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NON-DISSIPATIVE HYDRODYNAMIC EQUATIONS BASED ON A NONLOCAL COLLISION INTEGRAL

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We consider a slightly non-uniform one-component gas with weak potential interaction. The basis of the investigation is the known kinetic equation in the case of small interaction which is truncated up to the second order of smallness. This equation contains a nonlocal collision integral of the second order in small interaction. In this paper we consider the hydrodynamic stage of the system evolution, and, in contrast to the standard hydrodynamics, we take into account the non-locality of the collision integral. We propose the following set of the reduced description parameters which are the densities of the conserved quantities: the particle number density, the momentum density, and the total energy density. It should be stressed that in contrast to the standard hydrodynamics, the kinetic energy is not conserved, and only the total system energy is conserved if the nonlocal collision integral is used. Definitions of the system velocity and temperature are proposed; it should be stressed that the proposed temperature definition is based on the total system energy rather than on the kinetic one. The hydrodynamics in the leading order in small gradients is investigated, and it is shown that the system one-particle distribution function in the leading-in-gradients order coincides with the Maxwellian one. Particle number density, velocity and temperature time evolution equations (hydrodynamic equations) are derived in the non-dissipative case. The leading-in-interaction orders of the obtained equations coincide with the corresponding equations in the framework of the standard hydrodynamics. The corrections of the first and second order in small interaction are also obtained.

Keywords: kinetic equation in the case of small interaction, nonlocal collision integral, hydrodynamic equations, temperature and velocity definitions.

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1. Introduction

The system under consideration is a one-component slightly non-uniform gas with weak potential interaction. The investigation is based on the basis of the known kinetic equation in the case of small interaction [1] which is truncated up to the second order in small interaction. It is important to stress that this equation contains a general non-local second-order collision integral and in the local approximation for the collision integral it coincides with the Landau–Vlasov kinetic equation.

The aim of the paper is to describe the hydrodynamic stage of the system evolution. The construction of system hydrodynamics on the basis of the Champan–Enskog method is well-known for systems which are described by the Boltzmann kinetic equation with a local collision integral (see the description in [2] and an example of application in [3]). However, a description of the system on the basis of a nonlocal collision integral is more precise. In fact, in the case of small density the non-local collision integral is described in [2]. In [2] the combined Chapman–Enskog–Grad method is proposed for deriving the kinetic coefficients with taking into account the non-locality of the collision integral. But it should be stressed that in [2] the case of small density rather than the case of small interaction is considered. It should also be noticed that usually the hydrodynamics of systems with weak interaction is built on the basis of the Landau kinetic equation with a local collision integral which does not contain the Vlasov term. In such a case, the system hydrodynamics is constructed similarly to the one based on the Boltzmann equation (see, for example, [4]).

In this paper the case of small interaction is considered, the non-locality of the collision integral and the self-consistent Vlasov term are taken into account.

In our previous paper [5] it is shown that the conserved quantities for the system under consideration are the particle number, momentum and total system energy. It is also shown that although the kinetic energy is conserved on the basis of the local collision integral, it is not conserved on the basis of the nonlocal collision integral. As is known, the system reduced description parameters should be chosen as the densities of the conserved quantities, so the following set of the reduced description parameters is proposed in this paper: the particle number density, the momentum density and the total energy density. Usually the temperature and velocity are used as the reduced description parameters instead of momentum and energy density. The corresponding velocity and temperature definitions are proposed in this paper, and it should be stressed that the temperature is defined here on the basis of the total system energy rather than of the kinetic one.

The system hydrodynamics is investigated on the basis of the proposed definitions of the reduced description parameters. The kinetic equation is solved in the leading-in-gradients order and corresponding hydrodynamic equations are derived. In fact, the non-dissipative system hydrodynamics is investigated in this paper. Such an investigation is the basis of our future research devoted to the dissipative hydrodynamics of the system under consideration.

