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ON PLASMON CONTRIBUTION TO THE HOT A_0 CONDENSATE

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In $SU(2)$ gluodynamics, the Debye gluon contribution $W_D(A_0)$ to the effective action of the temporal gauge field component (we consider A_0 = const) is calculated at high temperature in the background R_ξ^{ext} gauge. It is shown that at $A_0 \neq 0$ the standard definition $k_0 = 0$, $|\vec{k}| \to 0$ corresponds to long distance correlations for the longitudinal in internal space gluons. The transversal gluons become screened by the A_0 background field. Therefore, they give zero contributions and have to be excluded from the correlation corrections. The total effective action accounting for the one-loop, two-loop, and correct $W_D(A_0)$ satisfies Nielsen's identity that proves gauge invariance of the A_0 condensation phenomenon.

Keywords: gluon, A_0 condensate, effective action, plasmon, Nielsen's identity, gauge invariance.

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1. Introduction

Investigation of QCD high temperature phase – quark-gluon plasma (QGP) – is a paramount problem nowadays. The order parameter of the phase transition here is Polyakov's loop. In the imaginary time formalism, it is the integral of the gluon field component A_0 along an imaginary time direction contour. This integral observable is not a solution to gluon field equations. Therefore, instead a related parameter, so-called A_0 condensate, A_0 = const, is also discussed. It is a constant part of the temporal gauge field component. In perturbation field theory, the $A_0 \neq 0$ has been determined in loop expansion of an effective action in the two-loop order [1, 2]. Different aspects of the condensation are discussed in the literature. Numerous references can be found, in particular, in review paper [3]. This classical field would be very essential for phenomenology. It is relatively simple to take it into account in actual calculations. It looks as a color chemical potential in finite temperature field theory. Influence of A_0 on various processes has also been discussed (see references in recent papers [4, 5]). In [6, 7] the gauge fixing independence (and hence a gauge invariance) of $A_0 \neq 0$ has been called in question. In particular, it was stated that the contributions of the plasmon diagrams to the effective action $W_D(A_0)$, which describe the long-range correlation corrections to the one-loop effective potential $W^{(1)}(A_0)$, cancel the part of the two-loop one, $W^{(2)}(A_0)$, and the $A_0 = 0$ must be detected. However, in contrast, in [8, 9] it has been shown that for the sum of the one-loop plus two-loop effective action the result $A_0 \neq 0$ followed. Just for this case Nielsen's identity holds. This, in accordance with the general theory (see, for example, [3]), means that the A_0 condensation is a gauge invariant phenomenon which is realized at two-loop level. The contradiction of these conclusions is obvious.

In the present paper we investigate the role of the plasmon diagrams in more details. To realize this, using the $SU(2)$ gluodynamics as an example, we calculate the plasmon contribution in a general relativistic R_ξ^{ext} gauge. We show that at $A_0 \neq 0$ the screening at low momenta of transversal color field modes takes place. Therefore they do not give contributions to the effective action of the correlation corrections. This is in contrast to the calculation procedures applied in the Feynman gauge in [6, 7]. Hence the cause for the discrepancy of the results [6, 7] and [3] becomes clear. We derive the correct expression for the plasmon contributions $W_D^{(3)}(A_0)$ (see (13)), which is gauge fixing independent. Nielsen's identity holds for the total effective action $W^{(tot)}(A_0) = W^{(1)}(A_0) + W_D^{(3)}(A_0) + W^{(2)}(A_0)$.

That proves the gauge invariance of the A_0 condensate.

In next section, we present a general theory of investigations and some previous results necessary for what follows. In section 3, we carry out actual calculations. The discussion of the results obtained and the difference between the cases of the $SU(2)$ and $SU(3)$ gluodynamics are given in the last section.

2. Consideration at two-loop order

Consider the total action for the configuration under consideration in the form

Let us consider $SU(2)$ gluodynamics in the Euclidean space time embedded in the background field $\overline{A_\mu^\alpha} = A_0\delta_{\mu 0}\delta^{\alpha 3} = const$ described by the Lagrangian

$$L = \frac{1}{4}(G^a_{\mu\nu})^2 + \frac{1}{2\xi}[(\overline{D}_\mu A_\mu)^a]^2 - \widetilde{C}\overline{D}_\mu D_\mu C \tag{1}$$

