ELECTROWEAK PHASE TRANSITION WITH MAGNETIZATION

P. Minaiev

Oles Honchar Dnipro National University, Dnipro, Ukraine

*e-mail: minaevp9595@gmail.com

We examine the theoretical and phenomenological properties of two-Higgs-doublet extensions of the Standard Model. The lightest neutral scalar state of the 2HDM is identified with the observed Standard Model Higgs boson, having a mass of $m_h = 125.26\,\mathrm{GeV}$.

The parameter space of this framework is extensive and accommodates a wide range of potential New Physics scenarios. At the same time, it is strongly constrained by both theoretical considerations and experimental data. In particular, models with an extended scalar sector must satisfy the Sakharov conditions in order to account for the observed baryon asymmetry of the Universe.

In this study, we investigate whether the Sakharov conditions can be realized within the 2HDM. Our focus is on identifying regions of the parameter space that allow for a first-order electroweak phase transition. Since the Standard Model itself fails to meet the Sakharov criteria, resolving this issue remains one of the key open problems in particle physics. Consequently, the search for viable extensions of the Standard Model that fulfill these conditions continues to be a timely and important direction of research.

Keywords: EWPT, 2HDM, symmetries, Lie groups, non-Abelian interactions

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1. Introduction

The Standard model explains the physics of high energy particles extremely good. It satisfies a lot of modern experimental data. The physics beyond Standard model is one of the modern steps of understanding the nature. One of unsolved problems of Standard model is baryogenesis. In this theory with mass of Higgs boson more 75 GeV the Sakharov conditions [1] are violated.

There are many reasons to think that the Two Higgs doublet model is a good extension of Standard model with the extra doublet of scalar fields. In the present paper, we show the way of simulations and finding the parametric space of the 2HDM where we can satisfy the Sakharov's conditions. We use a high temperature effective potential for the scalar sector of the model with spontaneous generation of magnetic fields [4, 5]. We are interested in the role of magnetic fields in evolution of early Univers and in processes near critical temperature for EWPT.

2. Effective potential of 2HDM

We consider a general CP-conserving 2HDM with a softly broken Z_2 symmetry, described by the following potential [2]:

$$
\begin{aligned}
V_{Higgs} = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right) + \\
& + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 + \\
& + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \frac{\lambda_5}{2}\left(\left(\Phi_1^\dagger \Phi_2\right)^2 + \left(\Phi_2^\dagger \Phi_1\right)^2\right), \qquad (1)
\end{aligned}
$$

where the term proportional to m_{12}^2 breaks the Z_2 symmetry. Φ_1 and Φ_2 are the pair of doublets of the scalar fields:

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix}, \tag{2}$$

with the vacuum expectation values $v_{1,2}$ and $v^{SM} = \sqrt{v_1^2 + v_2^2} = 246.22$ GeV.

$$< \Phi_1 >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, < \Phi_2 >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \tag{3}$$

We calculate parameters m_{11} and m_{22} from equations for stability of vacuum:

$$\begin{aligned} 0 &= \frac{\partial V_{Higgs}}{\partial \Phi_1} = 2m_{11}^2 v_1 - 2m_{12}^2 v_2 + \lambda_1 v_1^3 + \lambda_3 v_2^2 v_1 + \lambda_4 v_1 v_2^2 + \lambda_5 v_1 v_2^2 \\ 0 &= \frac{\partial V_{Higgs}}{\partial \Phi_2} = 2m_{22}^2 v_2 - 2m_{12}^2 v_1 + \lambda_2 v_2^3 + \lambda_3 v_1^2 v_2 + \lambda_4 v_1^2 v_2 + \lambda_5 v_1^2 v_2 \end{aligned} \tag{4}$$

We use usual notation for the mixing angles:

$$tan\beta = \frac{v_2}{v_1}, \quad tan2\alpha = \frac{2(-m_{12}^2 + v_1 v_2(\lambda_3 + \lambda_4 + \lambda_5))}{m_{12}^2(v_2/v_1 - v_1/v_2) + \lambda_1 v_1^2 - \lambda_2 v_2^2}. \tag{5}$$

