ISSN 2616-8685. JOURNAL OF PHYSICS AND ELECTRONICS Vol. 26(2), 2018
DOI 10.15421/331818

RELAXATION PROCESSES IN COMPLETELY IONIZED PLASMA IN GENERALIZED LORENTZ MODEL

S.A. Sokolovsky[1], A.I. Sokolovsky[2*], I. S. Kravchuk[2], O.A. Grinishin[2]

[1] *Prydniprovska State Academy of Civil Engineering and Architecture, Dnipro, Ukraine*
[2] *Oles Honchar Dnipro National University, Dnipro, Ukraine*
[*] *e-mail: alexander.i.sokolovsky@gmail.com*

On the basis of the Landau kinetic equation a generalized Lorentz model is proposed, which contrary to the standard model, considers ion system as an equilibrium one. For electron system kinetic equation of the Fokker-Planck type is obtained. In the Bogolyubov method of the reduced description, which is based on his idea of the functional hypothesis, basic equations for electron hydrodynamics construction with account for temperature and macroscopic velocity relaxation processes (kinetic modes of the system) is elaborated. The obtained equations are analyzed near the end of the relaxation processes when the theory has an additional small parameter. The main in small gradients approximation is studied in details, it corresponds to the description of relaxation processes in a spatially uniform case. The obtained equations are approximately solved by the method of truncated expansion in the Sonine polynomials. The velocity and temperature relaxation coefficients are discussed in one- and two-polynomial approximation. As a result the relaxation coefficients are calculated in one-polynomial approximation.

Keywords: method of the reduced description, generalized Lorentz model, kinetic modes, plasma hydrodynamics, relaxation coefficients, one-polynomial approximation.

Received 18.11.2018; Received in revised form 02.12.2018; Accepted 15.12.2018

1. Introduction

Investigation of nonequilibrium processes in a system with account of relaxation processes is an actual modern problem of kinetic theory (see [1, 2] and also a review [3]). Relaxation processes in the narrow sense of the word mean processes that can be observed in spatially uniform states of a system too. In the vicinity of the equilibrium these processes are described by kinetic modes of the system. The problem of their investigation is that the theory has not a small parameter, which could help to investigate these processes. The most important problem of such type is the description of states that precede in time to states in which relaxation processes already have finished. In fact, we are talking about the formation of well-studied nonequilibrium states in the process of evolution. In particular, in a many-component system, for example in plasma, we speak about the processes of alignment of macroscopic velocities and temperature of the components. The pioneering study of such processes was carried out by Landau [4] on the basis of the idea that the local equilibrium state is quickly established in the components of the system (see also the known paper [5]). Another important class of tasks is the problem of dissipative flow formation in hydrodynamics. The pioneering study of such processes was performed by Grad [6] on the basis of using nonequilibrium distribution functions which are taken in the form of truncated expansion in the Hermite orthogonal polynomials. A certain step forward in the study of relaxation processes was our idea of investigating them in the vicinity of the completion of these processes with introducing a new small parameter in the theory (see review [3]). This method has been used in a number of papers for the study of completely ionized plasma [7], subsystems of polarons and phonons of solids [8].

The complexity of such systems is a certain obstacle in this research. That is why in this paper we deal with introducing a generalization of the Lorentz model (see, for example, [9]) and advancing the research into the role of relaxation processes in spatially non-uniform plasma in the framework of this model.

The work is structured as follows. Section 2 formulates a generalized Lorentz model. In Section 3 based on the Bogolyubov reduced description method (see review [5]) the basic equations of hydrodynamics of the electron subsystem of plasma are formulated with taking

into account relaxation processes. In Section 4 a perturbation theory is developed to study the spatially uniform states of the system at the end of the relaxation processes. In Section 5 we are talking about an approximate solution of the integral equations of the theory by the method of the truncated expansion in Sonine polynomials. In Section 6 the coefficients of electron temperature and velocity relaxation are calculated.

2. Generalized Lorentz model

Completely ionized electron-ion plasma is investigated. The Landau kinetic equation in the presence of a constant homogeneous external electric field E_n lies in the basis of our consideration

$$\frac{\partial \mathrm{f}_{ap}(x,t)}{\partial t} = -\frac{p_n}{m_a}\frac{\partial \mathrm{f}_{ap}(x,t)}{\partial x_n} - e_a E_n \frac{\partial \mathrm{f}_{ap}(x,t)}{\partial p_n} = I_{ap}(\mathrm{f}(x,t)). \tag{1}$$

Its collision integral is given by the formula (see, for example, [4, 10])

$$I_{ap}(\mathrm{f}) = 2\pi e_a^2 L \sum_b e_b^2 \frac{\partial}{\partial p_n}\int d^3p' \left\{ \frac{\partial \mathrm{f}_{ap}}{\partial p_{al}}\mathrm{f}_{bp'} - \mathrm{f}_{ap}\frac{\partial \mathrm{f}_{bp'}}{\partial p'_l} \right\} S_{nl}\left(\frac{p}{m_a} - \frac{p'}{m_b}\right) \tag{2}$$

where

$$S_{nl}(u) \equiv (u^2\delta_{nl} - u_n u_l)/u^3 \tag{3}$$

and L is a constant which is called the Coulomb logarithm. Indices a,b run meanings e,i indicating electron and ion components; $e_e = -e$, $e_i = ez$ ($e>0$, z is the charge number of the ion). The component distribution function $\mathrm{f}_{ap}(x,t)$ is normalized by the condition

$$\int d^3p \mathrm{f}_{ap}(x,t) = n_a(x,t) \tag{4}$$

where $n_a(x,t)$ are densities of number of particles of components.

