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AN INTEGRAL EQUATION TECHNIQUE FOR THE ANALYSIS OF PHASED ARRAY ANTENNA WITH MATCHING STEP DISCONTINUITIES

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Applying the integral equation method of overlapping partial domains and the Schwartz alternating method to solving an electromagnetic wave diffraction problem is considered in this paper. The infinite rectangular waveguide phased array antenna scanning in H plane which waveguides have step matching discontinuities is represented. The whole field definition domain is sliced into three overlapping partial domains. The system of integral representations for unknown E_y components of the electrical field vector in each domain is set up using Green's functions. Unknown functions in each domain are presented as orthogonal eigenfunction series. Using Galerkin's procedure, the system of integral representations is reduced to the system of linear equations for unknown expansion coefficients. For Schwartz method the system of integral representation is solved using iterative methods. The dependences of the reflection coefficient magnitude and phase on the value of scan angle are obtained. The comparison of obtained results for particular cases with known ones is performed.

Keywords: integral equations, Green's functions, Schwartz alternating method, phased array antenna, rectangular waveguide.

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1. Introduction

Waveguide structures with step discontinuities play a significant role in designing microwave transformers, filters, and couplers. One of the effective approaches for treating diffraction problems in waveguide structures is the mode matching technique (MMT). In paper [1] an analysis for computing the transmission characteristics of a cascaded H-plane discontinuity in a rectangular waveguide is presented. An optimum double-plane three-step transformer for a P- to X-band waveguide is designed in [2].

In papers [3, 4] the Finite Element Method (FEM) to determine reflection and transmission coefficients of rectangular waveguide junction discontinuities is presented.

The integral equation method is also widely used for solving diffraction problems in waveguides [6-9]. The paper [6] is devoted to the analysis of electromagnetic wave diffraction on waveguide phased array antennas (PAA) using both integral equation and mode matching method. The Schwartz alternating method is applied to solve diffraction problems in papers [7, 8]. The method consists of dividing a whole field definition domain into simple overlapping partial domains, whose Green's functions are known. Through the use of Green's functions, the initial problem is reduced to a Fredholm integral equation of second kind that is solved by iteration method. According to overlapping partial domain method [9] the resulting integral equations are solved with the Galerkin's method.

In the present paper a novel approach within the overlapping partial domain method for solving electromagnetic wave diffraction problems on waveguide step discontinuities is proposed. A problem for matching steps in apertures of waveguide PAA is considered. In order to obtain the problem solution, a system of integral equations is reduced to a system of linear equations for unknown expansion coefficients.

2. Formulation of the problem

In order to consider the main features of the proposed approach, we solve a problem in which a three-dimensional vector Helmholtz equation is reduced to two-dimensional scalar one. We consider the infinite phased array antenna constituted by rectangular waveguides, whose apertures have matching steps. Since the field in radiation region has periodic character, one can take into account a unit PAA cell located at the origin. The PAA waveguides are excited by an incident wave of type H_{10}. As shown in [6], if beam scanning is performed in H plane and waveguide walls, that are normal to the electrical field vector, have infinitesimal thickness, only an E_y component of electrical field satisfying the two-dimensional Helmholtz equation has to be found.

According to the procedure described in [10], we divide the whole field definition domain of the selected PAA cell into three overlapping partial domains (Fig. 1).

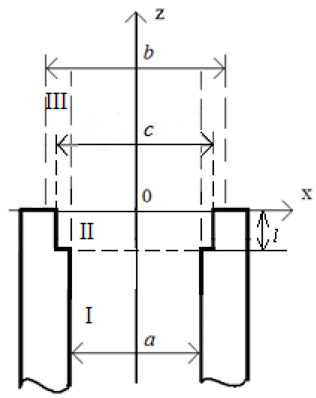


Fig.1. Unit cell of infinite parallel plate phased antenna array.

Domain I: $-a/2 \le x \le a/2$, $-\infty \le z \le \infty$. Domain II: $-c/2 \le x \le c/2$, $l \le z \le \infty$. Domain III: $-b/2 \le x \le b/2$, $0 \le z \le \infty$. The H_{10} wave is excited in domain I at $z \to -\infty$. Suppose that the Green's functions of domains I, II and III are known. Then, we can set up a system of integral equations of fields for each domain using the Green's second identity:

