

**Dedicated to the memory of Professor F.I. Kolomoitsev – the initiator
of the presented subjects**

**APPROXIMATE AND STRICT METHODS FOR THE CALCULATION
OF RODE ANGLE ANTENNAS OF MICROWAVE RANGE**

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Approximate and rigorous methods and results of calculations of electrical characteristics of rod arbitrarily curved antennas of the microwave range are investigated with the purpose of widening their varieties and increasing the accuracy of calculations of electrical parameters. An approximate calculation of the antennas is performed by the equivalent long line method, rigorous – by the integral equation method with respect to the distribution of the complex current at the antenna. An algorithm for the step-by-step development of curved rod antennas with included various concentrated loads and excitation nodes at arbitrary points are proposed. It is concluded that approximate methods are useful for a preliminary rough estimate of the input parameters of the antennas, and also as an initial approximation for their parametric synthesis and analysis. But strict methods can significantly expand the variety of studied antennas and improve the accuracy of calculations of their electrical parameters. A method is proposed for obtaining a stable solution of an integral equation by diagonalizing the matrix of coefficients of a system of equations. Basing on the results of the investigation of these antennas, new designs have been developed, a number of which have been introduced into production and operation. It is noted that for the first time in world practice the method of an integral equation described in this article for calculating such antennas has outpaced the development of known analogous methods and corresponding computer programs.

Keywords: arbitrarily curved linear antenna, concentrated load and excitation node, equivalent long line method, integral equation method.

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1. Introduction

When designing antennas and antenna arrays for modern means of telecommunications, radiolocation, radio navigation, radio astronomy, aerospace systems and other branches of science and technology, questions arise that need to be addressed, for example, what type, design, antenna configuration and what material is better to use when designing this antenna. How to choose optimal value of the correcting loading plugged in antenna and coordinate of her including? What should be the nature of the corrective load included in the antenna – capacitive, inductive, complex or active? How do the parameters of the corrective loads included in the antenna depend on the initial dimensions, configuration, and type of antenna (vibratory whip, loop, spiral or other)? How does the choice of the type and design of the antenna affect the current distribution (CD) on it, its complex input impedance, and the standing wave voltage factor (VSWR), the pass band, the directional pattern (DP), its polarization characteristics and other parameters. Early results of antenna research and development published in the works of famous scientists A.A. Pistolkors, G.Z. Eisenberg and other authors were obtained on the basis of an approximate estimate, assumptions or experimental results and could not fully answer the questions posed above.

This article presents a comparative analysis of approximate and rigorous methods of calculation and investigation of arbitrarily curved linear vibrator antennas with concentrated loads (CL) in branches included to improve their operational electrical and mechanical characteristics. A rigorous, more accurate method is based on the solution of integral equations (IE) in a rigorous formulation with respect to the unknown value of complex current on the antenna using a computer [1-5]. In the latter case, as a result of determining the complex value of CD along the antenna, it is possible to find its other necessary parameters.

2. Stages of creating linear randomly bent antennas and antenna systems with corrective loads

Consider the sequence and stages of development of a wide class of arbitrarily curved small-sized, multifrequency, broadband antennas with controlled DP and other antennas with improved structural and electrical characteristics. In **the first stage**, we formulate the initial data (ID) and technical requirements (TR) for the design and configuration of the antenna, as well as its electrodynamics and radio engineering parameters such as CD, VSWR, DP, efficiency and others, and set external characteristics such as impedance feeder at the antenna input, environmental parameters, etc. In **the second one**, we carry out a structurally parametric synthesis of the antenna with CL, which consists in determining the antenna type, its configuration, the list and the CL type included at various points of the antenna (the approximate approximation). In **the third stage**, if necessary, we develop or improve the mathematical model of the antenna synthesized at the second stage with switched on antennas CL and perform the refined analysis with a more rigorous method, for example, IE and optimization of the values CLs and the places of their inclusion in the antenna. In **step 4a**, after verifying the execution of the specified IDs and TRs. we make two decisions to go **to step 5** to prepare the ID and technical documentation for the subsequent manufacture of a new antenna that meets the specified requirements. Or **go to steps 4b and 4c** to select other ways of satisfying IDs and TRs. If the procedures in **steps 4b and 4c** do not lead to positive results, we resort to more radical solutions, namely, the transition **to 4g or 4d** with the subsequent repetition of all the previous procedures (**stages 1, 2, 3 and 4**).

In univariate analysis and optimization, we determine the antenna parameters for a certain fixed set value of internal and external parameters, and for multivariate analysis of the antenna parameters, we perform some internal and external parameters. Optimization of the antenna parameters is performed by searching for an extremum (minimum or maximum) of some objective function $F(X)$ within the domain XD of the vector of controlled parameters X [4]. We solve the problem by the method of mathematical programming and formulate in the form

$$\text{extr } F(X), X \in XD. \quad (1)$$

The optimization domain (XD) is defined by a set of constraints of the type of inequalities $\phi(X) \geq 0$ and the type of equalities $\psi(X) = 0$, that is,

$$\{X \in XP; \phi(X) \geq 0, \psi(X) = 0\} \quad (2)$$

where XP is the n -dimensional space of controllable X parameters such as d , the antenna bending angles α and β , coordinate of the h plugging in antenna correcting loading of Z .