The paper is organized as follows. In Sec. 2 the basic equations of the theory are given, in Sec. 3 the non-dissipative system hydrodynamics is investigated and in Sec. 4 conclusions are made.

2. Basic equations of the theory

The basis of investigation is the kinetic equation in the case of small potential interaction which is truncated up to the second order in small interaction [1]:

$$\frac{\partial f(\chi_1,t)}{\partial t} = -\frac{p_{1n}}{m}\frac{\partial f(\chi_1,t)}{\partial x_{1n}} + \frac{\partial f(\chi_1,t)}{\partial p_{1n}}\frac{\partial}{\partial x_{1n}}\int d\chi_2 V_{12} f(\chi_2) + I(\chi_1,f),$$

$$\chi = \{\mathbf{x},\mathbf{p}\},\ V_{12} \equiv V(|\mathbf{x}_1 - \mathbf{x}_2|),$$

$$I(\chi_1,f) = \frac{\partial}{\partial p_{1n}}\int d\chi_2 F_{12n}\int_{-\infty}^{0} d\tau F_{12l\tau}\left(\frac{\partial}{\partial p_{1l}} - \frac{\tau}{m}\frac{\partial}{\partial x_{1l}} - \frac{\partial}{\partial p_{2l}} + \frac{\tau}{m}\frac{\partial}{\partial x_{2l}}\right) f(\chi_1) f(\chi_2), \tag{1}$$

$$F_{12n} = -\frac{\partial V_{12}}{\partial x_{1n}},\ F_{12l\tau} = -\frac{\partial}{\partial x_{1l}} V\left(\left|\mathbf{x}_1 - \mathbf{x}_2 + \frac{\tau}{m}(\mathbf{p}_1 - \mathbf{p}_2)\right|\right)$$

where $V(|\mathbf{x}|)$ is the pair system potential, m is the mass of a particle, $f(\chi,t)$ is the one-particle distribution function, and $I(\chi_1,f)$ is the general nonlocal second-order collision integral. In the local approximation $I(\chi_1,f)$ coincides with the known Landau collision integral.

The particle number, momentum, kinetic energy and potential energy densities are given by standard definitions in terms of the one-particle distribution function:

$$n(\mathbf{x},t) = \int d\mathbf{p} f(\chi,t),\ \pi_n(\mathbf{x},t) = \int d\mathbf{p} p_n f(\chi,t),\ \varepsilon_{\text{kin}}(\mathbf{x},t) = \int d\mathbf{p}(p^2/2m) f(\chi,t),$$
$$\varepsilon_{\text{pot}}(\mathbf{x}_1,t) = \int d\chi_2\ f_2(\chi_1,\chi_2,f(t)) V_{12}/2, \tag{2}$$

here the result [1] for the two-particle distribution function should be taken into account:

$$f_2(\chi_1,\chi_2,f)=f(\chi_1)f(\chi_2)-$$
$$-\int_{-\infty}^{0}d\tau F_{12l\tau}\left(\frac{\partial}{\partial p_{1l}}-\frac{\tau}{m}\frac{\partial}{\partial x_{1l}}-\frac{\partial}{\partial p_{2l}}+\frac{\tau}{m}\frac{\partial}{\partial x_{2l}}\right)f(\chi_1)f(\chi_2)+O\left(\lambda^2\right), \tag{3}$$

the small parameter λ describes the smallness of the potential interaction.

The system is assumed to be weakly non-uniform on the scale of the radius of particle interaction, so we can expand $f(\chi_2)$ into a series in $\mathbf{x}_2-\mathbf{x}_1$:

$$f(\chi_2)=f(\mathbf{x}_1,p_2)+\frac{\partial f(\mathbf{x}_1,p_2)}{\partial x_{1n}}x_{21n}+\frac{1}{2}\frac{\partial^2 f(\mathbf{x}_1,p_2)}{\partial x_{1n}\partial x_{1l}}x_{21n}x_{21l}+O\left(g^3\right),$$
$$\mathbf{x}_{21}\equiv\mathbf{x}_2-\mathbf{x}_1, \tag{4}$$

the small parameter g describes the smallness of the gradients of a one-particle distribution function. It should be stressed that the same idea was used in [2] in the derivation of the non-local collision integral in the case of small density.