$$\begin{cases} E_{\mathrm{I}}(x,z) = E_{inc}(x,z) + \int\limits_{l}^{0} E_{\mathrm{II}}(x',z') \dfrac{\partial G_{\mathrm{I}}(x,z;x',z')}{\partial \vec{n}'} dz' + \\ \qquad + \int\limits_{0}^{\infty} E_{\mathrm{III}}(x',z') \dfrac{\partial G_{\mathrm{I}}(x,z;x',z')}{\partial \vec{n}'} dz'; \\ E_{\mathrm{II}}(x,z) = \int\limits_{-\frac{a}{2}}^{\frac{a}{2}} E_{\mathrm{I}}(x',z') \dfrac{\partial G_{\mathrm{II}}(x,z;x',z')}{\partial \vec{n}'} dx' + \\ \qquad + \int\limits_{0}^{\infty} E_{\mathrm{III}}(x',z') \dfrac{\partial G_{\mathrm{II}}(x,z;x',z')}{\partial \vec{n}'} dz'; \\ E_{\mathrm{III}}(x,z) = \int\limits_{-\frac{c}{2}}^{\frac{c}{2}} E_{\mathrm{II}}(x',z') \dfrac{\partial G_{\mathrm{III}}(x,z;x',z')}{\partial \vec{n}'} dx'. \end{cases} \qquad (1)$$

Here: x and z are coordinates of the observation point, x' and z' are coordinates of the source point, G_{I}, G_{II}, G_{III} are the Green's functions of domains I, II, and III, $\vec{n}'$ denotes an outward unit normal vector to a partial domain boundary surface, a prime symbol denotes that the differentiation is performed at source points. Green's functions are represented in a sourcewize form [11]:

$$G_{I,II}\left(x,z;x',z'\right)=\sum_{\mu=1}^{\infty}\phi_{\mu}^{I,II}\left(x\right)\phi_{\mu}^{I,II}\left(x'\right)f_{\mu}^{I,II}\left(z,z'\right). \tag{2}$$

Here: $\phi_{\mu}^{I,II}\left(x\right)$ are normalized orthogonal waveguide eigenfunctions of a corresponding domain described in [7]. Index μ denotes number of waveguide mode, for the domain I μ=*q*, for domain II μ=*p*, α is waveguide width, for domain I α=*a*, for domain II α=*c*. The Green's functions for domain I and II depending on longitudinal coordinates have the form:

$$f_{q}^{I}\left(z,z'\right)=\frac{1}{2j\gamma_{q}^{I}}\exp\left(-j\gamma_{q}^{I}\left|z-z'\right|\right). \tag{3}$$

$$f_{p}^{II}\left(z,z'\right)=\frac{1}{j\gamma_{p}^{II}}\begin{cases}\exp\left(-j\gamma_{p}^{II}\left(z'-l\right)\right)\mathrm{sh}\left(j\gamma_{p}^{II}\left(z-l\right)\right), & z'<z\\ \exp\left(-j\gamma_{p}^{II}\left(z-l\right)\right)\mathrm{sh}\left(j\gamma_{p}^{II}\left(z'-l\right)\right), & z>z'\end{cases}. \tag{4}$$

Here: $\gamma_{\mu}^{I,II}=-j\sqrt{\left(\frac{\mu\pi}{\alpha}\right)^{2}-\left(\frac{2\pi}{\lambda}\right)^{2}}$ are longitudinal propagation coefficients.

Because of periodic character of the PAA excitement the Green's function for domain III is represented as a series of "Floquet" harmonics [6]:

$$G_{III}\left(x,z;x',z'\right)=\sum_{m=-\infty}^{\infty}\psi_{m}\left(x\right)\psi_{m}^{*}\left(x'\right)\frac{\exp\left(-j\Gamma_{m}z\right)}{j\Gamma_{m}}\mathrm{sh}\left(j\Gamma_{m}z'\right). \tag{5}$$

Here: $\psi_{m}\left(x\right)$ are normalized orthogonal waveguide eigenfunctions of the domain III, symbol "*" denotes complex conjugation, $\Gamma_{m}=-j\sqrt{\left(\frac{kb\sin\theta+2m\pi}{b}\right)^{2}-\left(\frac{2\pi}{\lambda}\right)^{2}}$ are longitudinal propagation coefficients for domain III. Incident wave is an H_{10} – wave with unit magnitude:

$$E_{inc}\left(x,z\right)=\phi_{1}^{I}\left(x\right)\exp\left(-j\gamma_{1}^{I}z\right). \tag{6}$$

As seen from (1) the field in one domain (except domain III) is defined by fields from two other domains. Thus, the system (1) cannot be reduced to a single integral equation for one unknown function. Obviously, one equation in system (1) can be eliminated by expressing it through other equations. However, this procedure is associated with a large number of analytical transformations, especially for three-dimensional problems.

