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INDUCED CHARGES AND QUARK POLARIZATION OPERATOR AT A_0 BACKGROUND

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The presence of the A_0 condensate in quark-gluon plasma leads to $Z(3)$ symmetry breaking and Furry's theorem violation. Due to this phenomenon the color charges are induced and other even-number diagrams are allowed. We calculate the first diagram of this series – tadpole diagram with one gluon line and find induced color charges in the plasma. Next, quark polarization operator in A_0 presence is calculated and partially investigated.

Keywords: quark-gluon plasma, A_0-condensate, Furry's theorem, induced color charge

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1. Introduction

The quark-gluon plasma is a new object in nature, which consists of quarks and gluons in the state of asymptotic freedom. Plasma exists at high temperatures and densities and is very unstable. Due to a number of unique processes, quark-gluon plasma is the main object of research nowadays. Despite such high interest, many phenomena are not investigated.

At low energies the interaction between quarks is so strong that they form a bound state. However, due to asymptotic freedom at high energies, the effective coupling becomes weak. Quarks are deconfined and form the quark-gluon plasma state. This is so-called deconfinement phase transition (DPT).

There is a number of processes absent in vacuum through the Furry theorem. This theorem states that Feynman diagrams with odd number of external photonic lines mutually cancel (and thus give a zero total contribution to any process amplitude). This is valid for vacuum, but in the presence of plasma, C-parity is violated and the theorem ceases to be satisfied.

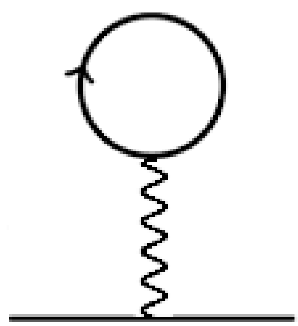

Fig. 1. Tadpole diagram.

The generally recognized $SU(3)$ global symmetry violation mechanism at high temperatures is the appearance of the so-called A_0 condensate. This leads to the appearance of the chromomagnetic mass and color charges.

The goal of this paper is to consider one of new type processes, so-called "tadpole" diagram, and to calculate the induced color charges and its corrections to quark polarization operator.

2. A_0 condensate

Quarks interact with electromagnetic field and gluons according to the form

$$L = \bar{\psi}^a \left[\gamma_\mu \left(\partial_\mu \delta^{ab} - ie_f A_\mu \delta^{ab} - ig \left(Q_\mu \frac{\lambda}{2} \right)^{ab} \right) - m_f \delta^{ab} \right] \psi^b \qquad (1)$$

where A_μ is the electromagnetic field potential, Q_μ is the gluon field potential, e_f is the electric charge of the quark with flavor f, m_f is the quark mass, **g** is charge of strong interaction, a, b are color indices.

A_0 condensate is a solution to field equations giving minimum to the effective potential for the gluon fields at high temperatures. It belongs to the center of the gauge group $A_0 = \frac{2\pi n}{\beta g N}, n \in Z_N, n = 0, 1, \dots N-1$.

Actually, the value of the A_0 has to be calculated from the full effective action with quantum corrections taken into account. Difference between $\langle A_0 \rangle$ and $2\pi n / \beta g N$ points is due to a to spontaneous gauge symmetry violation [1].

The QCD Lagrangian in the relativistic background gauge has the form

$$\begin{aligned}
L = {} & \frac{1}{4}(G^a_{\mu\nu})^2 + \frac{1}{2\xi}(D^B_\mu Q^a_\mu)^2 + \bar{\chi} D^B_\mu D_\mu \chi + \bar{\psi}^a(\gamma_\mu + \mathrm{i}m)\psi^a \\
& + \mathrm{i}g\bar{\psi}^a \gamma_\mu (A^c_\mu + Q^c_\mu)(t^c)^a_b \psi^b, \\
G^a_{\mu\nu} = {} & (D^B_\mu)^{ab} Q^b_\nu - (D^B_\nu)^{ab} Q^b_\mu - g f^{abc} Q^b_\mu Q^c_\nu, \\
(D^B_\mu)^{ab} = {} & \delta^{ab}\partial_\mu + g f^{abc} A^c_\mu, \\
(D_\mu)^{ab} = {} & \delta^{ab}\partial_\mu + g f^{abc}(Q^c_\mu + A^c_\mu), \\
A^c_\mu = {} & \delta_{\mu 0}(\delta^{c3} A^3_0 + \delta^{c8} A^8_0),
\end{aligned} \qquad (2)$$

where $(t^c)^a_b$ are $SU(3)$ generators, Q^a_μ is a quantized field, f^{abc} are structure constants, $\chi, \bar{\chi}$ are ghost fields.