On the basis of (1)–(4) it is shown in [5] that the conserved quantities of the system under consideration are the particle number, momentum and total energy of the system, the total energy is the sum of the kinetic and potential energies of the system. It is shown in [5] that, in contrast to the theory based on the local collision integral, the kinetic energy of the system is not conserved. The following time evolution equations for the densities of the conserved quantities are obtained in [5]:

$$\varepsilon(\mathbf{x}_1,t)=\varepsilon_{\text{pot}}(\mathbf{x}_1,t)+\varepsilon_{\text{kin}}(\mathbf{x}_1,t),$$
$$\frac{\partial n(\mathbf{x},t)}{\partial t}=-\frac{1}{m}\frac{\partial\pi_n}{\partial x_n},\ \frac{\partial\pi_n(\mathbf{x}_1,t)}{\partial t}=-\frac{\partial t_{nl}}{\partial x_{1l}},\ \frac{\partial\varepsilon(\mathbf{x}_1,t)}{\partial t}=-\frac{\partial q_l}{\partial x_{1l}}, \tag{5}$$

the expressions for the momentum and total energy fluxes obtained in [5] with the help of the following expressions

$$p_{1n}=\frac{p_{1n}+p_{2n}}{2}+\frac{p_{1n}-p_{2n}}{2},\ p_{2n}=\frac{p_{1n}+p_{2n}}{2}-\frac{p_{1n}-p_{2n}}{2} \tag{6}$$

can be rewritten as

$$t_{nl}=\int d\mathbf{p}_1\frac{p_{1l}p_{1n}}{m}f(\chi_1)+\frac{1}{2}\delta_{nl}n^2V_0+$$
$$+\frac{m}{2\pi^2}\int d\mathbf{p}_1d\mathbf{p}_2f(\chi_1)f(x_1,p_2)\left(\frac{2p_{12l}p_{12n}-p_{12}^2\delta_{nl}}{p_{12}^4}A-\frac{\delta_{nl}}{p_{12}^2}B\right)+O(\lambda^1g^2,\lambda^2g^1,\lambda^3),$$
$$q_l=\int d\mathbf{p}_1\frac{p_1^2}{2m}\frac{p_{1l}}{m}f(\chi_1)+\frac{1}{m}V_0\pi_l+$$
$$+\frac{1}{4\pi^2}\int d\mathbf{p}_1d\mathbf{p}_2f(\chi_1)f(x_1,p_2)\left(p_{1n}+p_{2n}\right)\frac{2p_{12l}p_{12n}-p_{12}^2\delta_{nl}}{p_{12}^4}A-$$
$$-\frac{1}{2\pi^2}\int d\mathbf{p}_1d\mathbf{p}_2f(\chi_1)f(x_1,p_2)\frac{p_{1l}+p_{2l}}{p_{12}^2}B+O(\lambda^1g^2,\lambda^2g^1,\lambda^3); \tag{7}$$
$$A\equiv\int_0^\infty dkk^3V(k)\frac{dV(k)}{dk},\qquad B\equiv\int_0^\infty dkk^2V(k)^2,\qquad V_0\equiv V(k=0);\quad \mathbf{p}_{12}\equiv\mathbf{p}_1-\mathbf{p}_2$$

where $V(k) = V(|\mathbf{k}|)$ is the Fourier transform of the pair system potential $V(|\mathbf{x}|)$:

$$V\left(|\mathbf{x}_1 - \mathbf{x}_2|\right) = (2\pi)^{-3} \int d^3k e^{i\vec{k}(\vec{x}_1 - \vec{x}_2)} V\left(k\right). \tag{8}$$

The results given in this section are the basis for the construction of system hydrodynamics.