The gauge field potential $A^a_\mu = Q^a_\mu + \overline{A}^a_\mu$ is decomposed into quantum and classical parts. The covariant derivative in Eq. (1) is $(D_\mu A_\mu)^{ab} = \partial_\mu \delta^{ab} - g\epsilon^{abc}\overline{A}^c_\mu$, $G^a_{\mu\nu} = \overline{D}_\mu Q^a_\nu - \overline{D}_\nu Q^a_\nu - Q^b_\mu Q^c_\nu$, g is a coupling constant, internal index a = 1,2,3. The Lagrangian of ghost fields $\overline{C}, C$ is determined by the background covariant derivative $\overline{D}_\mu(\overline{A})$ and the total one $D_\mu(\overline{A}+Q)$. As in [2, 8], we introduce the "charged basis" of fields:

$$A^0_\mu = A^3_\mu, A^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm iA^2_\mu), \qquad C^0 = C^3, C^\pm = \frac{1}{\sqrt{2}}(C^1 \pm iC^2). \tag{2}$$

In this basis a scalar product is $x^a y^a = x^+ y^- = x^+ y^- + x^- y^+ + x^0 y^0$ and the structure constants are: $\epsilon^{abc} = 1$ for $a = +$, $b = -$, $c = 0$. Feynman rules are the usual ones for the theory at finite temperature with modification: in the background field $\overline{A}^a_\mu$ a sum over frequencies $\frac{1}{\beta}\Sigma(k_0 = 2\pi/\beta)$ should be replaced by $\frac{1}{\beta}\Sigma(k_0 = 2\pi/\beta \pm g\overline{A}_0^a)$ in all loops of the fields $Q^\pm_\mu, C^\pm$. This frequency shift must be done not only in propagators but also in three particle vertex [6]. The effective action $W(A_0)$ is given as a functional integral over fields with a compact imaginary time direction $0 \le t \le 1/T = \beta$:

$$\exp\left[-W(\overline{A}_0)VT\right] = N\int DQDCD\widetilde{C}\exp[-\int_0^\beta d\tau \int d^3x(L-QJ)], \tag{3}$$

where N is T-independent normalization factor, V is a space volume, J is an external source. The effective action up to two-loop order reads:

$$W(x) = W^{(1)}(x) + W^{(2)}(x), \qquad \beta^4 W^{(1)}(x) = \frac{2}{3}\pi^2[B_4(0) + 2B_4(x/2)],$$
$$\beta^4 W^{(2)}(x) = \frac{1}{2}g^2[B_2^2(x/2) + 2B_2(x/2)B_2(0)] + \frac{2}{3}g^2(1-\xi)B_3(x/2)B_1(x/2) \tag{4}$$

where we introduce the dimensionless variable $x = \frac{gA_0}{\pi T}$ and

$$B_1(x) = x - \frac{1}{2}\epsilon(x), \qquad B_2(x) = x^2 - |x| + \frac{1}{6},$$
$$B_3(x) = x^3 - \frac{3}{2}\epsilon(x)x^2 + \frac{1}{2}x, \quad B_4(x) = x^4 - 2|x|^3 + x^2 - \frac{1}{30}, \tag{5}$$

are the Bernoulli polynomials, $\epsilon(x) = x/|x|$. For $\xi = 1$ it has been calculated in [6] (for $SU(3)$ theory see [2, 8]).

As we see, $W(x)$ is ξ-dependent. This point served as an origin for doubts in the gauge invariance of the gluon field condensation phenomenon. As we mentioned in Introduction, this problem has been solved within the Nielsen identity method in [8, 9]. So here we restrict ourselves to considering the plasmon contribution for ξ to be an arbitrary number.

3. Plasmon contribution

To be consistent, first we calculate the plasmon contribution in a way developed in [6, 7] for the value of $\xi = 1$. As it is well known [10, 11], the plasmon contribution to the effective action W_D is to be properly accounted for by summing the ring diagrams with leading infrared singularities, which are present in propagator $\sim 1/k^2$. To do that, the difference $\Delta\pi$ between the infrared limit of the one-loop polarization tensors at finite temperature and the zero temperature ones for all the gluon fields should be computed.

Now, let us calculate $W_D(A_0)$. By using the Feynman rules described above and taking into account the explicit form of the gluon propagator in the basis (2),

$$D^{ab}_{\mu\nu} = \frac{\delta_{\mu\nu}\delta^{ab}}{(\kappa^a)^2} - (1-\xi)\frac{\kappa^a_\mu \kappa^b_\nu}{(\kappa^a\kappa^a)^2}, \qquad \frac{\delta^{ab}}{(\kappa^a)^2} = \left[\frac{\delta^{+-}}{(\kappa^+)^2}, \frac{\delta^{-+}}{(\kappa^-)^2}, \frac{\delta^{00}}{(\kappa^0)^2}\right] \tag{6}$$

where $(k^\pm)^2 = (k_0 \pm g\overline{A}_0)^2 + \vec{k}^2$, we obtain:

$$W_D(\overline{A}_0, \xi) = -\frac{1}{2}\sum_{k_0 = 2\pi nT}\int\frac{d^3k}{(2\pi)^3}\sum_{a,b=(+,-,0)}\frac{(\Pi^{ab}(0))^2(D^{ba}(k))^2}{1-(\Pi^{ab}(0))(D^{ba}(k))}, \tag{7}$$

where $\Pi^{ab}(0) = \Delta\pi^{ab}_{00}(\vec{k})|_{\vec{k}\to 0}$ is the asymptotic form of the one-loop polarization tensor and limit $|\vec{k}_0| \to 0$ is to be calculated in a way depending on the definition of "infrared mass shell" at $T \neq 0, A_0 \neq 0$.