Further we will assume that ions form an equilibrium system in a state of rest with temperature T_0

$$\mathrm{f}_{ip}(x,t) = \mathrm{f}_{ip}^{eq}, \quad \mathrm{f}_{ip}^{eq} = w_{ip}, \qquad w_{ip} \equiv \frac{n_0}{(2\pi M T_0)^{3/2}} e^{-\frac{p^2}{2MT_0}}. \tag{5}$$

(n_0 is ion density, M is an ion mass). The ion-ion and electron-electron interaction are neglected, that is, we introduce a generalized Lorentz model. Electron distribution function is denoted here by $\mathrm{f}_p(x,t)$. The kinetic equation for this function takes the form

$$\frac{\partial \mathrm{f}_p(x,t)}{\partial t} = -\frac{p_n}{m}\frac{\partial \mathrm{f}_p(x,t)}{\partial x_n} + eE_n\frac{\partial \mathrm{f}_p(x,t)}{\partial p_n} + I_p(\mathrm{f}(x,t)) \tag{6}$$

where the ion-electron collision integral is given by the formula

$$I_p(\mathrm{f}) = 2\pi e^4 z^2 L \frac{\partial}{\partial p_n}\int d^3p' \left\{ \frac{\partial \mathrm{f}_p}{\partial p_l} w_{ip'} - \mathrm{f}_p \frac{\partial w_{ip'}}{\partial p'_l} \right\} S_{nl}\left(\frac{p}{m} - \frac{p'}{M}\right). \tag{7}$$

(m is electron mass). Let us introduce the notation

$$D_{nl}(p) = 2\pi\alpha \int d^3p' w_{ip'} S_{nl}\left(\frac{p}{m} - \frac{p'}{M}\right), \qquad \alpha = e^4 z^2 L \tag{8}$$

and take into account that

$$\int d^3p' \frac{\partial w_{ip'}}{\partial p'_l} S_{nl}\left(\frac{p}{m}-\frac{p'}{M}\right) = -\int d^3p' w_{ip'} \frac{\partial}{\partial p'_l} S_{nl}\left(\frac{p}{m}-\frac{p'}{M}\right) =$$
$$= \frac{m}{M}\frac{\partial}{\partial p_l}\int d^3p' w_{ip'} S_{nl}\left(\frac{p}{m}-\frac{p'}{M}\right).$$

Then collision integral (7) can be written in the form

$$I_p(\mathrm{f}) = -\frac{\partial}{\partial p_n}\left(A_n(p)\mathrm{f}_p - D_{nl}(p)\frac{\partial \mathrm{f}_p}{\partial p_l}\right) \tag{9}$$

where we denote

$$A_n(p) = \frac{m}{M}\frac{\partial D_{nl}(p)}{\partial p_l}. \tag{10}$$

We obtain a kinetic equation of the Fokker–Planck type. According to standard terms, quantities $D_{nl}(p)$, $A_n(p)$ are called the diffusion coefficient in the momentum space and the friction force (see about this class of equations, for example, in [9, 10]).

Note that in equilibrium the distribution function of electrons f_p is given by the Maxwell distribution

$$\mathrm{f}_p^{eq} = w_p, \qquad w_p \equiv \frac{n}{(2\pi m T_0)^{3/2}} e^{-\frac{p^2}{2mT_0}} \tag{11}$$

because according to (7) the formula

$$I_p(w) = 0 \tag{12}$$

is true as a consequence of the identity $u_l S_{nl}(u) = 0$. In terms of the notation (9) it means that

$$A_n(p) = -D_{nl}(p)\frac{p_l}{mT_0} \tag{13}$$

is true. So, finally the collision integral (9) takes the form

$$I_p(\mathrm{f}) = \frac{\partial}{\partial p_n}\left(D_{nl}(p)\left(\frac{\partial \mathrm{f}_p}{\partial p_l} + \frac{p_l}{mT_0}\mathrm{f}_p\right)\right) \tag{14}$$

The function $D_{nl}(p)$ defined in (8) has a fairly simple structure, which simplifies our further research. In particular, it has a simple dependence on dimensional quantities. This allows us to find out the structure of dependence on the dimensional values of all objects of the theory.

3. Basic equations of the reduced description method for electron hydrodynamics

Let us construct the basic equations of hydrodynamics in the presence of relaxation processes of the component temperature and velocity equalizing in the generalized Lorentz model. We proceed from the mass, energy, and momentum conservation laws, which, in accordance with the kinetic equation (6), (14), take the form

$$\frac{\partial \sigma(x,t)}{\partial t} = -\frac{\partial \pi_n(x,t)}{\partial x_n}, \qquad \frac{\partial \pi_l(x,t)}{\partial t} = -\frac{\partial t_{nl}(x,\mathrm{f}(t))}{\partial x_n} - \frac{e}{m}E_l\sigma(x,t) + R_l(x,\mathrm{f}(t)),$$
$$\frac{\partial \varepsilon(x,t)}{\partial t} = -\frac{\partial q_n(x,\mathrm{f}(t))}{\partial x_n} - \frac{e}{m}E_n\pi_n(x,t) + R_0(x,\mathrm{f}(t)) \tag{15}$$

where mass $\sigma(x,t)$, momentum $\pi_l(x,t)$ and energy $\varepsilon(x,t)$ densities according to (4) are defined by the formulas

$$\sigma(x,t) = m\int d^3p \mathrm{f}_p(x,t), \qquad \pi_l(x,t) = \int d^3p p_l \mathrm{f}_p(x,t),$$
$$\varepsilon(x,t) = \int d^3p \varepsilon_p \mathrm{f}_p(x,t) \qquad (\varepsilon_p = p^2/2m) \tag{16}$$

(along with the mass density it is convenient to use the density of the number of particles $n = \sigma/m$). Energy $q_n(x,\mathrm{f})$ and momentum $t_{nl}(x,\mathrm{f})$ flux densities and sources $R_0(x,\mathrm{f})$ and $R_l(x,\mathrm{f})$ are given by the expressions

$$t_{ln}(x,\mathrm{f}) \equiv \frac{1}{m}\int d^3p p_l p_n \mathrm{f}_p(x), \qquad q_n(x,\mathrm{f}) \equiv \frac{1}{m}\int d^3p \varepsilon_p p_n \mathrm{f}_p(x);$$
$$R_0(x,\mathrm{f}) \equiv \int d^3p \varepsilon_p I_p(x,\mathrm{f}), \qquad R_l(x,\mathrm{f}) \equiv \int d^3p p_l I_p(x,\mathrm{f}). \tag{17}$$

Electron component temperature $T(x,t)$ and velocity $u_n(x,t)$ are the basic hydrodynamic parameters and defined by usual formulas

$$\varepsilon(x,t) = \frac{3}{2m}\sigma(x,t)T(x,t) + \frac{1}{2}\sigma(x,t)u(x,t)^2, \qquad \pi_l(x,t) = \sigma(x,t)u_l(x,t) \tag{18}$$

because electron-electron interaction is neglected. They are based on the local equilibrium idea according to which

$$\varepsilon = \int d^3p \varepsilon_p w_{p-mu}, \qquad \pi_l = \int d^3p p_l w_{p-mu} \tag{19}$$

where the Maxwell distribution w_p defined by (11) is used.