In order to obtain a solution for system (1) we use the following approach. We represent unknown functions for each domain as a series of orthogonal eigenfunctions with unknown expansion coefficients, which have physical meanings of transmission and reflection coefficients. Thus, unknown functions take the following form:

$$\begin{aligned}
&E_{\mathrm{I}}\left(x,z\right)=\phi_{1}^{I}\left(x\right)\exp\left(-j\gamma_{1}^{I}\left(z-l\right)\right)+\sum_{q=1}^{\infty}R_{q}^{I}\phi_{q}^{I}\left(x\right)\exp\left(j\gamma_{q}^{I}\left(z-l\right)\right);\\
&E_{\mathrm{II}}\left(x,z\right)=\sum_{p=1}^{\infty}T_{p}^{II}\phi_{p}^{II}\left(x\right)\exp\left(-j\gamma_{p}^{II}\left(z-l\right)\right)+\sum_{p=1}^{\infty}R_{p}^{II}\phi_{p}^{II}\left(x\right)\exp\left(j\gamma_{p}^{II}z\right);\\
&E_{\mathrm{III}}\left(x,z\right)=\sum_{m=-\infty}^{\infty}T_{m}^{III}\psi_{m}\left(x\right)\exp\left(-j\Gamma_{m}z\right).
\end{aligned} \tag{7}$$

We substitute these representations into system (1) and fix coordinates of source and observation points. Wherein, field in domain II has to be found at two observation points with coordinates $z=l$ and $z=0$. Then using a property of eigenfunctions orthogonality the system (1) is reduced to a system of linear equations for unknown expansion coefficients. The obtained system can be solved using any direct method after limiting the number of unknowns to a finite value. The modulus of reflection coefficient of an incident H_{10} wave is determined by value of the R^{I}_1 coefficient.

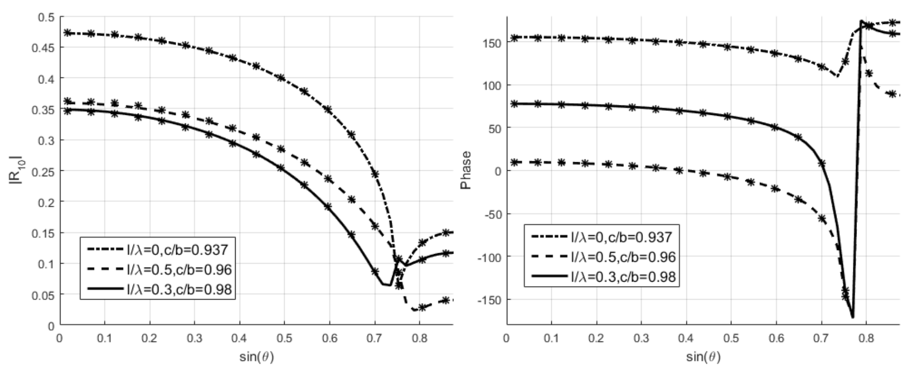
Applying the Schwartz alternating method to described problem leads to a system of linear equations, which can be solved using iterative methods. However, direct applying of the Schwartz algorithm to the considered problem does not allow to obtain a convergent solution. Thus, it is necessary to take advantage of the optimal iteration method for Schwartz algorithm introduced in paper [7]. Thereby, system of linear equations for unknown expansion coefficients can be represented in the next matrix form:

$$\mathbf{X}^{\langle i\rangle} = \mathbf{X}^{\langle i-1\rangle} + \beta\cdot\boldsymbol{\alpha}\cdot\mathbf{X}^{\langle i-1\rangle} - \beta\cdot\mathbf{X}^{\langle i-1\rangle} + \beta\cdot\mathbf{B}. \tag{8}$$

Here: i is the order of iteration, $\mathbf{X}$ is row matrix with unknown coefficients and $\mathbf{X}^{<1>}=\mathbf{B}$, $\mathbf{B}$ is row matrix of free terms of system of linear equations, $\boldsymbol{\alpha}$ can be determined as $\boldsymbol{\alpha}=\mathbf{I}-\mathbf{A}$, where $\mathbf{I}$ is a square unit matrix and $\mathbf{A}$ is the main matrix of the system, β can be determined as $1/\|\mathbf{A}\|$.