For further consideration it is convenient to introduce charged basis of the gluon field

$$\begin{aligned}
& \pi^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm \pi \mathrm{i} A^2_\mu), \qquad \pi^0_\mu = A^3_\mu, \qquad \eta_\mu = A^8_\mu, \\
& K^\pm_\mu = \frac{1}{\sqrt{2}}(A^4_\mu \pm \pi \mathrm{i} A^5_\mu), \qquad \bar{K}^\pm_\mu = \frac{1}{\sqrt{2}}(A^6_\mu \pm \pi \mathrm{i} A^7_\mu).
\end{aligned} \qquad (3)$$

The basis splits into 3 subgroups $\pi^\pm_\mu, \pi^0_\mu$, $K^\pm_\mu, \eta_\mu$ and $\bar{K}^\pm_\mu, \eta_\mu$ which are important for what follows.

In this basis, in the Lagrangian momentum space the background fields A_0^3 and A_0^8 are included as constant shifts.

3. Induced color charge

In this section, we calculate the induced color charge generated by a tadpole diagram. In charged basis, we have two components of the induced charge for the shifts A_0^3 and A_0^8.

First we calculate the contribution for the case $(A_0)_\mu^a = A_0\delta_{\mu 4}\delta^{a3}$

$$Q_{ind}^3 = 4gA_0\sum_{p_4}\int\frac{d^3p}{(2\pi)^3}Tr\left[\frac{\lambda^3}{2}\gamma_4\frac{\hat{p}_\sigma\gamma_\sigma + m}{\hat{p}^2+m^2}\right], \tag{4}$$

where $\hat{p} = (p_4 = p_4 \pm A_0, \mathbf{p}), p_4 = 2\pi T(l+1/2), l = 0, \pm 1, \ldots$

When calculating the trace, two terms appear: the first with $p_4 + A_0$, and the second with $p_4 - A_0$, due to the structure of $SU(3)$ generators. The result is

$$Q_{ind}^3 = -2\int\frac{d^3p}{(2\pi)^3}\sum_{p_4}\left[\frac{(p_4+A_0)}{(p_4+A_0)^2+\varepsilon_{\mathbf{p}}^2}+\frac{(p_4-A_0)}{(p_4-A_0)^2+\varepsilon_{\mathbf{p}}^2}\right]. \tag{5}$$

where $\varepsilon_{\mathbf{p}}^2 = \mathbf{p}^2 + m^2$. Also, we make the replacement $p_4 \to -p_4$ in the second sum

$$Q_{ind}^3 = -4\int\frac{d^3p}{(2\pi)^3}\sum_{p_4}\frac{(p_4+A_0)}{(p_4+A_0)^2+\varepsilon_{\mathbf{p}}^2}. \tag{6}$$

To calculate the sum we use the following representation [4]

$$Q_{ind}^3 = \int\frac{d^3p}{(2\pi)^3}\frac{\beta}{\pi\mathrm{i}}\oint_C \tan\frac{\beta\omega}{2}\frac{(\omega+A_0)}{(\omega+A_0)^2+\varepsilon_{\mathbf{p}}^2}d\omega. \tag{7}$$

The integrand function has two purely imaginary poles of the first order. We use residues to find its value

$$Q_{ind}^3 = 2\beta\int\frac{d^3p}{(2\pi)^3}\sum_{j=1}^{2}Res\left[\tan\frac{\beta\omega}{2}\frac{(\omega+A_0)}{(\omega+A_0)^2+\varepsilon_{\mathbf{p}}^2},\omega_j\right]. \tag{8}$$

The result is

$$Q_{ind}^3 = -\int\frac{d^3p}{(2\pi)^3}\frac{2\beta\sin\beta A_0}{\cos\beta A_0+\cosh\beta\varepsilon_{\mathbf{p}}}. \tag{9}$$

In what follows, we calculate the integral in the high-temperature limit $T\to\infty$ $(\beta\to 0)$. In this case $|\mathbf{p}| \gg m$, so we use

$$\varepsilon_{\mathbf{p}} = \sqrt{\mathbf{p}^2+m^2} \approx |\mathbf{p}|. \tag{10}$$