Equations (15) give the equations of hydrodynamics if we express their right-hand sides via independent variables (parameters of the reduced description)

$$\xi_\mu(x,t): \quad \xi_0(x,t) \equiv T(x,t), \quad \xi_l(x,t) \equiv u_l(x,t), \quad \xi_4(x,t) \equiv \sigma(x,t), \tag{20}$$

which describe a state of the system in hydrodynamics. This can be done through placing the Bogolyubov idea of the functional hypothesis

$$\mathrm{f}_p(x,t) \xrightarrow[t \gg \tau_0]{} \mathrm{f}_p(x,\xi(t)) \tag{21}$$

in the basis of consideration. According to this idea the distribution function $\mathrm{f}_p(x,t)$ becomes a functional $\mathrm{f}_p(x,\xi)$ of variables $\xi_\mu(x)$ at the times much greater than a certain characteristic time τ_0. This relation is the basis of the Bogolyubov formulation of the Chapman–Enskog method, which is a special case of his method of reduced description. In accordance with (16) and (18) the reduced description parameters are expressed in terms of some moments of the distribution function. Since the distribution function is determined by all moments, such a description is a reduced one.

In the system under consideration the electron component temperature and velocity relaxation is observed

$$T(x,t) \xrightarrow[t \gg \tau_T]{} T_0, \qquad u_l(x,t) \xrightarrow[t \gg \tau_u]{} -\kappa E_l \tag{22}$$

where τ_T and τ_u are corresponding relaxation times, κ is electron mobility. The time τ_0 in the functional hypothesis (21) is assumed to fulfill the condition $\tau_0 \ll \tau_T, \tau_u$, hence the relaxation processes in the situation under consideration are still taking place. This does not fit into the standard Chapman–Enskog method, in which the only small parameter is

the parameter of the smallness g of the gradients of the reduced description parameters that is introduced by the estimates

$$\partial^s \xi_\mu(x) / \partial x_{n_1} ... \partial x_{n_s} \sim g^s, \qquad g = l_{\text{fp}} / L. \tag{23}$$

Here l_{fp} is the mean free path, L is the characteristic size of the non-homogeneity of the electron distribution in space. Zero approximation in the gradients $\mathrm{f}_p^{(0)}$ for the distribution function $\mathrm{f}_p(x,\xi)$ describes relaxation in a spatially uniform case. The distribution function $\mathrm{f}_p^{(0)}$ satisfies the integral equation, whose solution cannot be found in the standard Chapman–Enskog method. In our work a new small parameter ε that allows us to calculate the distribution function $\mathrm{f}_p^{(0)}$ is introduced.

As a result, the equations of hydrodynamics can be written in the form

$$\frac{\partial \xi_\mu(x,t)}{\partial t} = L_\mu(x, \mathrm{f}(\xi(t))) \tag{24}$$

where $L_\mu(x,\mathrm{f})$ is a functional of $\mathrm{f}_p(x)$. In accordance with (15) and (18), the exact form of these equations is given by the formulas

$$\begin{gathered} \frac{\partial \sigma}{\partial t} = -\frac{\partial \sigma u_l}{\partial x_l}, \qquad \frac{\partial u_l}{\partial t} = \frac{1}{\sigma} u_l \frac{\partial \sigma u_m}{\partial x_m} - \frac{1}{\sigma}\frac{\partial t_{lm}}{\partial x_m} + \frac{1}{\sigma}(R_l - enE_l), \\ \frac{\partial T}{\partial t} = \frac{3T - mu^2}{3\sigma}\frac{\partial \sigma u_l}{\partial x_l} + \frac{2}{3n}\frac{\partial t_{lm}}{\partial x_m} u_l - \frac{2}{3n}\frac{\partial q_l}{\partial x_l} + \frac{2}{3n}\left(R_0 - enR_l u_l\right). \end{gathered} \tag{25}$$

For the distribution function at the reduced description $\mathrm{f}_p(x,\xi)$ the equation

$$\sum_\mu \int d^3x' \frac{\delta \mathrm{f}_p(x,\xi)}{\delta \xi_\mu(x')} L_\mu(x', \mathrm{f}(\xi)) = -\frac{p_n}{m}\frac{\partial \mathrm{f}_p(x,\xi)}{\partial x_n} + eE_n \frac{\partial \mathrm{f}_p(x,\xi)}{\partial p_n} + I_p(x, \mathrm{f}(\xi)) \tag{26}$$

is obtained from the kinetic equation (6) with taking into account the functional hypothesis (22). Definitions (16) and (18) of the parameters of the reduced description $\xi_\mu(x)$ give additional conditions for this equation

$$\begin{gathered} \int d^3p \mathrm{f}_p(x,\xi)\varepsilon_p = \frac{3}{2} n(x)T(x) + \frac{1}{2}\sigma(x)u(x)^2, \\ \int d^3p \mathrm{f}_p(x,\xi) p_l = \sigma(x) u_l(x), \qquad \int d^3p \mathrm{f}_p(x,\xi) = n(x). \end{gathered} \tag{27}$$

To construct the equations of hydrodynamics, it is necessary to solve equation (26) with the relations (27) in perturbation theory in gradients based on the estimations (23).

