of this object we start with $b = 0$ case (see, for example, [3] which is used in what follows). In Appendixes A_1, A_2 of it the Feynman rules and the integral representations for the Bernoulli polynomials are placed. For us, the QCD condensates generated by fermion loops are needed: $c_1 = g(A_0^3 + A_0^8/\sqrt{3})/2$, $c_2 = g(-A_0^3 + A_0^8/\sqrt{3})/2, c_3 = g(-A_0^8/\sqrt{3})$. Here, 3 and 8 are internal SU(3) group color indexes.

The basic integral representation for $B_4(\frac{C\beta}{2\pi})$, $\beta = 1/T$ reads (we changed notation $k_i \to p_i$ to be in correspondence with [3]),

$$\int dp \ln p_c^2 = \frac{2\pi^2}{3\beta^4} B_4(\frac{C\beta}{2\pi}). \tag{7}$$

Here, $\int dp = \frac{1}{\beta}\sum_{p_0}\int \frac{d^3p}{(2\pi)^3}$, and $p_c^2 = (p_0 + c)^2 + \vec{p}^2$. Summation over p_0 runs from minus to plus infinity with the values $p_0 = \frac{2\pi l}{\beta}$ for bosons, $p_0 = \frac{2\pi(l+1/2)}{\beta}$ for fermions and the value of c = c_i.

To obtain formulas of eq.(3) we have to make the substitutions in the integral representation for $B_4(c_i, \vec{p})$: $\int dp \to \int \frac{dp_3}{(2\pi)}\frac{gH}{(2\pi)^2}\sum_{(n,\sigma)}$, where $n = 0, 1, 2, ...$ is a Landau level number and $\sigma = \pm1$ is spin number. This is in accordance with energy spectrum of charged spin $1/2$ particle in constant magnetic field: $\epsilon_p^2 = p_3^2 + gH((2n+1) - \sigma) + m^2$. p_3 is momentum along field direction $H_3 = const$. Here the kind of magnetic field is insufficient. We assume that all the generated magnetic fields are directed along third axis in coordinate space. As it is occurred, for parallel fields the minimum of the effective potential is lower. In the ground state $n = 0, \sigma = 1$, and the particle energy is $\epsilon_p^2 = p_3^2 + m^2$. This is so called Low landau level (LLL) approximation. In strong fields it significantly simplifies calculations and gives very good results. Therefore, it will be used in what follows.

Our next steps are the following. First, we calculate the derivative with respect to ϵ_p^2 of the $B_4(c_i, \epsilon_p^2)$. Second, we sum up the series over p_0. Then we integrate over ϵ_p^2 and obtain the potential of the effective produced interactions between magnetic and A_0 fields. This nontrivial procedure results in the new type effective vertexes for high temperature QGP. Finally, we summarize our results and discuss prospects for future researches.

3. Calculation series over p_0

Now, we calculate the temperature sum over p_0 proceeding in two steps. First we calculate derivative of the $B_4(c_i, \epsilon_p^2)$ eq.(3) with respect to ϵ_p^2 which in a magnetic field background is realized by substituting $\vec{p}^2 \rightarrow \epsilon_p^2$. Remind that $\epsilon_p^2 = p_3^2 + gH((2n+1) - \sigma)$ and in LLL approximation $n = 0, \sigma = 1, \epsilon_p^2 = p_3^2 + m_q^2 = \epsilon_{pL}^2, m_q$ is quark mass. Below we write the parameter ϵ_p^2 for all cases, where it is clear.

We write for the left hand side of $B_4(c_i, \epsilon_p^2) = I_0$ and obtain

$$I_1 = \frac{dI_0}{d\epsilon_p^2} = \frac{1}{\beta} \sum_{p_0} \int \frac{dp_3}{(2\pi)} \frac{gH}{(2\pi)^2} \sum_{(n,\sigma)} \frac{1}{(p_0 + C)^2 + \epsilon_p^2}. \tag{8}$$

Here, C stands for one of c_i written above and p_0 corresponds to fermions. To calculate series over p_0 we use the standard integral representation for fermions

$$\frac{1}{\beta} \sum_{p_0} f(p_0) = -\frac{1}{2} \sum_{j} Res[f(\omega) \tan(\frac{\beta\omega}{2})], \tag{9}$$

where summation is over poles ω_j.