Now, we are going to calculate Eq. (7) in three ways. First was proposed in [6, 7]. Let us consider the standard definition of Π^{ab}

$$\Pi^{ab} = \Delta\pi^{ab}_{00}(k)|_{k_0=0,|(\vec{k})|\to 0} = -m_D^2, \tag{8}$$

where $m_D^2 = \frac{2}{3}g^2T^2$ is the Debye mass squared [6] and all x-dependent terms are omitted as in [6, 7]. Substituting expressions (6) and performing integration we get

$$W_d^{(1)} = \frac{g^2}{6\beta^4}\left[-\frac{g}{6\sqrt{\pi\sigma}} + \frac{1}{4\sigma}\lambda^{-1}(1-\sigma)^2 - \frac{1}{4\sigma}\lambda^{+}(1+\sigma)^2 + \frac{x}{3}(3-\xi)\right] + f(n \neq 0), \tag{9}$$

where

$$\sigma = \left(1 + \frac{6(1-\xi)x^2\pi^2}{g^2}\right)^{1/2}; \lambda^{\pm} = \left(x^2 + \frac{g^2}{3\pi^2}(1 \pm \sigma)\right)^{1/2} \tag{10}$$

and the explicit expression for Π_{00} was used. As usually (see [6, 7]) only the zero mode (n = 0), is picked out and the non-static mode contributions are denoted as $f(n \neq 0)$. As we see, $W_D^{(1)}$ is ξ-dependent and for $\xi = 1$ it coincides with the results of [6, 7]. In contrast to statements of these papers, one can conclude that the applied procedure results in the gauge variant expression. It is easy to verify using Eqs. (4), (9) that in the sum $W^{(2)} + W_D^{(1)} = W_{tot}^{(1)}$ the linear terms are cancelled, as in the Feynman gauge. So, following the idea of [6, 7], we would conclude that there is no A_0 condensation (in order $\sim g^2$).

However, the ξ-dependence of W_D may call a doubt on reliability of the latter conclusion. On theoretical grounds, it is well known that the plasmon contributions are gauge invariant either in QED or QCD [10, 11]. So, this functional must be taken into account together with (4). ξ-dependent terms in W_D have the order $\sim g^2$.

But the Nielsen identity holds for the functional (4) alone [8, 9]. Hence it immediately follows, the properties described by the sum W_{tot} occur to be gauge dependent. In particular, this concerns the result $A_0 = 0$ of [6, 7].

It is not difficult to find the origin for the gauge dependence of $W_D^{(1)}$. It comes out from the definition of the infrared mass shell in the case of $A_0 \neq 0$ used in these papers. Really, as it is well known, the plasmon corrections sum up the singular infrared contributions of the tree-level propagators and have the order g^3 in coupling constant. However, from Eq. (6) it follows that only for the longitudinal modes in the internal space ($a = b = 0$) the standard definition: $k_0 = 0, |\vec{k}| \to 0$ reproduces the infrared divergence of $D^{00}(k)$. At arbitrary $A_0 \neq 0$ for transversal modes ($a, b = \pm$) this limit is a regular one.

To have a singular infrared contribution, a slightly modified definition of infrared mass shell for transversal modes should be introduced: $(k_0 \pm gA_0) = 0, |\vec{k}| \to 0$. With this definition used the infrared singularity of the $D_{00}^{+-} \sim 1/|\vec{k}|^2$ is reproduced and its gauge invariance is obvious. To incorporate this definition in Eq. (**7**), it is necessary to calculate the components $\Pi_{00}^{+-}(A_0 \neq 0, k)$ of the one-loop polarization tensor.

Omitting standard one-loop calculations, let us write down the final result,

$$-\Pi_{00}^{+-}|_{(k_0 \pm gA_0)=0, |\vec{k_0}| \to 0} = 2g^2T^2 \cdot (B_2(0) + B_2(x/2)),\ -\Pi_{00}^{00}|_{k_0=0, |\vec{k}| \to 0} = 4g^2T^2B_2(x/2)). \tag{11}$$

Substituting expressions (11) into Eq. (**7**) and integrating over momentum space, we obtain

$$W_D^{(2)}(A_0) = -\frac{2g^3T^4}{3\pi}[B_2^{3/2}(x/2) + 2[\frac{1}{2}B_2(x/2) + B_2(0))]^{3/2}]. \tag{12}$$

This nonanalytic term has the order $\sim g^3$ and is gauge invariant, as it should be for plasmon diagram corrections [10].

