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DETERMINATION OF THE CONDUCTIVITY OF NON-METALLIC POWDER MATERIALS BY THE EDDY CURRENT METHOD

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A theoretical analysis of the interaction of the eddy current sensor field with powder particles makes it possible to calculate the particle conductivity using the measured value of the sensor added active resistance. Relative to its own reactance, the value of this resistance is proportional to powder density, frequency of the probing field, electrical conductivity of particles and square of their diameter. Particles of spherical shape and cylindrical shape, the height of which is equal to the diameter of the base, are considered. The analysis allows to explain the experimentally observed different character of the resistance frequency dependences for powder and corresponding solid material when measuring by the same sensor.

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1. Introduction

Conductive powder materials are used in power engineering, electrochemical and electronics industries, ore processing plants. Measurement of the powder conductivity is the complex problem. Traditionally used contact measurements have the low accuracy primarily due to variability of powder density both in the powder sample volume and in close proximity to the measuring electrodes, on contact place with them. Sufficient frequently there are not the same volume and surface conductivities of the particles, and it is the cause of additional error [1]. In this connection noncontact eddy current measurements of the conductive properties of materials with a discrete structure are more promising [2, 3]. According to our experiments, in this case significantly higher repeatability of results is observed [4]. At the same time these measurements require theoretical foundation. It is necessary to analyze the interaction primarily the uniform magnetic field of a solenoidal coil with discrete conductive particles of powder.

2. Main part

1. The interaction of the solenoidal coil field with nonmagnetic conductive particles of a powder

We assume that particles have the shape of sphere with diameter D_0 or short cylinder with a height D_0 that is equal to the base diameter. Specific electrical conductivity of the material of the particles is σ_0. The powder is poured into the thin-walled cylinder having a bottom (sampler) made of dielectric material. The sampler is positioned within the solenoidal coil in the space of uniform magnetic field (Fig. 1*a*).

The powder particles are located close to each other. We separate the parallelepiped inside the sampler whose bases lie in the planes of the sampler bases, and the lateral edges are parallel to the axis *z*. The cross section of the parallelepiped in the plane *xy* passing through the centers of particles is shown in Fig. 1*b*. Wherein the filling factor of the sampler for dense matrix arrangement of a spherical particles is $\eta_s = \pi/6$ and for particles in the form of short cylinder is $\eta_c = \pi/4$.

The number of particles in the sampler in this case is given by

$$N_\Sigma = \frac{\pi D_2^2 H_2}{4 D_0^3}. \quad (1)$$

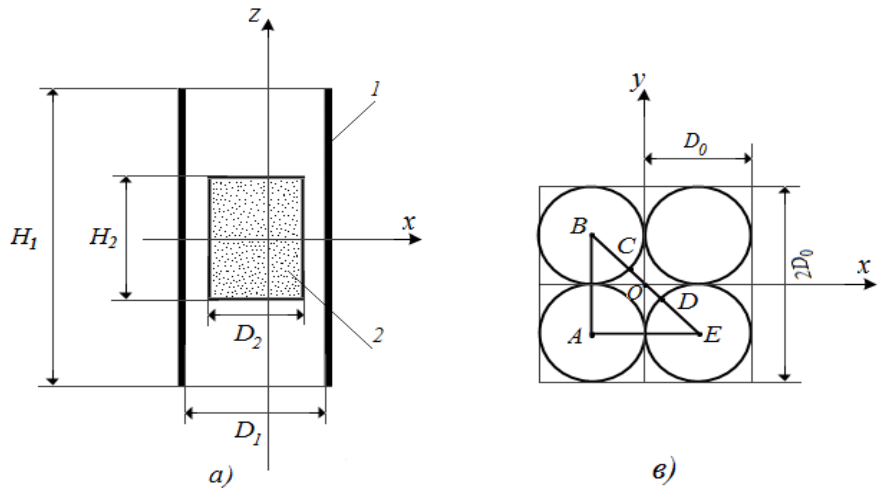

Fig. 1. Solenoidal coil (1) contained the sampler (2) filled with powder:
***H*$_1$ and *D*$_1$ are height and interior diameter of the solenoidal coil, *H*$_2$ and *D*$_2$ are height and interior diameter of the cylindrical sampler (*a*); powder particles arrangement in section *xy*, perpendicular to the solenoid axis for dense matrix particles packing (*b*).**