3. Non-dissipative system hydrodynamics

As is known, the hydrodynamic reduced description parameters should be chosen as the densities of the conserved quantities. In what follows the hydrodynamics based on the local collision integral is called as the standard hydrodynamics. It should be stressed that in the framework of standard hydrodynamics the following reduced description parameters are chosen: the particle number density, the momentum density and the kinetic energy density because the particle number, the momentum and the kinetic energy are conserved on the basis of the nonlocal collision integral. But in the problem under consideration we cannot use the kinetic energy density as the reduced description parameter because the kinetic energy is not conserved on the basis of the nonlocal collision integral. So we propose the following set of the reduced description parameters: the particle number density, momentum density and total energy density of the system.

As is known, in the standard hydrodynamics the system velocity and temperature are often used as the reduced description parameters instead of the momentum density and the kinetic energy density. For example, the well-known Maxwellian distribution function is usually written in terms of the system velocity and temperature. So it is important to introduce the velocity and the temperature for the problem under consideration. The definitions of the reduced description parameters in terms of the one-particle distribution function should be the same as in the equilibrium case. As is known [6], in equilibrium we have

$$\pi_l^{\text{eq}} = mn\upsilon_l, \quad \varepsilon_{\text{kin}}^{\text{eq}} = \frac{3}{2}nT + \frac{1}{2}mn\upsilon^2, \quad \varepsilon_{\text{pot}}^{\text{eq}} = \frac{1}{2}n^2 \int d\mathbf{r} V(r) e^{-V(r)/T} \tag{9}$$

where $\boldsymbol{\upsilon}$ is the system velocity and T is the system temperature. Here and in what follows, the temperature is written in energy units. The expression for $\varepsilon_{\text{pot}}^{\text{eq}}$ in (9) should be truncated up to the second order of smallness in small interaction because the basic equation of the theory (1) is also truncated up to the second-in-interaction order. Obviously,

$$e^{-V(r)/T} = 1 - \frac{V(r)}{T} + O(\lambda^2), \tag{10}$$

So we have the following definitions of the particle number density, velocity and temperature of the system:

$$\begin{gathered} n(\mathbf{x},t) = \int d\mathbf{p} f(\chi,t), \quad \pi_n(\mathbf{x},t) = mn(\mathbf{x},t)\upsilon_n(\mathbf{x},t) = \int d\mathbf{p} p_n f(\chi,t), \\ \varepsilon(\mathbf{x}_1,t) = \frac{3}{2}n(\mathbf{x}_1,t)T(\mathbf{x}_1,t) + \frac{1}{2}mn(\mathbf{x}_1,t)\upsilon^2(\mathbf{x}_1,t) + \frac{n^2(\mathbf{x}_1,t)}{2}\int d\mathbf{r} V(r) - \\ -\frac{n^2(\mathbf{x}_1,t)}{2T(\mathbf{x}_1,t)}\int d\mathbf{r} V^2(r) = \int d\mathbf{p}_1 \frac{p_1^2}{2m} f(\chi_1,t) + \int d\mathbf{p}_1 d\chi_2 f_2(\chi_1,\chi_2,f(t)) V_{12}, \end{gathered} \tag{11}$$

the last expression in (11) is a consequence of (9), (10), (2), and (5). Expressions (11) are the definitions of the system temperature and velocity. Moreover, these expressions are additional conditions to the kinetic equation (1). As can be seen, these conditions contain the two-particle distribution function, so the result (3) with account for (4) should also be taken into account in (11). It should be stressed that the system temperature is defined on the basis of the total system energy rather than of the kinetic one.