And as a result we can write masses of scalar particles:

$$\begin{aligned} m_h^2 &= M^2 cos^2(\alpha - \beta) + v^2 \left(\lambda_1 sin^2\alpha cos^2\beta + \lambda_2 cos^2\alpha sin^2\beta - \frac{\lambda_3 + \lambda_4 + \lambda_5}{2} sin2\alpha sin2\beta \right); \\ m_H^2 &= M^2 sin^2(\alpha - \beta) + v^2 \left(\lambda_1 cos^2\alpha cos^2\beta + \lambda_2 sin^2\alpha sin^2\beta + \frac{\lambda_3 + \lambda_4 + \lambda_5}{2} sin2\alpha sin2\beta \right); \\ m_A^2 &= M^2 - \lambda_5 v^2; \\ m_{H^\pm}^2 &= M^2 - \frac{\lambda_4 + \lambda_5}{2} v^2, \end{aligned} \tag{6}$$

where $M^2 = m_{12}^2/(sin\beta cos\beta)$ and $v^2 = v_1^2 + v_2^2$.

To include finite temperature part and interaction with magnetic fields to full potential we should add one loop part [3]. For one charged scalar degree of freedom with mass m and charge e we have general expression:

$$V = -\frac{1}{16\pi^2} \sum_{-\infty}^{\infty} \int_0^{\infty} \frac{ds}{s^3} e^{-(m^2 s + n^2/4sT^2)} \frac{eHs}{sh(eHs)}, \tag{7}$$

where H - magnetic fields strength and T- temperature.

For our calculations we present the next notions:

1) For charged scalar degree of freedom:

$$V^{Charged}(m,H,T) = V_0^{Charged}(m,H) + V_T^{Charged}(m,T,H),$$
$$V_0^{Charged}(m,H) = -\frac{1}{16\pi^2}\int\limits_0^\infty \frac{ds}{s^3} e^{-m^2 s}\frac{eHs}{sh(eHs)},$$
$$V_T^{Charged}(m,T,H) = -\frac{1}{8\pi^2}\sum_{n=1}^{\infty}\int\limits_0^\infty \frac{ds}{s^3} e^{-(m^2 s + n^2/4sT^2)}\frac{eHs}{sh(eHs)} \tag{8}$$

2) For neutral scalar degree of freedom:

$$V^{Neutral}(m,T) = V_0^{Neutral}(m) + V_T^{Neutral}(m,T),$$
$$V_0^{Neutral}(m) = -\frac{1}{16\pi^2}\int\limits_0^\infty \frac{ds}{s^3} e^{-m^2 s},$$
$$V_T^{Neutral}(m,T) = -\frac{1}{8\pi^2}\sum_{n=1}^{\infty}\int\limits_0^\infty \frac{ds}{s^3} e^{-(m^2 s + n^2/4sT^2)}. \tag{9}$$

For temperature part we have the next physical conditions:

$$V^{Charged}(m,H=0,T) = V^{Neutral}(m,T)$$
$$V_T^{Charged}(m,H,T=0) = V_T^{Neutral}(m,T=0) = 0 \tag{10}$$

To use formulas above we make Mellin's transformations and use definition of K-function (11) to find high temperature asymptotic(see Appendix A) and for $n=0$ we have one loop contribution, which should be renormalized with adding counter terms to final potential.

$$\int\limits_0^\infty dx x^{n-1}\,\mathrm{exp}\left(-ax-\frac{b}{x}\right) = 2\left(\frac{b}{a}\right)^{n/2} K_n(2\sqrt{ab}) \tag{11}$$

To make research about electroweak transition we use full potential as function of temperature, magnetic fields and the vacuum expectation values. The potential (1) we write in terms of free parameters - λ_i, v_1, v_2 and m_{12}. We are interested in behavior of potential as dependence on increasing the temperature.

As a results we have the following expression:

$$V(T,H,v_1,v_2) = \frac{H^2}{2} + V_{Higss}(v_1,v_2) + \sum_{A,h,H} V_T^{Neutral}(m,T) + \sum_{H^\pm} V_{HT}^{Charged}(m,H,T) +$$
$$+ \sum_{particles} V_{CT}(v_1,v_2,T,H), \tag{12}$$

where $V_{CT}(v_1, v_2, T, H)$ is the corresponding counter terms.The location of the minimum is the same as for the tree level potential. The tree level masses are preserved at $H = 0$ and $T = 0$.