In this case, since A_0^3 has a finite value, it is suppressed by $\beta \to 0$. In this case, large momenta give significant contribution. Consequently, the integral gets the following form

$$Q_{ind}^3 = -2\beta \int \frac{d^3 p}{(2\pi)^3} \frac{1}{1+\cosh \beta |\mathbf{p}|}. \tag{11}$$

When passing to spherical coordinates, we change the lower limit of integration for p from 0 to m to eliminate the divergence at 0. This does not significantly affects the integral value,

$$Q_{ind}^3 = -\frac{\beta}{\pi^2} \int_m^\infty \frac{p^2 dp}{1+\cosh \beta p}. \tag{12}$$

The result is

$$Q_{ind}^3 = -\frac{2}{\beta^2} \left\{ Li_2(e^{-m\beta}) m\beta \left[-m\beta - 4\ln(1+e^{-m\beta}) + m\beta \tanh \frac{m\beta}{2} \right] \right\} \tag{13}$$

where

$$Li_s(z) = \sum_{k=0}^{\infty} \frac{z^k}{k^s}. \tag{14}$$

The final result is obtained after the expansion of (13) with $\beta \to 0$

$$Q_{ind}^3 = gA_0 \left[\frac{4}{3}\beta^{-2} - \frac{2m^3}{3\pi^2}\beta + O(\beta^3) \right]. \tag{15}$$

More complicated situation appears in the case $(A_0)_\mu^a = A_0 \delta_{\mu 4} \delta^{a8}$, since λ^8 generator has three diagonal components

$$Q_{ind}^8 = 4gA_0 \sum_{p_4} \int \frac{d^3 p}{(2\pi)^3} Tr \left[\frac{\lambda_8}{\sqrt{3}} \gamma_4 \frac{\hat{p}_\sigma \gamma_\sigma + m}{\hat{p}^2 + m^2} \right]. \tag{16}$$

After doing similar calculations we obtain the following result

$$Q_{ind}^8 = -\frac{4\beta}{\sqrt{3}} \int \frac{d^3 p}{(2\pi)^3} \left[\frac{\sin \beta A_0}{\cos \beta A_0 + \cosh \beta \varepsilon_{\mathbf{p}}} + \frac{\sin 2\beta A_0}{\cos 2\beta A_0 + \cosh \beta \varepsilon_{\mathbf{p}}} \right]. \tag{17}$$

However, in the high temperature limit $\beta \to 0$, both integrals acquire the same form

$$Q_{ind}^8 = \frac{4\beta}{\sqrt{3}\pi^2} \int_m^\infty \frac{p^2 dp}{1+\cosh \beta p}. \tag{18}$$

After integration we obtain

$$Q_{ind}^{8}=\frac{4}{\sqrt{3}\pi^{2}\beta^{2}}\left\{-4Li_{2}(e^{-m\beta})+m\beta\left[m\beta+4\ln(1+e^{-m\beta})-m\beta\tanh\frac{m\beta}{2}\right]\right\}. \quad (19)$$

The final result is

$$Q_{ind}^{8}=gA_{0}\left[\frac{16}{3\sqrt{3}}\beta^{-2}-\frac{8m^{3}}{3\sqrt{3}\pi^{2}}\beta+O(\beta^{3})\right]. \quad (20)$$

As we see from (15) and (20), the induced color charge in the first term does not depend on mass and depends on temperature as $\sim T^{2}$. At the same time, the second term depends on mass and temperature as $\sim m^{3}T^{-1}$. When passing to the high-temperature limit, only the first term is significant and therefore the quark-gluon plasma acquires the spontaneous charge in the case $m=0$, also.

Thus, one of the consequences of the the A_{0} condensate presence is the $Z(3)$ symmetry and the C-parity violation, which leads to the induction of color charge in the plasma.

4. Quark self-energy

Now let us calculate a quark self-energy in the presence of the A_0 condensate. As is known, the particle spectra are determined by the Schwinger-Dyson equation

$$G^{-1}=\hat{p}+m-\Sigma, \quad (21)$$

where Σ is polarization operator.

Let us calculate it in one-loop approximation. For this, in addition to the tadpole diagram, we also need to consider the next one shown in the figure.

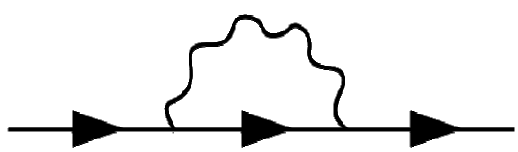

Fig. 2. One-loop quark self-energy.