4. Construction of the perturbation theory for solving the equations of the theory

The basis of this work is the idea of studying hydrodynamics taking into account the relaxation phenomena near their completion, when the quantities $T(x,t) - T_0$ and $u_n(x,t) + \mu E_n(x)$ in accordance with (23) are small

$$\tau(x,t) \equiv T(x,t) - T_0 \sim \varepsilon, \qquad u_n(x,t) + \mu E_n(x) \sim \varepsilon \tag{28}$$

where ε is a small parameter introduced formally. To simplify the consideration, we will assume that the electric field is weak. Therefore, according to (23) and (28) the estimates

$$u_l(x,t)\sim\varepsilon,\qquad \tau(x,t)\sim\varepsilon,\qquad E_l\sim\varepsilon;\qquad \sigma\sim g^0,\qquad \frac{\partial\sigma}{\partial x_l}\sim g;$$
$$\frac{\partial u_l(x,t)}{\partial x_m}\sim g\varepsilon,\qquad \frac{\partial T(x,t)}{\partial x_l}\sim g\varepsilon \tag{29}$$

are true. The solution of equation (26), with taking into account (27), is sought in the form of a double series in the gradients of parameters $\xi_\mu(x)$ and the small parameter ε

$$\mathrm{f}_p(x,\xi)=\mathrm{f}_p^{(0)}+\mathrm{f}_p^{(1)}+O(g^2),$$
$$\mathrm{f}_p^{(0)}=\mathrm{f}_p^{(0,0)}+\mathrm{f}_p^{(0,1)}+\mathrm{f}_p^{(0,2)}+O(g^0\varepsilon^3),\qquad \mathrm{f}_p^{(1)}=\mathrm{f}_p^{(1,0)}+\mathrm{f}_p^{(1,1)}+O(g^1\varepsilon^2) \tag{30}$$

where a quantity $\mathrm{f}_p^{(m,n)}\sim g^m\varepsilon^n$. Similarly the contributions of the perturbation theory to flows, sources, and collision integral $q_l(x,\mathrm{f})$, $t_{lm}(x,\mathrm{f})$, $R_0(x,\mathrm{f})$, $R_l(x,\mathrm{f})$, $I_p(x,\mathrm{f})$ are denoted. In this paper we restrict ourselves to considering the basic (zero) approximation in gradients, which describes the evolution of a spatially uniform system, that is, only the relaxation phenomena in the narrow sense. The results of the previous section allow us to construct a theory of relaxation processes in a spatially non-uniform case (in other words, hydrodynamics taking into account relaxation processes), it will be done in another work.

Distribution of the gradient-zero approximation $\mathrm{f}_p^{(0)}$, in accordance with (25) and (26), satisfies the nonlinear differential equation

$$\frac{\partial\mathrm{f}_p^{(0)}}{\partial\upsilon_l}\frac{1}{\sigma}\{R_l(\mathrm{f}^{(0)})-neE_l\}+\frac{\partial\mathrm{f}_p^{(0)}}{\partial T}\frac{2}{3n}\{R_0(\mathrm{f}^{(0)})-R_l(\mathrm{f}^{(0)})u_l\}+eE_l\frac{\partial\mathrm{f}_p^{(0)}}{\partial p_l}=I_p(\mathrm{f}^{(0)}) \tag{31}$$

with additional conditions

$$\int d^3p\mathrm{f}_p^{(0)}\varepsilon_p=\frac{3}{2}nT+\frac{1}{2}\sigma u^2,\qquad \int d^3p\mathrm{f}_p^{(0)}p_l=\sigma u_l,\qquad m\int d^3p\mathrm{f}_p^{(0)}=\sigma. \tag{32}$$

It was noted above that methods for solving such nonlinear equations were not developed. In this work it is proposed to investigate them near the end of the relaxation, when one can use the perturbation theory in parameter ε. The contributions of the order ε^0 to (31) and (32) have the form

$$\frac{\partial\mathrm{f}_p^{(0,1)}}{\partial u_l}\frac{1}{\sigma}R_l(\mathrm{f}^{(0,0)})+\frac{\partial\mathrm{f}_p^{(0,0)}}{\partial u_l}\frac{1}{\sigma}R_l(\mathrm{f}^{(0,1)})-\frac{\partial\mathrm{f}_p^{(0,0)}}{\partial u_l}neE_l+$$
$$+\frac{\partial\mathrm{f}_p^{(0,1)}}{\partial T}\frac{2}{3n}R_0(\mathrm{f}^{(0,0)})+\frac{\partial\mathrm{f}_p^{(0,0)}}{\partial T}\frac{2}{3n}R_0(\mathrm{f}^{(0,1)})-\frac{\partial\mathrm{f}_p^{(0,0)}}{\partial T}\frac{2}{3n}R_l(\mathrm{f}^{(0,0)})\upsilon_l=I_p(\mathrm{f}^{(0,0)});$$
$$\int d^3p\mathrm{f}_p^{(0,0)}\varepsilon_p=\frac{3}{2}nT_0,\qquad \int d^3p\mathrm{f}_p^{(0,0)}p_l=0,\qquad m\int d^3p\mathrm{f}_p^{(0,0)}=\sigma. \tag{33}$$

According to (12), the collision integral $I_p(\mathrm{f})$ with the Maxwell distribution w_p defined in (11) equals to zero. The definition of sources in the conservation laws (17) shows that they with w_p also equal to zero

$$R_0(w)=0,\qquad R_l(w)=0. \tag{34}$$

On this basis it is established that the solution of equations (33) is given by the Maxwell distribution w_p from (11)

$$\mathrm{f}_p^{(0,0)} = w_p . \tag{35}$$

Simultaneously from formulas (20) we find the flows

$$t_{ln}^{(0,0)} \equiv t_{ln}(x, \mathrm{f}^{(0,0)}) = \frac{1}{m}\int d^3 p p_l p_n \mathrm{f}_p^{(0,0)} = nT_0\delta_{nl},$$
$$q_n^{(0,0)} \equiv q_n(x, \mathrm{f}^{(0,0)}) = \frac{1}{m}\int d^3 p \varepsilon_p p_n \mathrm{f}_p^{(0,0)} = 0 . \tag{36}$$

In the first order in the parameter ε from equations (31) and (32) taking into account (12) and (35) we obtain

$$\frac{\partial \mathrm{f}_p^{(0,1)}}{\partial u_l}\frac{1}{\sigma}R_l(\mathrm{f}^{(0,1)}) - \frac{\partial \mathrm{f}_p^{(0,1)}}{\partial u_l}\frac{e}{m}E_l + \frac{\partial \mathrm{f}_p^{(0,1)}}{\partial T}\frac{2}{3n}R_0(\mathrm{f}^{(0,1)}) + eE_l\frac{\partial \mathrm{f}_p^{(0,0)}}{\partial p_l} = I_p(\mathrm{f}^{(0,1)});$$
$$\int d^3 p \mathrm{f}_p^{(0,1)}\varepsilon_p = \frac{3}{2}n\tau , \quad \int d^3 p \mathrm{f}_p^{(0,1)} p_l = \sigma u_l , \quad \int d^3 p \mathrm{f}_p^{(0,1)} = 0 . \tag{37}$$