In our case, the poles are, $\omega_1 = -C + i\epsilon_p,\ \ \omega_2 = -C - i\epsilon_p$. By using these values we obtain the result

$$I_2 = \frac{1}{2\epsilon_p} (\frac{\sinh(\epsilon_p \beta)}{\cos(C\beta) + \cosh(\epsilon_p \beta)} - 1). \tag{10}$$

In this expression we have subtracted 1 to separate a zero temperature contribution. In fact, in this expression $\epsilon_p = (p_3^2 + m^2)^{1/2}$ because we turn to the LLL approximation and not calculated complete series over n, σ.

4. Conclusions

As final step, we integrate over ϵ_p^2 as a parameter. The result is

$$I_{2F} = -\epsilon_p + \frac{1}{\beta} \ln[\cosh(\epsilon_p \beta) + \cos(C\beta)]. \tag{11}$$

This expression has to be inserted in the integral over dp_3 eq. (8). We obtain,

$$V_{H,A_0} = \frac{gH}{(2\pi)^2} \int \frac{dp_3}{(2\pi)} (-\epsilon_p + \frac{1}{\beta} \ln[\cosh(\epsilon_p \beta) + \cos(C\beta)]), \tag{12}$$

where $C = c_i$ written in the second section. It describes effective interactions of different kind magnetic fields H^3, H^8 and usual magnetic field H with A_0 condensates in high temperature QGP.

Derived potential generates new type effective vertexes and produces new effects in heavy ion collisions. Investigation of them will be reported in other place.

References

1. Fei Gao, Joannis Papavassiliou, and Jan M. Pawlowski. Fully coupled functional equations for the quark sector of QCD. *Phys. Rev. D*, 103:094013, 2021.
2. Nathan Weiss. Wilson line in finite-temperature gauge theories. *Phys. Rev. D*, 25:2667–2672, 1982.
3. V.V. Skalozub and I.V. Chub. 2-Loop Contribution of Quarks to the Condensate of the Gluon Field at Finite Temperatures. *Physics of Atomic Nuclei*, 57:324, 1994.
4. Vladimir Skalozub. A_0 condensation, Nielsen's identity and effective potential of order parameter . *Physics of Elementary Particles and Atomic Nuclei. Letters*, 18(7):738–745, 2021.
5. Vladimir Skalozub and Michael Bordag. Color ferromagnetic vacuum state at finite temperature. *Nucl. Phys. B*, 576:430–444, 2000.
6. V. Demchik, V. Skalozub. Spontaneous magnetization of a vacuum in the hot Universe and intergalactic magnetic fields. *Physics of Elementary Particles and Atomic Nuclei*, 46(1): 5 – 47, 2015.
7. V. V. Skalozub. Condensation of gauge fields at high temperature Dnipro, Lira Ltd. 2021, 156 p. ISBN 978-966-981-581-1
8. V. I. Demchik, N. V. Kolomoyets, and V. V. Skalozub. Spatial structure of the Polyakov loop in external chromomagnetic field in lattice SU(2) gluodynamics. *Visnyk Dnipropetrovskogo universytetu. Fizyka. Radioelectronika*, 21(2):13–17, 2013.
9. V. Demchik and V. Skalozub. Spontaneous creation of chromomagnetic field and A_0-condensate at high temperature on a lattice. *J. Phys.*, A41:164051, 2008.
10. V. Skalozub. Spontaneous magnetization of a vacuum in high temperature gluodynamics (two-loop approximation). *J. Phys. Electon.* 30(2), 3 – 9, 2022. ArXiV: 2305. 00757 17 May v 2 2023 [hep-ph].
11. P. Cea, L. Cosmai. Probing the nonperturbative dynamics of SU(2) vacuum. *Phys. Rev. D* 60, 094506, 1999.
12. P. Cea, L. Cosmai. The lattice Schrodinger functional and the background field effective action. *Phys. Lett.* B392, 177 –181, 1997.
13. V. Skalozub, P. Minaiev. Magnetized quark-gluon plasma at the LHC. *Phys. Part. Nucl. Letts* 15(6), 568 – 575, 2018.
14. V. Skalozub, A. Turinov. Induced color charge and quark propagation at Polyakov's loop background. *J. Phys. Electon.* 30(1), 3 – 10, 2022.
15. M. Bordag, V. Skalozub. Effective potential of gluodynamics in background of Polyakov loop and colormagnetic field. *Eur. Phys. J. C*, 82:390, 2022.
16. G. K. Savvidy. Infrared Instability of the Vacuum State of Gauge Theories and Asymptotic Freedom. *Phys. Lett. B*, 71:133, 1977.
17. N. K. Nielsen and P. Olesen. Unstable Yang-Mills Field Mode. *Nucl. Phys. B*, 144(2-3):376–396, 1978.
18. V. V. Skalozub. Restoration of Spontaneously Broken Symmetry in a Magnetic-Field.