That has been done in [2] for a chemical potential μ. We use these results in the case $N=3$ for QCD. As it follows from the explicit form of the Feynman rules [4],

$$\Sigma(q)=\frac{4g^{2}}{3\beta}\sum_{p_4}\int\frac{d^{3}p}{(2\pi)^{3}}\mathcal{D}_{\mu\nu}(p-q)\gamma_{\mu}G(p)\Gamma_{\nu}(p,q\,|\,p-q), \quad (22)$$

where the bare functions have the form

$$\Gamma_{\mu}^{0}(p,q\,|\,p-q)=\gamma_{\mu},\qquad G^{0}(p)=\frac{-i\gamma_{\mu}\hat{p}_{\mu}+m}{\hat{p}^{2}+m^{2}},\qquad \mathcal{D}_{\mu\nu}^{0}(p)=\frac{\delta_{\mu\nu}}{p^{2}}. \quad (23)$$

We sum over the spinor indices and obtain

$$\Sigma(q)=\frac{8g^2}{3\beta}\sum_{p_4}\int\frac{d^3p}{(2\pi)^3}\frac{\mathrm{i}\gamma_\mu\hat{p}_\mu+2m}{(\hat{p}^2+m^2)(p-q)^2}. \tag{24}$$

After that we calculate the temperature sum

$$\Sigma(q)=-\frac{2g^2}{3\pi\mathrm{i}}\int\frac{d^3p}{(2\pi)^3}\oint_C\tan\frac{\beta\omega}{2}\frac{\mathrm{i}(\omega\gamma_4+\mathbf{pq})+2m}{[(\omega-A_0)^2+\varepsilon_{\mathbf{p}}^2][(\omega-q_4)^2+|\mathbf{p}-\mathbf{q}|^2]}. \tag{25}$$

Also, we introduce the convenient notations for Bose and Fermi occupation number

$$n_{\mathbf{p}}^B=\frac{1}{e^{\beta|\mathbf{p}|}-1},\qquad n_{\mathbf{p}}^{\pm}=\frac{1}{e^{\beta(\varepsilon_{\mathbf{p}}\mp\mathrm{i}A_0)}+1}. \tag{26}$$

The result is found to be

$$\begin{aligned}\Sigma(q)=-\frac{4g^2}{3}\int\frac{d^3p}{(2\pi)^3}&\\ \times\Bigg\{&\frac{\pi n_{\mathbf{p}}^+}{\varepsilon_{\mathbf{p}}}\frac{\gamma_4\varepsilon_{\mathbf{p}}+\mathrm{i}\boldsymbol{\gamma}\mathbf{p}+2m}{(q_4+\mathrm{i}\varepsilon_{\mathbf{p}}-A_0)^2+|\mathbf{q}-\mathbf{p}|^2}\\ &+\frac{\pi n_{\mathbf{p}}^B}{|\mathbf{p}|}\frac{[|\mathbf{p}|+\mathrm{i}(A_0-q_4)]\gamma_4+\mathrm{i}\boldsymbol{\gamma}(\mathbf{p}-\mathbf{q})-2m}{(q_4-A_0+\mathrm{i}|\mathbf{p}|)^2+\varepsilon_{\mathbf{p}-\mathbf{q}}^2}\\ &-\frac{\pi n_{\mathbf{p}}^-}{\varepsilon_{\mathbf{p}}}\frac{\gamma_4\varepsilon_{\mathbf{p}}+\mathrm{i}\boldsymbol{\gamma}\mathbf{p}-2m}{(q_4+\mathrm{i}\varepsilon_{\mathbf{p}}+A_0)^2+|\mathbf{q}-\mathbf{p}|^2}\\ &-\frac{\pi n_{\mathbf{p}}^B}{|\mathbf{p}|}\frac{[|\mathbf{p}|-\mathrm{i}(A_0+q_4)]\gamma_4+\mathrm{i}\boldsymbol{\gamma}(\mathbf{p}-\mathbf{q})-2m}{(q_4+A_0+\mathrm{i}|\mathbf{p}|)^2+\varepsilon_{\mathbf{p}-\mathbf{q}}^2}\Bigg\}.\end{aligned} \tag{27}$$