We describe the hydrodynamic stage of system evolution, so we write the functional hypothesis

$$f(\chi,t)=f_p(\mathbf{x},t)\xrightarrow[t\gg\tau_0]{}f_p\left(\mathbf{x},n(t),\upsilon_n(t),T(t)\right). \tag{12}$$

The distribution function f_p is sought in a perturbation theory in the small parameter g:

$$f_p(\mathbf{x},t)=f_p^{(0)}(\mathbf{x},t)+f_p^{(1)}(\mathbf{x},t)+O\left(g^2\right),\ f_p^{(n)}(\mathbf{x},t)\sim g^n. \tag{13}$$

On the basis of the functional hypothesis (12) the kinetic equation (1) can be rewritten as

$$\begin{gathered}\int d\mathbf{x}'\frac{\delta f_p(\mathbf{x})}{\delta n(\mathbf{x}')}\frac{\partial n(\mathbf{x}')}{\partial t}+\int d\mathbf{x}'\frac{\delta f_p(\mathbf{x})}{\delta T(\mathbf{x}')}\frac{\partial T(\mathbf{x}')}{\partial t}+\int d\mathbf{x}'\frac{\delta f_p(\mathbf{x})}{\delta \upsilon_l(\mathbf{x}')}\frac{\partial \upsilon_l(\mathbf{x}')}{\partial t}=\\ =-\frac{p_{1n}}{m}\frac{\partial f(\chi_1,t)}{\partial x_{1n}}+\frac{\partial f(\chi_1,t)}{\partial p_{1n}}\int d\chi_2\frac{\partial V_{12}}{\partial x_{1n}}f(\chi_2)+I(\chi_1,f).\end{gathered} \tag{14}$$

On the basis of (8) it can be shown that the expression for $\varepsilon(\mathbf{x},t)$ in (11) can be rewritten as:

$$\varepsilon(\mathbf{x}_1,t)=\frac{3}{2}n(\mathbf{x}_1,t)T(\mathbf{x}_1,t)+\frac{1}{2}mn(\mathbf{x}_1,t)\upsilon^2(\mathbf{x}_1,t)+\frac{n^2(\mathbf{x}_1,t)}{2}V_0-\frac{n^2(\mathbf{x}_1,t)}{4\pi^2T(\mathbf{x}_1,t)}B. \tag{15}$$

From (15) and (11) we have

$$\begin{gathered}\frac{\partial n}{\partial t}=-\frac{\partial n\upsilon_n}{\partial x_{1n}},\ \frac{\partial \upsilon_n}{\partial t}=\frac{1}{mn}\frac{\partial \pi_n}{\partial t}+\frac{\upsilon_n}{n}\frac{\partial n\upsilon_l}{\partial x_{1l}},\\ \frac{\partial T}{\partial t}=\frac{2}{3n}\frac{\frac{\partial \varepsilon}{\partial t}-\left(\frac{3T}{2}+\frac{m\upsilon^2}{2}+nV_0-\frac{n}{2\pi^2T}B\right)\frac{\partial n}{\partial t}-mn\upsilon_n\frac{\partial \upsilon_n}{\partial t}}{1+\frac{n}{6\pi^2T^2}B}.\end{gathered} \tag{16}$$

It should be noticed that all the quantities of the theory should be truncated up to the second order in small interaction, so

$$\begin{gathered}\frac{\partial T}{\partial t}=\frac{2}{3n}\left[\frac{\partial \varepsilon}{\partial t}-\left(\frac{3T}{2}+\frac{m\upsilon^2}{2}+nV_0-\frac{n}{2\pi^2T}B\right)\frac{\partial n}{\partial t}-mn\upsilon_n\frac{\partial \upsilon_n}{\partial t}\right]\times\\ \times\left[1-\frac{n}{6\pi^2T^2}B+O\left(\lambda^4\right)\right].\end{gathered} \tag{17}$$