For reasons of rotational invariance the solution of these equations has the structure

$$\mathrm{f}_p^{(0,1)} = w_p\{A_p p_l u_l + B_p\tau + C_p p_l E_l\} \tag{38}$$

where A_p, B_p, C_p are some scalar functions. Substituting this function into equation (37) with account for the independence of the variables u_l, τ, E_l in (38) gives equations for the quantities A_p, B_p, C_p with additional conditions to them

$$\hat{K}A_p p_l = \lambda_u A_p p_l , \quad \langle \varepsilon_p A_p\rangle = \frac{3}{2}n ; \quad \hat{K}B_p = \lambda_T B_p , \quad \langle \varepsilon_p B_p\rangle = \frac{3}{2}n , \quad \langle B_p\rangle = 0 ;$$
$$\hat{K}C_p p_l = \left(\nu A_p - \frac{e\beta}{m}\right)p_l , \quad \langle \varepsilon_p C_p\rangle = 0 . \tag{39}$$

Hereafter for an arbitrary quantity a_p we denote

$$\langle a_p\rangle = \int d^3 p w_p a_p \tag{40}$$

as well as introduce coefficients λ_u, λ_T, and ν by the formulas

$$\lambda_u\delta_{nl} = -\frac{1}{\sigma}R_n(wAp_l) , \quad \lambda_T = -\frac{2}{3n}R_0(wB) , \quad \nu\delta_{ln} = -\frac{1}{\sigma}R_l(wCp_n) + \frac{e}{m}\delta_{nl} . \tag{41}$$

For further it is convenient to use the operator $\hat{K}$

$$I_p(wa) = -w_p\hat{K}a_p \tag{42}$$

which, according to (14), is given by the formula

$$\hat{K}a_p = -\left(\frac{\partial}{\partial p_n} - \frac{p_n}{mT_0}\right)D_{nl}(p)\frac{\partial a_p}{\partial p_l} . \tag{43}$$

With the help of the operator $\hat{K}$ one can introduce the bilinear form (brackets)

$$\{a_p, b_p\} = \int d^3 p w_p a_p \hat{K} b_p = \int d^3 p w_p D_{nl}(p)\frac{\partial a_p}{\partial p_n}\frac{\partial b_p}{\partial p_l} , \tag{44}$$

which has important properties

$$\{a_p, b_p\} = \{b_p, a_p\} , \qquad \{a_p, a_p\} \geq 0 , \qquad \{a_p, c\} = 0 \tag{45}$$

(a_p, b_p are arbitrary functions, c is an arbitrary constant).

Sources (17) in the conservation laws (15) are expressed in terms of this form by relations

$$R_0(wa) = -\{\varepsilon_p, a_p\}, \qquad R_l(wa) = -\{p_l, a_p\}. \tag{46}$$

Herewith the coefficients λ_u, λ_T, and ν according to (42) are given by the formulas

$$\lambda_u = \frac{1}{3\sigma}\{p_n, A_p p_n\}, \quad \lambda_T = \frac{2}{3n}\{\varepsilon_p, B_p\}, \quad \nu = \frac{1}{3\sigma}\{p_l, C_p p_l\} + \frac{e}{m}. \tag{47}$$

The meaning of these coefficients is understood from the time equations for the reduced description parameters (28), which in the order $g^0\varepsilon^1$ of the perturbation theory, with taking into account (38) and (46), give

$$\left(\frac{\partial\sigma}{\partial t}\right)^{(0,1)} = 0, \qquad \left(\frac{\partial u_l}{\partial t}\right)^{(0,1)} = -\lambda_u u_l + \nu E_l, \qquad \left(\frac{\partial T}{\partial t}\right)^{(0,1)} = -\lambda_T \tau. \tag{48}$$

Hereafter we call λ_u and λ_T the relaxation coefficients. From formulas (45) and identities

$$\lambda_u \langle A_p^2 p^2 \rangle = \{A_p p_l, A_p p_l\}, \qquad \lambda_T \langle B_p^2 \rangle = \{B_p, B_p\}, \tag{49}$$

which follow from equations (38) and definition (44), it is clear that these coefficients are positive. They describe the relaxation (22) with characteristic times

$$\tau_u = \lambda_u^{-1}, \qquad \tau_T = \lambda_T^{-1}. \tag{50}$$

Note that the relations (47) do not make the equations (39) nonlinear. Formulas (47) are the consequences of equations (39). Functions $A_p p_l$ and B_p are the operator $\hat{K}$ eigenfunctions, which correspond to their eigenvalues λ_u and λ_T. In this sense eigenvalues are functions of their eigenfunctions.

Note that the operator $\hat{K}$ is a Hermitian one when the scalar product is defined by the formula

$$(a_p, b_p) = \int d^3 p w_p a_p^* b_p. \tag{51}$$

This is evident from the transformations

$$\begin{aligned}(a_p, \hat{K} b_p) = \int d^3 p w_p a_p^* \hat{K} b_p = \{a_p^*, b_p\} = \{b_p, a_p^*\} = \{b_p^*, a_p\}^* = \left(\int d^3 p w_p b_p^* \hat{K} a_p\right)^* = \\ = (b_p, \hat{K} a_p)^* = (\hat{K} a_p, b_p),\end{aligned} \tag{52}$$

which take into account relations (44) and (45). This property of the operator $\hat{K}$ can be used in the analysis of approximate methods for solving equations (39).