To simplify further calculations we introduce two new functions

$$\Sigma(q)=\mathrm{i}\gamma_\mu K_\mu(q)+mZ(q), \tag{28}$$

and found them separately. Using $Tr\Sigma(q)/4=mZ(q)$ we find $Z(q)$

$$\begin{aligned}Z(q)=-\frac{4g^2}{3}\int\frac{d^3p}{\pi^3}\frac{n_{\mathbf{p}}^B}{|\mathbf{p}|}\Bigg[&\frac{1}{(q_4-A_0+\mathrm{i}|\mathbf{p}|)^2+\varepsilon_{\mathbf{p}-\mathbf{q}}^2}\\ &-\frac{1}{(q_4+A_0+\mathrm{i}|\mathbf{p}|)^2+\varepsilon_{\mathbf{p}-\mathbf{q}}^2}\Bigg].\end{aligned} \tag{29}$$

Similarly, for $Tr\gamma_4\Sigma(q)/4=\mathrm{i}K_4(q)$ we obtain the function $K_4(q)$

$$K_4(q) = -mZ(q) + \frac{2\mathrm{i}g^2}{3}\int \frac{d^3p}{\pi^3}\left[\frac{n_{\mathbf{p}}^{+}}{(q_4 - A_0 + \mathrm{i}\,|\,\mathbf{p}\,|)^2 + |\,\mathbf{q}-\mathbf{p}\,|^2} - \left[h.c(A_0 \to -A_0)\right]\right] \tag{30}$$

And from $Tr\gamma_n\Sigma(q)/4 = \mathrm{i}K_n(q)$ we find the vector $K_n(q)$ $(n = 1,2,3)$

$$K_n(q) = \frac{2g^2}{3}\int \frac{d^3p}{\pi^3}\frac{p_n}{\varepsilon_{\mathbf{p}}}\left[\frac{n_{\mathbf{p}}^{+}}{(q_4 - A_0 + \mathrm{i}\,|\,\mathbf{p}\,|)^2 + |\,\mathbf{q}-\mathbf{p}\,|^2} - \left[h.c(A_0 \to -A_0)\right]\right]. \tag{31}$$

As we see, unlike previous calculations with chemical potential μ, in the case of A_0 it is included as the real shifts of the zero momentum component. Therefore, it also changes the frequencies of modes in the environment and affects the stability of spectra.

Next we consider the contribution of the induced color charge, which was not calculated already. For the tadpole diagram we have the expression.

$$\Sigma^{tp}(q) = \frac{4g^2}{3\beta}\int \frac{d^4k}{(2\pi)^4}\sum_{p_4}\int \frac{d^3p}{(2\pi)^3} \times \mathcal{D}_{\mu\nu}(k + m_{gl}^3)\gamma_\mu Tr\left[G(p)\Gamma_\nu(p,q\,|\,p-q)\right]. \tag{32}$$

From the equality of initial and final momenta $q = q'$ it is obvious that $k = 0$. The second integral gives Q_{ind}^3, which was calculated above. The result is

$$\Sigma^{tp} = \frac{16g^2}{3\beta(m_{gl}^3)^2}Q_{ind}^3, \tag{33}$$

where $(m_{gl}^3)^2 \sim g^4T^2$ is the dynamical gluon mass in the plasma. It enters the gluon propagator as phenomenological parameter which is not calculated here. Using Q_{ind}^3 expansion (15) we obtain

$$\Sigma^{tp} = \frac{A_0}{g}\left[\frac{64}{9}\beta^{-1} - \frac{32m^3}{9\pi^2}\beta^2 + O(\beta^5)\right]. \tag{34}$$

As we see, the induced color charge gives correction to quark masses. Moreover, the first term depends on temperature as $\sim Tg^{-1}$. The next term is of the order $\sim T^{-2}$ and is next to the leading at high temperature.

5. Discussion

In this paper, we have considered the quark-gluon plasma at A_0 background. We used the charged basis, which made it possible to consider the condensate as the constant shifts of the zero momentum component. We calculated the induced color charge for the cases $(A_0)_\mu^a = (A_0)_\mu \delta^{a3}$ and $(A_0)_\mu^a = (A_0)_\mu \delta^{a8}$.

Also, we calculated the proper diagram Fig. 2 in the presence of the A_0 condensate. We also have considered the induced charge – the tadpole diagram, which was not yet calculated. It turns out that the induced color charge brings new corrections to quark masses, which were calculated. These corrections depend on temperature as $\sim Tg^{-1}$ at high temperature limit.

Since the first term of induced color charge factorization does not depend on mass, it is induced even in case $m = 0$. Taking into account that the generation of the A_0 condensate is an order parameter of deconfinement, it is clear that in all previous papers considering the state of the quark-gluon plasma without taking into account the induced charge (for example [2]) not only the contribution of the tadpole diagram was not considered, but also the charge conservation law was violated.

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