Below we will consider the application of approximate and rigorous methods to the calculation and optimization of the investigated antennas. Approximate methods assume that the current distribution along the antenna is known in advance and obeys the harmonic (sinusoidal) law. In this case, the phase of the current along the wire of the antenna is constant, and in the "nodes" (i.e., at the minimum current values) changes abruptly by 180° . Similar approaches are known as the equivalent long line method (ELL), the method of induced electromotive forces, and others. Essentially, these methods already have an error in the results due to the approximate specification of the current distribution in the antenna.

3. Approximate calculation of whip antenna with loads

After formulating the ID and TR (Section 2) **in step 1** to the given electrically short antenna, we perform its structurally parametric synthesis (**stage 2**), that is, we determine the antenna type and design, the values CL and the switching points $(Z_1 \dots Z_M)$ in the initial approximation. In the case when CL are inductive, we determine the discrete set of values $L = \{L_1, L_2, \dots, L_M\}$ of these loads included in the arms of the antenna to provide a serial resonance mode ($X_{in} = 0$) at its input. Each load value L_j of a given set corresponds to the set of coordinates of its inclusion h_{Lj} : $H = \{h_{L_1}, h_{L_2}, \dots, h_{L_M}\}$. The load inclusion coordinates h_{Lj} correspond to the set $\{0 \dots d\}$, where d is the arm length of the antenna. The problem of determining the values L_j of the required loads is solved using the approximate ELL method, representing the symmetric antenna (Fig. 1) as a two-wire line with loads $(Z_1 \dots Z_j)$ included in the line branches (antenna) and load Z at the end of the line [4]. For the case of the line open on the end of equivalent to this antenna, resistance at the end of line and complete reactance of antenna has a form

$$Z_{load}^{open} = 2i X_{open} - 2i W_{open} \operatorname{ctg}[k(d - h_x)] \quad (3)$$

where $W_{open} = 120 \left[\ln(d/r_a) - 1 \right]$ is the wave impedance of the line open at the ends or the equivalent antenna; h_x are the coordinates of the inclusion of corrective loads X_{open} in the antenna.

Considering that the condition for matching this antenna with the feeder is the zero of the reactive component of the antenna input resistance, we obtain a formula for calculating the value of the reactance of the corrective load X_{open} necessary for matching the antenna with the feeder

$$X_{open} = i \frac{W_{open}}{2} \left\{ \operatorname{ctg}[k(d - h_x)] - \operatorname{tg}(kh_x) \right\}. \quad (4)$$

Note that in the above-described stage of antenna development using an approximate method it is impossible to obtain the necessary parameters, such as complex input resistance and reflection coefficient, VSWR, DP, efficiency parameter, and others.

4. Calculation of the current distribution and the input resistance of antennas by the integral equation method

We perform a deeper analysis of the antenna under investigation in a rigorous formulation using the IE method, which first determines on a computer by a numerical method the complex current distribution on an arbitrarily curved antenna (Fig. 1) in accordance with expression (5).

$$\int_{-d}^{+d} I(s') \left\{ \partial / \partial s' \left[G(s, s') - G(-s, s') \right] - k^2 \int_{-s}^s G(\xi, s') (\bar{\xi}, \bar{s}') d\xi \right\} ds' +$$

$$+ i\omega \varepsilon_a \int_{-s}^s I_\xi(\xi) \sum_{v=1}^M Z_v \delta(\xi - h_v) d\xi = i\omega \varepsilon_a \int_{-s}^s E^{exc}(\xi) d\xi, \quad (-d \leq s \leq d) \quad (5)$$

where $G(s, s') = e^{-ik\Delta} / \Delta$, are Green's function; $E_s^{exc}(s)$ is antenna excitation function; s, ξ are points of observation on the surface of the conductor; s' are source points on the axis of the conductor; Δ is the distance between the source point and the observation point; $\vec{\xi}, \vec{s}'$ are the unit vectors tangent to the surface and the axis of the conductor, respectively, at the observation point and at the source point; Z_v are values of the line impedances included in the conductor at distances h_v ; $\delta(\xi - h_v)$ is Dirac delta function; M is the number of impedance elements included in the arm of the antenna at distances h_v .

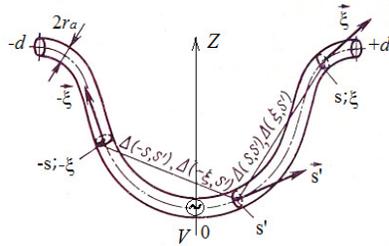


Fig. 1. An arbitrarily curved linear antenna, symmetrical about the axis 0 - Z

When developing a computer program for calculating the current distribution, a method is proposed for “diagonalizing” the matrix of the system of equations to improve its conditionality and to obtain a stable solution of the problem for the current with considering [1-3, 5-10].