5. Method of the truncated expansion in the Sonine polynomials for calculating the theory objects in spatially uniform state

At approximate solution of equations (39) by an expansion in orthogonal polynomials it is expedient to use the Sonine polynomials $S_q^\alpha(x)$ ($q = 0,1,2,\ldots$, α is a real number), which are defined by conditions

$$\int_0^{+\infty} dx\, x^\alpha e^{-x} S_q^\alpha(x) S_{q'}^\alpha(x) = \delta_{qq'} \frac{\Gamma(q+\alpha+1)}{q!}, \quad S_q^\alpha(x) = \sum_{0 \le s \le q} a_s x^s, \quad a_q = (-1)^q \tag{53}$$

(see, for example, [9, 11]). This in particular gives

$$S_0^\alpha(x) = 1, \quad S_1^\alpha(x) = -x + 1 + \alpha, \quad S_2^\alpha(x) = \{x^2 - 2(2+\alpha)x + (2+\alpha)(1+\alpha)\}/2. \tag{54}$$

The condition of orthonormalization (53) can be written in the form of the average (40) with the Maxwell distribution

$$\langle \varepsilon_p^{\alpha-1/2} S_q^{\alpha}(\beta\varepsilon_p) S_{q'}^{\alpha}(\beta\varepsilon_p)\rangle = \frac{n}{\beta^{\alpha-1/2}} \frac{2\Gamma(q+\alpha+1)}{\pi^{1/2} q!}\delta_{qq'} \qquad (\beta \equiv T_0^{-1}), \tag{55}$$

which indicates the expediency of using Sonine polynomials.

Let us find the functions A_p, B_p, and C_p in the form of expansions

$$A_p = \sum_{q=0}^{\infty} a_q S_q^{3/2}(\beta\varepsilon_p), \quad B_p = \sum_{q=0}^{\infty} b_q S_q^{1/2}(\beta\varepsilon_p), \quad C_p = \sum_{q=0}^{\infty} c_q S_q^{3/2}(\beta\varepsilon_p), \tag{56}$$

for which the orthonortmalization conditions take the form

$$\begin{aligned} &\langle p^2 S_q^{3/2}(\beta\varepsilon_p) S_{q'}^{3/2}(\beta\varepsilon_p)\rangle = x_q \delta_{qq'}, \qquad && x_q \equiv \frac{4mn}{\beta}\frac{\Gamma(q+5/2)}{\pi^{1/2} q!}, \\ &\langle S_q^{1/2}(\beta\varepsilon_p) S_{q'}^{1/2}(\beta\varepsilon_p)\rangle = y_q \delta_{qq'}, && y_q \equiv 2n\frac{\Gamma(q+3/2)}{\pi^{1/2} q!}. \end{aligned} \tag{57}$$

Herewith the additional conditions (39) get a simple form

$$a_0 = \beta, \quad b_0 = 0, \quad b_1 = -\beta, \quad c_0 = 0. \tag{58}$$

The equations for the coefficients a_q and b_q will be found as it follows. We substitute the expansions (56) into equations (39), multiply them by $p_l S_q^{3/2}(\beta\varepsilon_p)$ and $S_q^{1/2}(\beta\varepsilon_p)$, correspondingly, take the average value with the distribution function w_p in the both sides and take into account (57). As a result we have

$$\sum_{q'=0}^{\infty} A_{qq'} a_{q'} = \lambda_u a_q x_q, \quad \sum_{q'=0}^{\infty} B_{qq'} b_{q'} = \lambda_T b_q y_q, \quad \sum_{q'=0}^{\infty} A_{qq'} c_{q'} = \nu a_q x_q - 3en^{1/2}\delta_{q,0}, \tag{59}$$

where the notations

$$A_{qq'} = \{p_l S_q^{3/2}(\beta\varepsilon_p), p_l S_{q'}^{3/2}(\beta\varepsilon_p)\}, \qquad B_{qq'} = \{S_q^{1/2}(\beta\varepsilon_p), S_{q'}^{1/2}(\beta\varepsilon_p)\}. \tag{60}$$

are introduced. The first two equations (59) can also be written in the form

$$\sum_{q'=0}^{\infty} \overline{A}_{qq'} a_{q'} x_{q'}^{1/2} = \lambda_u a_q x_q^{1/2}, \qquad \sum_{q'=0}^{\infty} \overline{B}_{qq'} b_{q'} y_{q'}^{1/2} = \lambda_T b_q y_q^{1/2} \tag{61}$$

where

$$\overline{A}_{qq'} \equiv A_{qq'}(x_q x_{q'})^{1/2}, \qquad \overline{B}_{qq'} \equiv B_{qq'}(y_q y_{q'})^{1/2}. \tag{62}$$

Equations (59) in the form (61) are the equations for the eigenvectors $a_q x_q^{1/2}$ and $b_q y_q^{1/2}$ and the corresponding eigenvalues λ_u and λ_T. Eigenvalues λ_u and λ_T formally are solutions of equations

$$\det \| \overline{A}_{qq'} - \lambda_u \delta_{qq'} \| = 0, \qquad \det \| \overline{B}_{qq'} - \lambda_T \delta_{qq'} \| = 0. \tag{63}$$

These equations have an infinite set of solutions, from which the minimum ones have to be chosen. Such solutions describe the slowest relaxation process.

Additional conditions (39) give the contribution (58) of the first polynomials to expansion (56). We will say that the solutions of equations (59) are calculated in the s-polynomial approximation if all the coefficients of the expansion (56) starting from

$(s+1)$-th are assumed to be zero. For example, equations (59) and their solutions in the approximation of s polynomials have the form

$$\sum_{q'=0}^{s-1} A_{qq'} a_{q'}^{[s]} = \lambda_u^{[s]} a_q^{[s]} x_q \quad (q = 0,...,s-1); \qquad A_p^{[s]} = \sum_{q=0}^{s-1} a_q^{[s]} S_q^{3/2}(\beta\varepsilon_p), \qquad a_0^{[s]} = \beta;$$
$$\sum_{q'=1}^{s} B_{qq'} b_{q'}^{[s]} = \lambda_T^{[s]} b_q^{[s]} y_q \quad (q = 1,...,s); \qquad B_p^{[s]} = \sum_{n=1}^{s} b_n^{[s]} S_n^{1/2}(\beta\varepsilon_p), \qquad b_1^{[s]} = -\beta. \tag{64}$$

Here as unknowns the quantities $a_n^{[s]}$ $(1 \le n \le s-1)$, $\lambda_u^{[s]}$ and $b_n^{[s]}$ $(2 \le n \le s)$, $\lambda_T^{[s]}$ should be considered. Further at quantities for simplicity we will omit an index s, which indicates the number of polynomials in the considered approximation.