In a conducting particle placed in magnetic field of a solenoid an eddy currents flowing along the closed paths are induced. We used the energy method of analysis [5]. Power of Joule losses in a single particle is

$$P_0 = \sigma_0 E_0^2 s_0 l_0 \tag{2}$$

where σ_0 is specific electrical conductivity of the particle material, E_0 is the strength of corresponding electric field in the particle, s_0 and l_0 are respectively cross-section area and length of the equivalent tube of eddy current in the conductive particle. Total power of the Joule losses when dense matrix arrangement of particles in the sampler is $P_\Sigma = N_\Sigma \cdot P_0$.

Replace the cylindrical sampler (Fig. 1*a*) by the solid cylinder made of conductive material. In this case strength $\dot{H}$ of solenoid harmonic field is described by Bessel equation which has the solution [6]:

$$\dot{H} = \dot{H}_0 \frac{I_0(k \cdot r)}{I_0(\beta)} \tag{3}$$

where I_0 is a modified cylindrical function of first kind and zero order, $\dot{H}_0$ is the strength value $\dot{H}$ on the cylinder surface, $|k| = \sqrt{\omega\sigma\mu_0}$ (ω is a circular frequency; σ is specific electrical conductivity of the cylinder material; μ_0, magnetic constant), r is the radial coordinate, $\beta = R \cdot |k|$ (R is a radius of the cylinder).

The vector of eddy current density in this case has only an angular component: $\dot{J} = k\dot{H}_0[I_1(kr)/I_0(\beta)]$ where I_1 is a modified cylindrical function of the first kind and first order.

We investigated the relative density of eddy current $\dot{J}_r = \dot{J}/\dot{J}_0$ where $\dot{J}_0 = \dot{J}$ when $r = D_2/2$, i.e. $\dot{J}_0$ is the current density on lateral surface of the cylinder. Then for absolute value of current density we obtain: $J_r = I_1(\beta \cdot r/R_2)/I_1(\beta)$.

Fig. 2 is the plot of J_r on the ratio $r/R_2 = 2r/D_2$. The calculations were performed for ω=40 MHz, $\sigma = 10^2$ Ohm$^{-1}\cdot$m^{-1}, and D_2=10 mm. For these parameters β=0.88. The linearity of the dependence J_r on r/R_2 improves when β decreasing. Herewith for $\beta<1$ the value of magnetic field strength inside the conductive cylinder is almost the same as the strength on its lateral surface.

For cylindrical particles with $D_0=0.1$ mm β is $0.88\cdot10^{-2}$.

Next we located the cylindrical particle inside the solenoidal coil so that its axis coincides with the solenoid axis (Fig. 3*a*).

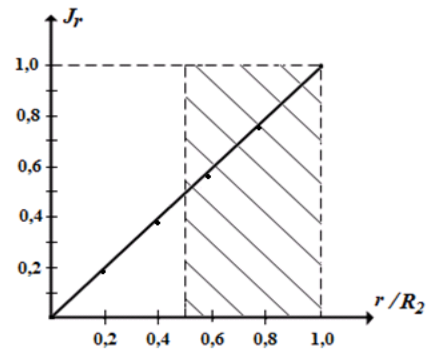

Fig. 2. Dependence J_r on r/R_2 for a conductive cylinder when β=0.88.