For calculations we use the following conditions:

$$\left.\frac{\partial V}{\partial H^2}\right|_{H=0,T=0,\phi_i=v_i} = \frac{1}{2};$$
$$\left.\frac{\partial V}{\partial \phi_i}\right|_{H=0,T=0,\phi_i=v_i} = 0;$$
$$\left.\frac{\partial^2 V}{\partial \phi_i \partial \phi_j}\right|_{H=0,T=0,\phi_i=v_i} = \left.\frac{\partial^2 V_{Higgs}}{\partial \phi_i \partial \phi_j}\right|_{H=0,T=0,\phi_i=v_i}. \tag{13}$$

This potential we should use to find parameters where the Sakharov's conditions are satisfied.

3. Discussion

The behavior of the electroweak phase transition (EWPT) in the presence of magnetic fields has been discussed. The role of magnetic fields in satisfying Sakharov's conditions within the framework of the CP-conserving two-Higgs-doublet model (2HDM) remains an open question. Algorithms for simulating points in the parameter space have been developed.

The main objective of this research is to identify regions of the parameter space of various CP-conserving 2HDMs that are not excluded by experimental data and that exhibit a strongly first-order electroweak phase transition, with the magnetic field included as a parameter. If such points corresponding to a first-order phase transition are found, the associated models may serve as viable candidates for explaining the observed matter-antimatter asymmetry of the Universe.

This search has been carried out by studying the minimum of the effective potential as a function of temperature and magnetic field strength for a wide range of model parameters.

We conclude that in the scenario where the Higgs boson discovered at the LHC is the lightest scalar in the 2HDM, the model can accommodate a strongly first-order electroweak phase transition.

4. Appendix A

In this appendix we show the exact calculation of Mellin's transformations for sums of K-function in finite temperature part of potential.

$$V_T^{Neutral}(m,T) = -\frac{1}{8\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{ds}{s^3}e^{-(m^2 s + n^2\beta^2/4s)} = -\frac{1}{\pi^2}m^2T^2\sum_{n=1}^{\infty}\frac{K_2(mn/T)}{n^2} \tag{14}$$

$$V_T^{Charged}(m,T,H) = -\frac{1}{8\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{ds}{s^3}e^{-(m^2s+n^2/4sT^2)}\frac{eHs}{sh(eHs)} = -\frac{1}{\pi^2}m^2T^2\sum_{n=1}^{\infty}\frac{K_2(mn/T)}{n^2}+$$
$$+\frac{1}{8\pi^2}\frac{(eH)^2}{3}\sum_{n=1}^{\infty}K_0(mn\beta) - \frac{1}{16\pi^2}\frac{7(eH)^4}{360}\frac{1}{m^4}\sum_{n=1}^{\infty}(m\beta)^2n^2K_2(mn\beta) \quad (15)$$

$$\sum_{n=1}^{\infty}\frac{K_2(n\omega)}{n^2} = \frac{\pi^4}{45\omega^2} - \frac{\pi^2}{12} + \frac{\pi\omega}{6} - \frac{\omega^2}{16}(\frac{3}{4} - \gamma + Log(\frac{4\pi}{\omega})) + \sum_{k=1}^{\infty}\frac{\omega^{2k}\zeta'(2-2k)}{2^{2k}(k-1)!(k+1)!} \quad (16)$$

$$\sum_{n=1}^{\infty}K_0(n\omega) = \frac{\pi}{2\omega} + \frac{\gamma - Log(\frac{4\pi}{\omega})}{2} + \sum_{n=1}^{\infty}\frac{\omega^{2n}\zeta'(-2n)}{2^{2n}(n!)^2} \quad (17)$$

$$\sum_{n=1}^{\infty}(\omega n)^2K_2(n\omega) = \frac{3\pi}{2\omega} - 1 + \frac{1}{8}\omega^4\zeta'(-4) \quad (18)$$

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