In the one-polynomial approximation the equations (59) give

$$\begin{aligned} &A_{00}a_0 = \lambda_u a_0 x_0, && a_0 = \beta, && A_p = a_0 S_0^{3/2}(\beta\varepsilon_p); \\ &B_{11}b_1 = \lambda_T b_1 y_1, && b_1 = -\beta, && B_p = b_1 S_1^{1/2}(\beta\varepsilon_p); \\ &A_{01}c_1 = \nu a_0 x_0 - 3en^{1/2}, && A_{11}c_1 = \nu a_1 x_1, && C_p = c_1 S_1^{3/2}(\beta\varepsilon_p), \end{aligned} \tag{65}$$

and therefore

$$A_p = \beta, \quad \lambda_u = \frac{\beta}{3mn}\{p_l, p_l\}; \quad B_p = -\beta S_1^{1/2}(\beta\varepsilon_p), \quad \lambda_T = \frac{2\beta^2}{3n}\{\varepsilon_p, \varepsilon_p\};$$
$$C_p = 0, \quad \nu = \frac{e}{m}. \tag{66}$$

(the same expressions are given by formulas (47)). The coefficient c_1 in (65) should be assumed to be zero, since in the one-polynomial approximation one must take $a_1 = 0$.

In the two-polynomial approximation the formulas (59) give the following equations for the quantities λ_u, a_1, λ_T, b_2 and functions A_p, B_p

$$\begin{cases} A_{00}a_0 + A_{01}a_1 = \lambda_\upsilon a_0 x_0, \\ A_{10}a_0 + A_{11}a_1 = \lambda_\upsilon a_1 x_1 \end{cases}, \qquad A_p = a_0 S_0^{3/2}(\beta\varepsilon_p) + a_1 S_1^{3/2}(\beta\varepsilon_p), \quad a_0 = \beta;$$
$$\begin{cases} B_{11}b_1 + B_{12}b_2 = \lambda_\tau b_1 y_1, \\ B_{21}b_1 + B_{22}b_2 = \lambda_\tau b_2 y_2 \end{cases}, \qquad B_p = b_1 S_1^{1/2}(\beta\varepsilon_p) + b_2 S_2^{1/2}(\beta\varepsilon_p), \quad b_1 = -\beta. \tag{67}$$

The set of equations (67) gives such an expression for the smallest root λ_u of two ones and the expression for the coefficient a_1

$$\lambda_u = \left\{(x_0A_{11} + x_1A_{00}) - [(x_0A_{11} + x_1A_{00})^2 - 4x_0x_1(A_{11}A_{00} - A_{01}A_{10})]^{1/2}\right\} / 2x_0x_1,$$
$$a_1 = (\lambda_\upsilon x_0 - A_{00})\beta / A_{01} \tag{68}$$

Also it gives such an expression for the smallest root λ_T of two ones and the expression for the coefficient b_2

$$\lambda_T = \left\{(y_1B_{22} + y_2B_{11}) - [(y_1B_{22} + y_2B_{11})^2 - 4y_1y_2(B_{11}B_{22} - B_{12}B_{21})]^{1/2}\right\} / 2y_1y_2$$
$$b_2 = (B_{11} - \lambda_T y_1)\beta / B_{12} \tag{69}$$

A detailed analysis of the two-polynomial approximation and the convergence of the method of the truncated expansion in the Sonine polynomials for the generalized Lorentz model will be carried out in another paper.

6. Calculation of the relaxation coefficients λ_u, λ_T in one-polynomial approximation

In accordance with expression (66), the brackets needed to calculate the relaxation coefficients λ_u and λ_T in the one-polynomial approximation are $\{p_l, p_l\}$ and $\{\varepsilon_p, \varepsilon_p\}$. Taking into account the definitions of the brackets (44), which include the function $D_{nl}(p)$ from (8), one has

$$\{a_p, b_p\} = 2\pi\alpha \int d^3p d^3p' w_p w_{ip'} S_{nl}\left(\frac{p}{m} - \frac{p'}{M}\right)\frac{\partial a_p}{\partial p_n}\frac{\partial b_p}{\partial p_l}. \tag{70}$$

Let us calculate a more general value, through which the brackets are expressed

$$I_{nl} \equiv \int d^3p_1 d^3p_2 w_{p_1} w_{ip_2} S_{nl}\left(\frac{p_1}{m} - \frac{p_2}{M}\right)\mathrm{f}(p_1, p_2). \tag{71}$$

In order to do this, let us introduce in the integral the velocities that are used in the two body problem

$$p_1 \equiv m\upsilon_1, \qquad p_2 \equiv M\upsilon_2; \qquad \upsilon_c \equiv \frac{m\upsilon_1 + M\upsilon_2}{m+M}, \qquad \upsilon \equiv \upsilon_1 - \upsilon_2,$$

Given the well-known identities of this problem $d^3\upsilon_1 d^3\upsilon_2 = d^3\upsilon d^3\upsilon_c$, we have

$$I_{nl} = (mM)^3 \int d^3\upsilon d^3\upsilon_c \frac{nn_0}{(2\pi T_0 m)^{3/2}(2\pi T_0 M)^{3/2}} e^{-\frac{(m+M)\upsilon_c^2}{2T_0}} e^{-\frac{mM\upsilon^2}{2(m+M)T_0}} S_{nl}(\upsilon)\times$$

$$\times \mathrm{f}(m\upsilon_1, M\upsilon_2).$$

Another substitution of variables

$$\upsilon_c \equiv \left(\frac{T_0}{m+M}\right)^{1/2} q_c, \qquad \upsilon \equiv \left(\frac{(m+M)T_0}{mM}\right)^{1/2} q$$

finally gives

$$I_{nl} = \frac{nn_0 m^{1/2}}{(1+\lambda^2)^{1/2} T_0^{1/2}} \int d^3q_c d^3q w_{q_c} w_q S_{nl}(q)\,\bar{\mathrm{f}}(q, q_c) \tag{72}$$

where

$$\bar{\mathrm{f}}(q, q_c) \equiv \mathrm{f}\left(\frac{(mT_0)^{1/2}}{(1+\mu^2)^{1/2}}(\mu q_c + q), \frac{(mT_0)^{1/2}}{(1+\mu^2)^{1/2}}\left(\frac{1}{\mu}q_c - q\right)\right); \quad w_q \equiv \frac{1}{(2\pi)^{3/2}} e^{-\frac{q^2}{2}}. \tag{73}$$

In the problem of completely ionized plasma the magnitude $\mu \equiv (m/M)^{1/2}$ is a small parameter, since the mass of an ion M is much larger than the electron mass m.