3. Numerical results

According to the described algorithm, the numerical calculation of the reflection coefficients in PAA waveguides was performed. Fig. 2 depicts the dependence of the reflection coefficient magnitude (a) and phase (b) on the value of steering phase shift sin(θ) for PAA with waveguide dimensions $b/\lambda=0.5714$, $a/b=0.937$ for different values of c/b and l/λ ratios.



(a) (b)

Fig. 2. The dependence of the reflection factor modulus (a) and phase (b) on the steering phase shift for PAA with b/λ=0.5714, a/b=0.937.

Symbol * on curves describes results obtained using the Schwartz algorithm. The results were obtained for three different matching step dimensions. Dimension $c/b=0.937$ represents the case of matching step absence since the value of c is equal to a. A comparison of the obtained result for the case of matching step absence with known results from [6] shows the correctness of the described algorithm.

Table 1 represents a convergence of the described method for different values of accounted modes M in domain III for PAA with dimensions $b/\lambda=0.5714$, $a/b=0.937$ $c/b=0.96$, and $l/\lambda=0.5$. The maximum value is limited by $M=32$ because further increasing of it affects on result accuracy less than 10^{-4}. The number of modes in domains I and II is defined as $Q=2M+1$.

Table 1

The results of the method convergence

sin(θ)	M=2	M=4	M=8	M=16	M=32
0.0525					
0.175					
0.35					
0.525					
0.7	0.1619	0.1610	0.1618	0.1617	0.1609
0.875	0.0405	0.0416	0.0407	0.0406	0.0413

The investigation of the step discontinuity effect on the value of an incident wave reflection coefficient in PAA waveguides allowed finding the optimum dimensions of a matching discontinuity, which can provide a uniform dependence of a reflection coefficient magnitude on the steering phase shift. Fig. 3 depicts the dependence of the reflection coefficient magnitude (a) and phase (b) on the value of *sin*(θ) for PAA with waveguide dimensions $b/\lambda=0.6724$, $a/b=0.88$, and $l/\lambda=0.3$ for different values of c/b.

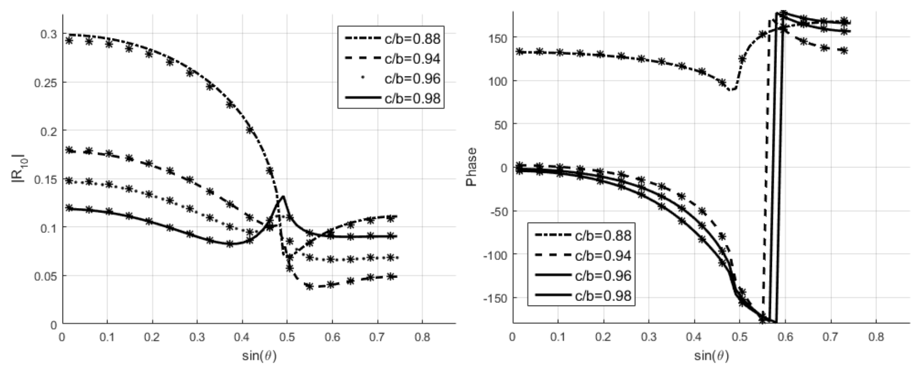


Fig.3. The dependence of the reflection factor modulus (a) and phase (b) on the steering phase shift for PAA with b/λ=0.6724, a/b=0.88, l/λ=0.3.

For comparison, dimensions of $c/b=0.88$ and $l/\lambda=0$ represent the case of matching step absence. The results show that introducing step notches in waveguide apertures can significantly improve a PAA matching in a wide range of scanning angles.

4. Conclusions

The approach within the overlapping partial domain method is considered in this paper. This approach allows solving diffraction problems in which a system of integral equations for unknown partial domain functions cannot be reduced to a single integral equation for one unknown function.

In the proposed approach a system of integral equations is reduced to a system of linear equations for all unknown field functions in each partial domain. Wherein, unknown functions are presented as series of orthogonal eigenfunctions. The solution of this system allows to obtain transmission and reflection coefficients in each partial domain. Also, the proposed approach reduces significantly an amount of required analytical transformations before performing numerical calculations, especially for three-dimensional problems.

Based on proposed approach the problem of electromagnetic wave diffraction on a waveguide PAA, whose waveguides have matching steps in its apertures, is solved in the paper. The dependences of reflection coefficient in unit cell of PAA on a steering phase shift were obtained for different sizes of matching step discontinuity. The second problem was a three-dimensional case: wave diffraction on a cascaded rectangular waveguide junction. The effect of intermediate waveguide length on the character of reflection coefficient frequency dependence has been analyzed.

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