Now, another question arises: How the condition $(k_0 \pm gA_0) = 0$ could be implemented in the imaginary time formalism? In fact, it is not possible at arbitrary values of A_0 because of the $k_0 = 2\pi Tn$, $n = 0, \pm 1, \pm 2, \ldots$ The only way is $gA_0 = 0$ for $n = 0$,

$gA_0 = 2\pi T$ for $n = -1$, $gA_0 = -2\pi T$ for $n = +1$. These are the values corresponding to the $Z(2)$ symmetry, which takes place for $SU(2)$ gauge fields with unbroken symmetry of the Lagrangian (1) at finite temperature. For these values of A_0 the values of $B_2(x/2) = 1/6$ and the squared bracket in Eq. (12) is the fixed number. This case corresponds to the effective action including the one-loop plus $W_D^{(2)}(A_0)$ which describes the $Z(2)$ phases without symmetry breaking. This is the exact meaning of the second considered definition of the infrared mass shell.

However, to investigate the spontaneous symmetry breaking we have to put A_0 value to be arbitrary and determine it from the minimum condition of the effective action. Since for this case the symmetry is broken, no infrared singularities present for the transversal propagators in Eq. (12). Therefore we have to drop the contribution of these modes in Eq. (6), which comes from the polarization tensor (11). Thus, the correct expression for the plasmon contribution is

$$W_D^3(A_0) = -\frac{2g^3T^4}{3\pi}B_2^{3/2}(x/2). \tag{13}$$

It comes from neutral gluon components of the internal space and obviously is gauge invariant. For this case the Nielsen's identity holds and the effective action $W_{tot} = W^{(1)}(A_0) + W^{(2)}(A_0) + W_D^{(3)}(A_0)$ has a non-trivial minimum $A_0 = const$. Thus, in accordance with general principles of this approach (see [8], [3]), we have to conclude that the A_0 condensation takes place at two-loop level and is a gauge invariant phenomenon.

4. **Discussion**

In the present paper we analyzed the role of the plasmon contributions to the effective action of the gluon condensate $A_0 = const$ at high temperature.

Our main result is two-fold. First we shown explicitly that the conclusion of [6, 7] derived from the effective action $W_{tot}^{(1)}(A_0, \xi = 1)$ (Eqs. (4), (9)) is inconsistent with Nielsen's identity and so occurs to be gauge non-invariant. We have calculated the gauge independent plasmon contribution $W_D^{(2)}(12)$, which takes into consideration the special definition of the infrared mass shell $k_0 \pm gA_0 = 0, |\vec{k}| \to 0$ for the transversal internal space gluons, which corresponds to the $Z(2)$ unbroken symmetry. This possibility has also been mentioned in [2]. But for broken symmetry at high temperature these modes become massive and should be excluded from the long-range corrections. As a result, the modes occur to be long range and contribute to the effective action. It is gauge fixing independent and satisfies Nielsen's identity. This effective action has a nontrivial minimum that implies the gauge invariance of the gluon condensation phenomenon as a whole. In this approach, the ξ-dependence of the minimum position simply means that there is a set of special unknown this moment diagrams, which contribution cancels the non-invariant terms. The cancelation does not change the minimum value of the effective action as well as other characteristics of particles.

The one of the applications of the results obtained is the early Universe before the electroweak phase transition. At high temperatures, the symmetry is restored and the W and Z bosons as well as photons convert into charged and neutral gluons and Abelian gauge fields from $U(1)$ gauge group. For the former fields, all the results obtained are relevant. That means the presence of the condensate generated in the weak sector of the Standard Model. Detailed investigation of this phenomenon will be present in other publication. As far as we know, it was not discussed elsewhere already. The detailed investigation of the plasmon contributions was not given in the review [3] or anywhere else. So, the present paper removes this shortcoming.

To complete, we note that the considered $SU(2)$ gluodynamics differs a little from the $SU(3)$ one. In the latter case, two background fields A_0^3 and A_0^8 corresponding to the commuting generators $\frac{\lambda^3}{2}$ and $\frac{\lambda^8}{2}$ are expected to be generated. In principle, some combinations of these fields could become massless. It may happen after the diagonalization of the non-diagonal matrix of charged gluon fields appearing from the one-loop polarization tensors entering Eq. (7). This possibility is assumed in [2]. It is realized in case when both of condensed fields are nonzero. However, as it is shown in [8], [9], and [3], at two-loop level only the condensate $A_0^3 = const$ is generated. Therefore, there are no massless (or unstable) charged modes at high temperature. As a result, the situation in $SU(3)$ gluodynamics is similar to the case studied here.

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