On the basis of (16) and (17) it is obvious that in the leading-in-gradients order the kinetic equation (14) takes the form

$$I^L(\chi_1, f) = 0 \tag{18}$$

where $I^L(\chi_1, f)$ is the local Landau collision integral. As is known, the solution of (18) is the Maxwellian distribution function

$$f_p^{(0)}(\mathbf{x},t) = \frac{n(\mathbf{x},t)}{\left(2\pi mT(\mathbf{x},t)\right)^{3/2}} \exp\left(-\frac{\left(\mathbf{p} - m\boldsymbol{\upsilon}(\mathbf{x},t)\right)^2}{2mT(\mathbf{x},t)}\right). \tag{19}$$

In the leading-in-gradients order the additional conditions (11) and (15) take the form

$$\begin{gathered} n(\mathbf{x},t) = \int d\mathbf{p} f_p^{(0)}(\mathbf{x},t), \quad mn(\mathbf{x},t)\upsilon_n(\mathbf{x},t) = \int d\mathbf{p} p_n f_p^{(0)}(\mathbf{x},t), \\ \frac{3}{2} n(\mathbf{x}_1,t)T(\mathbf{x}_1,t) + \frac{1}{2} mn(\mathbf{x}_1,t)\upsilon^2(\mathbf{x}_1,t) + \frac{n^2(\mathbf{x}_1,t)}{2} V_0 - \\ -\frac{n^2(\mathbf{x}_1,t)}{4\pi^2 T(\mathbf{x}_1,t)} B = \int d\mathbf{p}_1 \frac{p_1^2}{2m} f(\chi_1,t) + \int d\mathbf{p}_1 d\chi_2 f_2^{(g^0)}(\chi_1,\chi_2,f(t)) V_{12}. \end{gathered} \tag{20}$$

By a straightforward calculation on the basis of (19) it can be shown that the first two expressions in (20) are valid. The third expression in (20) on the basis of (2)–(4) can be rewritten as

$$\begin{gathered} \frac{3}{2} nT + \frac{1}{2} mn\upsilon^2 + \frac{n^2}{2} V_0 - \frac{n^2}{4\pi^2 T} B = \\ = \int d\mathbf{p}_1 \frac{p_1^2}{2m} f_{p1}^{(0)} + \frac{n^2}{2} V_0 - \frac{m}{2\pi^2} \int d\mathbf{p}_1 d\mathbf{p}_2 f_{p1}^{(0)} f_{p2}^{(0)} \frac{1}{p_{12}^2} B, \\ f_{p1} = f(\chi_1,t), \quad f_{p2} = f(\mathbf{x}_1, p_2, t), \end{gathered} \tag{21}$$

here for simplicity the dependence of the reduced description parameters on $\mathbf{x}_1$ and t is not stressed. By a straightforward calculation on the basis of (19) it can be shown that (21) is valid; the following change of variables should be made during the integration over $d\mathbf{p}_1$ and $d\mathbf{p}_2$:

$$\mathbf{y} = \mathbf{p}_1 + \mathbf{p}_2, \ \mathbf{z} = \mathbf{p}_1 - \mathbf{p}_2 = \mathbf{p}_{12}. \tag{22}$$

So in the leading-in-gradients order the Maxwellian distribution function (19) satisfies both the kinetic equation and the additional conditions. So the Maxwellian distribution function is the system distribution function in the leading-in-gradients order. On the basis of (19), (16), (17), (5), and (7), with the help of the substitution (22), one can obtain the following hydrodynamic equations truncated up to the second order in λ:

$$\partial_t n = -n\frac{\partial \upsilon_n}{\partial x_n} - \upsilon_n \frac{\partial n}{\partial x_n},$$

$$\frac{\partial \upsilon_n}{\partial t} = -\upsilon_l \frac{\partial \upsilon_n}{\partial x_l} + \left[-\frac{T}{nm} - \frac{1}{m}V_0 + + \frac{1}{2\pi^2 mT}\left(B + \frac{1}{3\pi^2}A\right)\right]\frac{\partial n}{\partial x_n} + \left[-\frac{1}{m} - \right.$$
$$\left. -\frac{n}{4\pi^2 mT^2}\left(B + \frac{1}{3}A\right)\right]\frac{\partial T}{\partial x_n} + O\left(g^1\lambda^3, g^2\right), \tag{23}$$