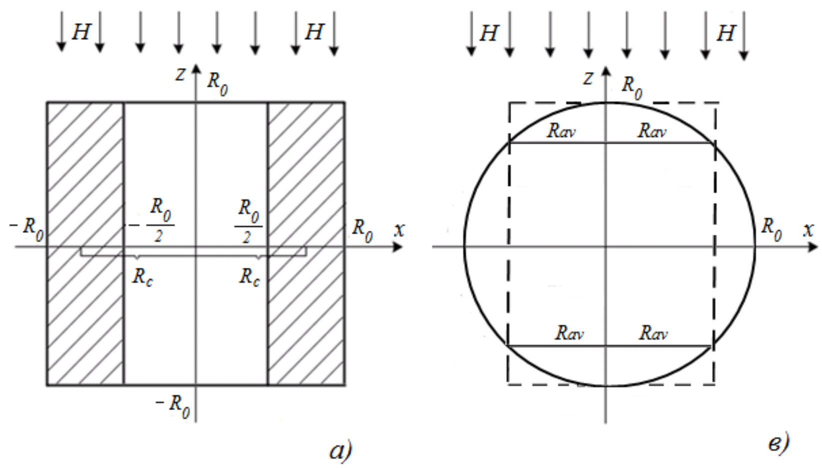

Fig. 3. Axial sections of the cylindrical particle (*a*) and spherical particle (*b*).

Obviously, the distribution of eddy current density in the solid conductive cylinder (Fig. 2) will remain for conductive cylinder particle too. Conserving the total current value, the area $0\leq r/R_0\leq1$ with actual distribution of its density can be replaced with the equivalent area $0.5\leq r/R_0\leq1$ where the current density is constant and equal to the maximal density. Cross-section of such equivalent tube of eddy current is shaded in Fig. 3*a*. Section area is $S_{0C}=D_0^2/4$.

We equated the current distribution in a spherical particle (Fig. 3*b*) with the distribution in a cylindrical particle taking into account the specificity of the eddy current flow in it. In this connection we studied the first quarter of the particle section (circle) for $x\geq0$, $z\geq0$. In this area the average radius is $R_{av}=\pi R_0/4$.

The same value R_{av} corresponds to the fourth quarter of the section for $x\geq0$, $z\leq0$. Thus the current model of a spherical particle can be replaced by the equivalent cylindrical model with a diameter $D_{0S}=2R_{av}=\pi\ D_0/4$ and height D_0. Such cylinder is bounded in cross-section by dotted line in Fig. 3*b*. Herewith the equivalent cross-sectional area of the eddy current with constant density is $S_{0C}=\pi D_0^2/16$ here. The equivalent eddy current tube length for the cylindrical particle (Fig. 3*a*) is defined as $l_{0C}=2\pi\cdot(R_0/2+R_0/4)=3\pi/4D_0$. The similar value for a spherical particle is $l_{0C}=3\pi^2/16D_0$.

Then, expression for the total power of Joule losses in all powder particles under their dense matrix packing can be written as follows. For cylindrical particles:

$$P_{\Sigma C}=\frac{3\pi^2}{64}D_2^2H_2\sigma_0E_0^2. \qquad (4)$$

For spherical particles the first multiplier in (4) is $3\pi^4/1024$.

Let us consider the effect of the density of particles spacing in a volume of sampler. The lower density of a granular material is so-called bulk density. The particles may not touch each other primarily due to the presence of electrostatic repulsion forces [7]. The density of a granular (powder) material is

$$N_\rho=\frac{M}{V}=\frac{N_\rho\cdot\rho_0\cdot V_0}{V} \qquad (5)$$

where M is a mass of powder in a sampler, V is a sampler volume, N_ρ is the quantity of powder particles at given density, ρ_0 is a density of particle material, V_0 is a particle volume.

Defining N_ρ from (5) and multiplying it by P_0 (2) with considering s_0 and l_0, for spherical particles we have

$$P_{\rho S}=\frac{9\pi^3}{512}\cdot\frac{\rho_\rho}{\rho_0}\cdot D_2^2H_2\sigma_0E_0^2. \qquad (6)$$

For cylindrical particles the first multiplier in (6) changes to $3\pi/16$.