The application of the IE method allows for deeper synthesis and analysis of the investigated antennas [4] in comparison with the approximate method considered above. Taking into account the current distribution, we find the complete set of necessary parameters, such as complex input resistance, VSWR, bandwidth, DP and others. As an example, consider the results of calculating the electrical parameters of a V-shaped antenna (Fig. 2) with arm length $d = 0.83\lambda$, three uniformly and symmetrically included in each arm capacitive loads, the resistance of each is $-i 300$ Ohm for the following initial and calculated data: $\alpha = 180^\circ$, $\beta = 135^\circ$, $h_v = 0$, $N = 24$, $N' = 1$, $\Omega = 11.6$, where Ω - antenna thickness parameter $\Omega = 4,6 \lg(2d / r_a)$.

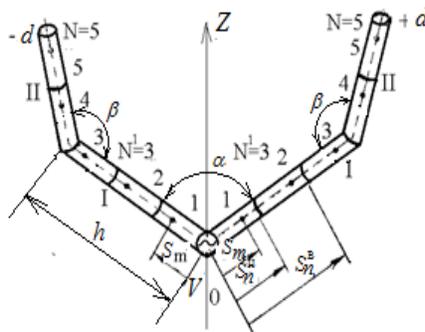


Fig. 2. Symmetrical curved whip vibrator, divided into 2 N segments of the partition

On the graphs of Fig. 3 the solid lines represent the dependences obtained with the aid of the expression (5) for such an antenna for the distribution of the module (Fig. 3, a.) and phase (Fig. 3, b). Analogous parameters for this antenna without the loads for comparison are shown in dotted lines. It follows from the graphs that the inclusion of capacitive loads in this antenna makes it possible to smoothly reduce the amplitude and phase of the complex current to its ends without sudden jumps, compared to the antenna without loads. Also, the inclusion of capacitive loads in the antenna prevents a sharp phase jump (approximately) of 180° in the minima (nodes) of the distribution of the current amplitude in the antenna without loads (Fig. 3, dotted lines). And it changes the standing wave mode in the antenna to the running current wave mode, that is, it substantially corrects the property of such an antenna and expands its frequency range [4]. At the input of this antenna with capacitive loads, the VSWR is significantly reduced in comparison with the antenna without loads, which makes it possible to improve the matching of the antenna with the transceiver radio equipment [11].

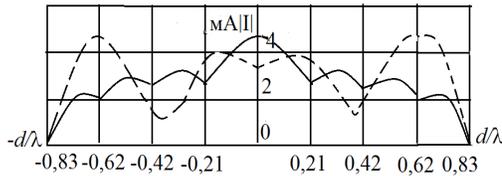


Fig. 3,a

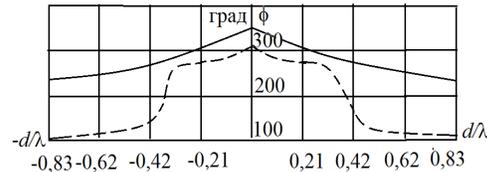


Fig. 3,b

Later, after the publication of the considered method [1–3], appropriate computer environments appeared and other similar works, for example, *NEC4WIN95*, *Super NEC*, *MMANA*, *FEKO*, *MICROWAVE VIZIO* and others similar to the *DISTRIBUTION* program, also based on solving integral linear or surface equations. From these programs, it follows that the results of developing an integral equation method for calculating arbitrarily bent antennas with different loads discussed above have been further developed in world practice.

4. Conclusions

1. An algorithm for the phased development of a wide range of arbitrarily bent wire pin and loop antennas with arbitrary concentrated loads and excitation nodes is included.
2. A comparative analysis of the possibilities and errors of the calculation of the approximate and rigorous methods of calculation of arbitrarily bent similar antennas. It was concluded that approximate methods can be useful, in particular, for a preliminary assessment of the input parameters of antennas, as well as an initial approximation for parametric synthesis and analysis of the antenna or antenna system being developed. If the accuracy of the calculation of antenna parameters by an approximate method is not sufficient, it is recommended to apply the more rigorous and more accurate method of the integral equation.
3. It is shown that the use of the Fredholm integral equations of the first kind for calculating such antennas, taking into account the Poklington integral equation, allows us to expand a variety of versions of the developed antennas.
4. Taking into account these features, a system of integral equations for curved antennas, the arm of which consists of two straight line segments, has been compiled and solved numerically. To solve the problem developed a computer program

DISTRIBUTION. When developing the program, the method of diagonalizing the matrix of the system of equations was applied to improve and restore its conditionality and improve the stability of the solution of the problem for the current.

5. With the use of this program, calculations and developments of a series of new pin and loop curved antennas for spacecraft and other objects, a number of which have been introduced into production and operation, have been performed [11].

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