$$\frac{\partial T}{\partial t} = \left[-\frac{2}{3}T + \frac{n}{9\pi^2 T}\left(B + \frac{1}{2}A\right)\right]\frac{\partial \upsilon_n}{\partial x_n} - -\upsilon_n \frac{\partial T}{\partial x_n} + O(g^1\lambda^3, g^2).$$

As can be seen, the obtained hydrodynamic equations (23) are truncated up to the linear order in small gradients and they are based on the Maxwellian distribution function. So the obtained equations (23) are the hydrodynamic equations of a non-dissipative hydrodynamics. It should be stressed that they explicitly contain the quantities A, B, and V_0 defined in (7) so the theory is valid if all these quantities are finite. If the integrals in (24) are infinite, then an artificial truncation of integration limits may be applied, such as, for example, the idea of the Coulomb logarithm [7]. If $V(k=0)$ is infinite, then a straightforward calculation of system modes on the basis of (23) fails, and most likely the system has not only hydrodynamic, but also the so-called plasma modes (see [8]). But the investigation of the system kinetic coefficients on the basis of (23) most likely will not fail because, as known, the Vlasov term has no effect on the system kinetic coefficients and it is logical to expect that the irregular term will vanish after substitution of (23) and (19) into the linear-in-gradients order of (14).

In the leading-in-interaction order the obtained hydrodynamic equations (23) coincide with the corresponding standard hydrodynamic equations, but the corrections of the first and second orders in small interaction are obtained. The non-dissipative hydrodynamics of the system under consideration is built: the distribution function and hydrodynamic equations are obtained in the non-dissipative case.

4. Conclusions

The non-dissipative hydrodynamics of a one-component slightly non-uniform gas with weak potential interaction is constructed in this paper. The paper is based on the kinetic equation in the case of weak interaction which is truncated up to the second-in-interaction order, and the second-order collision integral is nonlocal. The non-locality of the collision integral is taken into account.

The following hydrodynamic reduced description parameters are proposed: the particles number density, the momentum density, and the total energy density because they are the densities of the conserved quantities; the total energy is the sum of the kinetic and potential energies. It should be stressed that, in contrast to the standard hydrodynamics which is based on the local collision integral, the kinetic energy density cannot be chosen as a reduced description parameter, because it is not conserved on the basis of the nonlocal collision integral. The corresponding temperature and velocity definitions in terms of the one-particle distribution function are proposed above, and the temperature is defined on the basis of the total system energy rather than of the kinetic

one. The definitions of the particle number density, the velocity and the temperature are additional conditions to the kinetic equation.

The non-dissipative system hydrodynamics is constructed. It is shown that in the leading-in-gradients order the one-particle distribution function is the Maxwellian one. The hydrodynamic equations are obtained in the linear-in-gradients order, i.e. the non-dissipative hydrodynamic equations are built. These equations contain quantities which are expressed in terms of the Fourier-transform of the system pair potential. If they are infinite, then the idea of artificial truncation of the limits of integration may be applied (see, for example, the Coulomb logarithm [7]). If the Fourier-transform $V(k=0)$ is infinite, then a straightforward calculation of the system modes based on the obtained hydrodynamic equations fails, and in such a case the system may have not only hydrodynamic, but also the so-called plasma modes (see the corresponding investigation for plasma in [8]). But it is expected that the system kinetic coefficients may be sought on the basis of the developed equations even if $V(k=0)$ is infinite because most likely the infinite terms vanish in the derivation of the kinetic equation of the linear-in-gradients order.

The results of this paper are the basis of our future investigation of the dissipative system hydrodynamics.

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