Soviet Journal of Nuclear Physics-USSR, 28(1):113–115, 1978.
19. A. O. Starinets, A. S. Vshivtsev, and V. C. Zhukovskii. Color Ferromagnetic State in SU (2) Gauge-Theory at Finite Temperature. *Phys. Lett. B*, 322(4):403–412, FEB 24, 1994.
20. Peter N. Meisinger and Michael C. Ogilvie. The Finite temperature SU(2) Savvidy model with a nontrivial Polyakov loop. *Phys. Rev. D*, 66:105006, 2002.
21. K. Enqvist and K. Kajantie. Hot gluon matter in a constant A_0-background. *Zeitschrift für Physik C Particles and Fields*, 47(2):291–295, 1990.
22. V. M. Belyaev and V. L. Eletsky. Two-loop free energy for finite temperature SU(3) gauge theory in a constant external field. *Zeitschrift für Physik C Particles and Fields*, 45(3):355 – 359, 1990.
23. V. M. Belyaev. Order parameter and effective potential. *Physics Letters B*, 254(1):153 – 157, 1991.
24. V. V. Skalozub. Gauge Independence of the Hot A_0-condensate. *Mod. Phys. Lett. A*, 7(31):2895–2903, 1992.
25. V. V. Skalozub. Nielsen's identity and gluon condensation at finite temperature *Phys. Rev. D*, 50:1150–1156, 1994.
26. V. V. Skalozub. Gauge invariance of the gluon field condensation phenomenon in finite tempwrature QCD *Int.J. Mod. phys. A*, 9:4747-4758, 1994.
27. R. Kobes, G. Kunstatter and A. Rebhann. Gauge dependence identities and their applications at finite temperature Nucl. Phys. B, 355(1):1-37, 1991.
28. I. S. Gradshteyn and I. M. Ryzhik. *Table of Integrals, Series and Products*. Academic Press, New York, 2007.
29. Milton Abramowitz and Irene A. Stegan. *Handbook of mathematical funktions* National bureau of standards applied mathematics series - 55, Issued June 1964.
30. M. Ninomiya and N. Sakai. Finite temperature behavior of a color ferromagnetic state in QCD. *Nuclear Physics B*, 190(2):316–324, 1981.
31. D. Ebert, V. Ch. Zhukovsky, and A. S. Vshivtsev. Thermodynamic potential with condensate fields in an SU(2) model of QCD. *Int. J. Mod. Phys.*, A13:1723–1742, 1998.
32. M. Bordag and V. Skalozub. A_0–condensation in quark-gluon plasma with finite baryon density. *Eur. Phys. J. C*, 81:998, 2021. ArXiv 2009.11734.
33. M. Bordag and V. Skalozub. Photon dispersion relations in A_0-background *Eur. Phys. J. Plus*, 134:289, 2019.