It is of interest to determine the maximum possible packing density of the spherical powder particles. Such package is achieved when in each pair of adjacent layers the upper layer is offset relative to the lower, so that the vertical projection of the center of each particle in the upper layer reaches a point O (see Fig.1*b*) of the lower layer. In this case, the layers pair thickness reduces. The total height of the pair of layers is $h_2=D_0(\sqrt{2}+1)/\sqrt{2}$. Compared to the original matrix arrangement of the particles, the height of each pair of layers decreases to $\lambda=2\sqrt{2}/(\sqrt{2}+1)$ times, that is approximately 17%. It entails a proportional increasing of the quantity of powder particles in sampler.

But now, the second layer of each pair is offset relative to the first layer by $D_0/2$, therefore the empty seats appear near the sampler interior surface. This somewhat reduces the filling factor of the sampler. We estimated the value of that reduction.

The number of pairs of layers stacked along height H_2 of the sampler is equal to $n=\sqrt{2}H_2/(\sqrt{2}+1)D_0$. The number of particles stacked along the sampler base length is equal to $m=\pi D_2/D_0$. But since the displacement of neighboring layers is carried out on $D_0/2$, the number of particles vacancies for each pair of layers will be $m_v=\pi D_2/2D_0$. The total number of vacancies:

$$Q=n\cdot m_v=\frac{\sqrt{2}\pi D_2H_2}{2(\sqrt{2}+1)D_0^2}. \qquad (7)$$

The relative decreasing of the sampler filling factor is therefore $\gamma=D_0/D_2$. Such decreasing can be neglected.

2. Connection between the eddy current sensor parameters and power of Joule losses in conductive powders

In the practice of eddy current measurements the active and reactive components of sensor impedance are used as informational parameters. So we need to find the connection between these parameters and power of Joule losses (6). Consider the equivalent scheme shown in Fig. 5 [8].

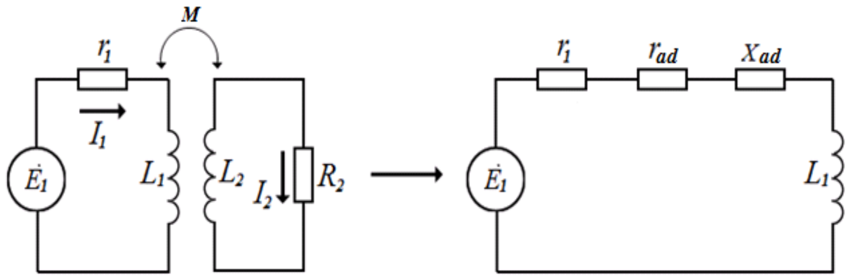

Fig. 4. Equivalent scheme representing the interaction of the solenoidal coil field and the field of eddy current induced in the particle of powder.

In this scheme $\dot{E}_1$ reflect the electromotive force specified the solenoidal coil current, L_1 is the solenoidal coil inductance, r_1 is the own resistance of the coil winding, L_2 is the inductance of the closed eddy current contour in powder particle, R_2 is the active resistance of particle material to the flowing current, M is the mutual inductance of L_1 and L_2.

In accordance with the Kirchhoff equations we obtain

$$I_1[(r_1+\alpha^2 R_2)+i\omega(L_1-\alpha^2 L_2)]=\dot{E}_1 \tag{8}$$

where $\alpha^2=\dfrac{\omega^2 M^2}{R_2^2+\omega^2 L_2^2}$.

Active resistance increasing and sensor inductance decreasing due to eddy current flowing in the particle described by equation (8), are represented in Fig. 4 as a circuit of additional active r_{ad} and inductive X_{ad} resistances. Herewith $r_{ad}=\alpha^2 R_2$.