Formulas (66) and relations (70)-(73) give the following expressions for the temperature and velocity relaxation coefficients in the one-polynomial approximation

$$\lambda_u = \frac{2^{5/2}\pi^{1/2}\alpha n_0}{3(1+\mu^2)^{1/2} m^{1/2} T_0^{3/2}}, \quad \lambda_T = \frac{2^{7/2}\pi^{1/2}\mu^2\alpha n_0}{3(1+\mu^2)^{3/2} m^{1/2} T_0^{3/2}} = \frac{2\mu^2}{1+\mu^2}\lambda_u \tag{74}$$

In paper [7] these values were calculated in the main approximation in the parameter μ. At the same time our expression for λ_u matches the expression [7], but our expression for λ_T gives the expression of [7] after the replacement $n_0 \to n_0(z+1)$. This result is expected since in [7] the dynamics of ions was more fully taken into account, but the calculations could be carried out only with the additional assumption of μ smallness.

7. Conclusions

In the paper based on the Landau's kinetic equation for a completely ionized plasma a generalized Lorentz model, which considers the system of ions to be equilibrium one, is

developed. As a result, the plasma electron system is described by a Fokker-Planck kinetic equation. In the Bogolyubov reduced description method the basic equations for constructing the hydrodynamics of electrons are developed. They take into account the temperature and macroscopic velocity relaxation processes. The equations of the theory are analyzed at the end of the relaxation processes with restricting ourselves by considering the basic in the gradients approximation, which corresponds to the description of relaxation processes in a spatially homogeneous case. The resulting equations are solved approximately by the method of the truncated expansion in the Sonine polynomials, limiting the consideration by one- and two-polynomial approximation. Finally the coefficients of relaxation of temperature and electron velocity are calculated in the one-polynomial approximation.

References

1. **Jou D.** Extended Irreversible Thermodynamics/ D. Jou, J. Casas-Vazques, G. Lebon. – New York: Springer, 2010. – 483 p.

2. **Navinder Singh.** Two-temperature model of nonequilibrium electron relaxation: a review / Singh Navinder // International Journal of Modern Physics B. – 2010. – Vol. 24, No. 9. – P. 1141 – 1158.

3. **Gorev, V. N.** New method in the theory of relaxation processes for non-uniform media / V.N. Gorev, S.A. Sokolovsky, A. I. Sokolovsky // Visnik Dnipropetrovs'kogo Unìversitetu. Seria Fizika, Radioelektronika. – 2016. – V. 25, Issue 24. – P. 83 – 93.

4. **Landau, L.D.** Kinetic equation in case of Coulomb interaction / L.D. Landau // ZhETF. – 1937. – Vol. 7. – P. 203-209 (in Russian).

5. **Braginsky, S.I.** Transport phenomena in completely ionixzed two-temperature plasma / S.I. Braginsky // ZhETF. – 1957. – Vol. 33. – P. 459 – 472 (in Russian).

6. **Grad, H.** On the kinetic theory of rarefied gases /H. Grad // Commun. on Pure and Appl. Math. – 1949 – Vol. 2, No. 4. – P. 331 – 407.

7. **Gorev, V.N.** Hydrodynamic, kinetic modes of plasma and relaxation damping of plasma oscillations / V.N. Gorev, A.I. Sokolovsky// Ukrainian Journal of Physics. – 2015. – Vol. 60, No. 3. – P. 232 – 246.

8. **Sokolovsky S.A.** Toward polaron kinetics in the Bogoliubov reduced description method / S.A. Sokolovsky // Theoretical and Mathematical Physics. – 2011. – Vol. 168, No. 2. – P. 1150 – 1164.

9. **Lifshitz E.M.** Physical Kinetics / E.M. Lifshitz, L.P. Pitaevskii. – Oxford: Pergamon Press, 1981, – 452 p.

10. **Akhiezer A.I**. Methods of Statistical Physics / A.I. Akhiezer, S.V. Peletminskii. – Oxford: Pergamon Press, 1981. – 450 p.

11. **Ferziger, J.H.** Mathematical Theory of Transport Processes in Gases / J.H. Ferziger, H.G. Kaper. – Amsterdam: North-Holland, 1972. – 554 p.

10. **Yukhnovsky, I.R.** Phase transitions in the vicinity of critical point gas–liquid / I.R. Yukhnovsky // Ukr. Journ. of Physics. Reviews. – 2015. – V.10, № 1. – P. 33 – 97. (in Ukrainian).

11. **Kocharovsky, Vitaly V.** Microscopic theory of phase transitions in a critical region / Vitaly V. Kocharovsky, Vladimir V. Kocharovsky // Physica Scripta. – 2015. – Vol. 90. – P. 108002 (23 pp.).

12. **Bulakhov, M.** Re-examining the quadratic approximation in theory of a weakly interacting Bose gas with condensate: the role of nonlocal interaction potentials / M. Bulakhov, A. Peletminskii, S. Peletminskii, Y. Slyusarenko and A. Sotnikov // arXiv: 1804.10398v1 [cond-mat.stat-mech].

13. **Abrikosov, A.A.** Methods of Quantum Field Theory in Statistical Physics / A.A. Abrikosov, L.P. Gorkov, I.E. Dzyaloshinskii. – New Jersey: Prentice-Hall, 1963. – 352 p.

14. **Lebedev, V.V.** Fluctuation Effects in Macrophysics. / V.V. Lebedev. – M.: MTsNMO, 2004. – 256 p. (in Russian).

15. **Bogoliubov, N.N.** On the theory of superfluidity / N.N. Bogoliubov // Journal of Physics. – 1947. –Vol. 11 (1). – P. 23 – 32.