The ratio of the currents flowing in the first and second circuits of the scheme may be expressed as

$$\frac{I_2}{I_1}=\frac{\omega M}{\sqrt{R_2^2+\omega^2 L_2^2}}(\cos\delta+i\sin\delta) \tag{9}$$

where tgδ=$R_2/\omega L_2$. We estimated the value of tgδ for a cylindrical particle whose section is shown in Fig. 3*a*. Resistance R_2 shown in the equivalent scheme equals $R_2=l_0/\sigma S_0=3\pi/\sigma D_0$. When D_0=0.1mm and $\sigma=10^2$ Ohm^{-1}m^{-1}, $R_2=9.42\cdot10^2$ Ohm. The value of inductance L_2 for a circular ring with the rectangular cross-section of width $D_0/4$ and height D_0 (see Fig. 3*a*) in accordance with [9] is $L_2=0.18\cdot10^{-10}$ Gn. When a frequency $\omega=2\pi\cdot4\cdot10^7$ Hz the value $\omega L_2=4.5\cdot10^{-3}$ Ohm. Thus $R_2\gg\omega L_2$ and $\delta=\pi/2$. This is true for a wide range of electrical conductivity of nonmetallic particles.

We divide (8) into the real and imaginary parts, obtain a square equation for the real part and replace in it I_1^2 for I_2^2. After that we come to

$$P_2\cdot\frac{R_2}{\omega^2 M^2}(r_1+r_{ad})^2=\mathrm{Re}^2\,\dot{E}_1^2 \tag{10}$$

where $P_2 = I_2^2 \cdot R_2$ is a power of Joule losses for the sensor (solenoidal coil) field in a single conductive particle of powder, when eddy current flow in it.

Substituting r_{ad} into (10) and taking into account that $R_2^2 » \omega^2 L_2^2$, we obtain

$$P_2 = \frac{r_{ad}}{(r_1 + r_{ad})^2} \mathrm{Re}^2\, \dot{E}_1 . \tag{11}$$

The value r_{ad} for a single powder particle is much less than r_1. Therefore for a single particle $P_2 = r_{ad} \cdot \mathrm{Re}^2 E_1 / r_1^2$. For the association of particles in the sampler

$$P_\Sigma = N_\rho \cdot P_2 = N_\rho \frac{\mathrm{Re}^2\, \dot{E}_1}{r_1^2} r_{ad} = \frac{\mathrm{Re}^2\, \dot{E}_1}{r_1^2} R_{ad} \tag{12}$$

where $R_{ad} = N_\rho \cdot r_{ad}$ is inactive resistance introduced by the association of conductive particles of the sampler in eddy current sensor.

Substituting P_Σ (12) instead of $P_{\rho s}$ in equation (6), we obtain for spherical particles the following expression:

$$\frac{\mathrm{Re}^2\, \dot{E}_1}{r_1^2} R_{ad} = \frac{9\pi^3}{512} \cdot \frac{\rho_\rho}{\rho_0} \cdot D_2^2 H_2 \sigma_0 E_0^2 . \tag{13}$$

For cylindrical particles the first multiplier in right-hand side of (13) is equal to $3\pi/16$.

3. Determination of the electrical conductivity of powder particles

To determine σ_0 we used the formula (13). Consider first the cylindrical particle shown in Fig.3*a*. The magnetic flux through the cross-section $S_2 = \pi D_0^2/4$ is equal to

$$\Phi_{2C} = \frac{\pi D_0^2}{4} \cdot \mu_0 H_z e^{i\omega t} . \tag{14}$$

Taking into account the electromagnetic field induced in the corresponding loop with diameter D_0, for amplitude of the electric field strength E_{2c} we obtain

$$E_{2C} = \frac{D_0}{4} \cdot \mu_0 \omega H_z . \tag{15}$$

Next, we compose the similar equation for a loop with a diameter $D_0/2$ and then average obtained value of strength E_{1c} and value E_{2c} (15) within the equivalent section of the eddy current flow with constant density (shaded area in Fig. 3*a*).Then for E_0 the first multiplier in the right-hand side of (15) is equal to $3D_0/16$.

The magnetic field strength of the solenoid in the area of uniformity is

$H_z = 10^2 I_1 W / \sqrt{D_1^2 + H_1^2}$ where I_1 is the current in a solenoid winding, W is the total number of turns, the factor 10^2 appears due to the transition to SI.

Expressing current I_1 as $I_1 = \mathrm{Re}\, \dot{\varepsilon}_1 / \omega L_1$ where L_1 is an inductance of the solenoid, we have as a result for powder consisting of the cylindrical particles

$$\frac{R_{ad}}{\omega L_1} = \frac{27 \cdot 10^4 \pi}{2^{12}} \cdot \frac{\rho_\rho}{\rho_0} \cdot \frac{\mu_0^2 \omega D_0^2 W^2}{Q_0^2 L_1} \cdot \frac{D_2^2 H_2}{D_1^2 + H_1^2} \sigma_0 \tag{16}$$

where $\dfrac{R_{ad}}{\omega L_1}=\dfrac{Q_0-Q_1}{Q_0 Q_1}$, $Q_0=\dfrac{\omega L_1}{r_1}$, and $Q_1=\dfrac{\omega L_1}{r_1+r_{ad}}$ are Q-factor values for eddy current sensor obtained in the measuring process: Q_0 is an own Q-factor, Q_1 is a Q-factor of the sensor with the sample. Relative added active resistance $R_{ad}/\omega L_1$ is usually used in eddy current testing. For spherical particles the first multiplier in the right-hand side of (16) is equal to $81\cdot10^4\pi^5/2^{21}$.

We calculate the specific conductivity of spherical powder particles. At the same time we assume that after compacting powder in the sampler, for example by vibration, its density is the highest. In accordance with above, the sampler filling factor for spherical particles by their dense matrix arrangement is $\eta=\pi/6$. The maximally dense packing is $\eta_{S\max}=\dfrac{\pi}{6}\cdot\dfrac{2\sqrt{2}}{1+\sqrt{2}}$ and $\rho_\rho=\rho_0\cdot\eta_{Smax}=0.61\cdot\rho_0$. We used the solenoidal coil and the sampler with parameters D_1=12mm, H_1=48 mm, W=24, D_2=10mm, H_2=20 mm for testing. The diameter of a spherical particle is D_0=0.1 mm. With these parameters, the inductance of the coil is $L_1=1.54\cdot10^{-6}$Gn [9]. The work frequency of the Q-meter is ω=40 MHz. We obtained the values Q_0=200 and Q_1=190 from the experiment by investigating the graphite powders when used eddy current sensor with specified parameters [10]. Then, in accordance with (16), specific electrical conductivity of the spherical powder particle is $\sigma_0=9.8\cdot10^4$ Ohm^{-1} m^{-1}. The obtained value refers to the range of graphite conductivities.

In experimental studies of graphite powders carried in a frequency range, we noticed the interesting feature. The value of relative added active resistance $R_r=R_{ad}/\omega L_1$ for the eddy current sensor with increasing frequency in the range up to 40 MHz was growing. At the same time for the solid graphite, value R_r decreases with increasing frequency.

It can be explained by the fact that for solid graphite the value $\beta=R_2\sqrt{\omega\sigma\mu_0}$ is large due to a significant radius of the eddy current loop, which is approximately coincides with the radius of sensor, and the operating point locates on the after extremum section of the impedance hodograph curve.

At the same time the value β for a single graphite particle is small because of the small particle dimensions, and therefore the small radius of eddy current loop in the particle. A large number of powder particles in the sampler makes significant total value R_r. However, β for a set of non-interacting particles remains small, the operating point locates in the before extremum section of the impedance hodograph curve, and value R_r with increasing frequency grows.

Conclusions

The analytical expressions were obtained that make it possible to calculate a conductivity σ_0 of powder particles using measured value of added active resistance $R_r=R_{ad}/\omega L_1$ of eddy current sensor (solenoidal coil). Herewith it is necessary in addition to define a powder density ρ_ρ and to evaluate the particles diameter D_0. The analysis has been performed for particles of cylindrical and spherical shapes.

The informational parameter R_r is in proportion to density of a powder ρ_p, frequency of the probing field ω, conductivity σ_0 of a particles and square of a particles diameter D_0^2.

Changing the measuring range of powder particles conductivity is realized by changing the geometric dimensions, number of turns of the sensor and frequency of the probing field.

Theoretical analysis conducted in this paper explains the experimentally observed different character of frequency dependences $R_r(\omega)$ for powder and corresponding solid material when measuring by the same sensor.

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