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TEMPERATURE AND VELOCITY RELAXATION IN PLASMA. SPECTRAL THEORY APPROACH

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The electron temperature and velocity relaxation of completely ionized plasma is studied on the basis of kinetic equation obtained from the Landau equation in a generalized Lorentz model. In this model contrary to the standard one ions form an equilibrium subsystem. Relaxation processes in the system are studied on the basis of spectral theory of the collision integral operator. This leads to an exact theory of relaxation processes of component temperatures and velocities equalizing. The relation of the developed theory with the Bogolyubov method of the reduced description of nonequilibrium systems is established, because the theory contains a proof of the relevant functional hypothesis, the idea of which is the basis of the Bogolyubov method. The temperature and velocity relaxation coefficients as eigenvalues of the collision integral operator are calculated by the method of truncated expansion of its eigenfunctions in the Sonine orthogonal polynomials. The coefficients are found in one- and two-polynomial approximation. As one can expect, convergence of this expansion is slow.

Keywords: plasma, generalized Lorentz model, relaxation coefficients, collision integral operator, spectral theory, one- and two-polynomial approximations, functional hypothesis.

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1. Introduction

This paper is devoted to the study of relaxation processes in completely ionized plasma. The plasma is considered in the generalized Lorentz model, in which the ion subsystem is assumed to be in equilibrium. The model is based on the Landau kinetic equation [1].

For the first time the problem of equalizing the component temperatures in a system was considered by Landau [1] (the velocity relaxation was studied analogously in [2]). His research was based on the mentioned kinetic equation [1] (see also [3]). In his investigation he assumed without proof that the plasma components quickly become equilibrium and are described by the Maxwell distribution functions. A related problem in polaron theory was considered by Bogolyubov and Bogolyubov (Jr.) in book [4] where solution of the kinetic equation for a polaron were discussed with the same assumption about the Maxwell distribution. Our paper avoids this assumption.

Contrary to our papers [5–7], which are based on the Bogolyubov reduced description method [8] (see also a review in book [3]), the present paper develops kinetics of the system through elaborating the spectral theory of the collision integral operator.

The paper is constructed as it follows. In the Section 2 the generalized Lorentz model is formulated following to [7]. The Section 3 discusses the spectral theory of the collision integral operator which is used to solve the kinetic equation of the theory. The Section 4 describes the method of truncated expansion in the Sonine polynomials for the spectral problem solution. Section 5 presents calculation of the relaxation coefficients in one- and two-polynomial approximation.

2. The generalized Lorentz model of plasma

Completely ionized electron-ion plasma is investigated. The Landau kinetic equation lies in the basis of our consideration (see, for example, [1, 3]). It is assumed that ions form an equilibrium system in a state of rest with temperature T_0 which are described by the Maxwell distribution

$$w_{ip} \equiv \frac{n_0}{(2\pi M T_0)^{3/2}} e^{-\frac{p^2}{2MT_0}} \tag{1}$$

(n_0 is ion density, M is an ion mass). Moreover, here ion-ion and electron-electron interactions are neglected. So, the investigation is based on the generalized Lorentz model [7]. Electron distribution function is denoted here by $f_p(x,t)$ and is normalized by the condition

$$\int d^3 p f_p(x,t) = n(x,t) \tag{2}$$

where $n(x,t)$ is density of number of electrons. The kinetic equation for this function takes the form

$$\partial_t f_p(x,t) = -\frac{p_n}{m}\frac{\partial f_p(x,t)}{\partial x_n} + I_p(f(x,t)) \tag{3}$$

where the ion-electron collision integral is given by the formula [7]

$$I_p(f) = \frac{\partial}{\partial p_n}\left(D_{nl}(p)\left(\frac{\partial f_p}{\partial p_l} + \frac{p_l}{mT_0} f_p\right)\right) \tag{4}$$

with function $D_{nl}(p)$ defined by the formula

$$D_{nl}(p) = C\int d^3 p' w_{ip'} S_{nl}\left(\frac{p}{m} - \frac{p'}{M}\right), \qquad C \equiv 2\pi e^4 z^2 L \tag{5}$$

(m is an electron mass, ez is an ion charge, L is the Coulomb logarithm) where

$$S_{nl}(u) \equiv (u^2\delta_{nl} - u_n u_l)/u^3 . \tag{6}$$

Equilibrium electron distribution function f_p is given by the Maxwell distribution

$$f_p^{eq} = w_p , \qquad w_p \equiv \frac{n}{(2\pi m T_0)^{3/2}} e^{-\frac{p^2}{2mT_0}} ; \qquad I_p(w) = 0 . \tag{7}$$

The function $D_{nl}(p)$ defined in (5) has a fairly simple structure, which simplifies our further research. In particular, it has a simple dependence on dimensional quantities. This allows finding out the structure of dependence on the dimensional values of all objects of the theory.

3. Dynamics of spatially uniform states of plasma

In this paper we will limit ourselves to research of spatially uniform states of the system, for which distribution function $f_p(x,t)$ does not depend on coordinates x_n. Let us introduce collision operator $\hat{K}$ by the definition

$$I_p(w\mathrm{f}) = -w_p \hat{K}\mathrm{f}_p \tag{8}$$

(f_p is an arbitrary function). It is symmetric and positively defined one at the definition of a scalar product by the formula

$$(\mathrm{f}_1, \mathrm{f}_2) = \int d^3 p w_p \mathrm{f}_{1p} \mathrm{f}_{2p} . \tag{9}$$

Therefore its eigenfunctions g_{ip} and the corresponding eigenvalues λ_i

$$\hat{K} g_{ip} = \lambda_i g_{ip} \tag{10}$$

have the properties

$$\lambda_i > 0, \qquad (g_i, g_{i'}) \equiv b_i \delta_{ii'} \tag{11}$$

(b_i are some values). Note, that the spectral theory of the collision integral operator $\hat{K}$ is used too in our paper [9] for the investigation of electron mobility in plasma and in paper [10] for the polaron theory in semiconductors.

Let us find a solution of (3) in the form

$$f_p = w_p (1 + g_p) . \tag{12}$$

The function g_p is normalized by the definition

$$\langle g_p \rangle = 0 \tag{13}$$

where the average value notation with the distribution w_p is introduced

$$\langle \mathrm{f}_p \rangle = \int d^3 p w_p \mathrm{f}_p . \tag{14}$$

Kinetic equation (3) takes the form

$$\partial_t g_p = -\hat{K} g_p \tag{15}$$

and can be solved in terms of eigenfunctions g_{ip} and eigenvalues λ_i of the operator $\hat{K}$

$$g_p = \sum_i c_i g_{ip} e^{-\lambda_i t} \tag{16}$$

with some coefficients defined by the initial condition for the distribution function $f_p(t=0) \equiv f_{p0}$. Eigenvalues λ_i define relaxation times $\tau_i \equiv \lambda_i^{-1}$ of the system. According to (13) and (16), average values of the eigenfunctions are equal to zero

$$\langle g_{ip} \rangle = 0 . \tag{17}$$

Let us study the evolution of the electron subsystem in terms of momentum π_n and energy ε densities

$$\pi_n \equiv \int d^3 p f_p p_n = \langle g_p p_n \rangle , \qquad \varepsilon \equiv \int d^3 p \varepsilon_p f_p = \langle g_p \varepsilon_p \rangle + \varepsilon_0 , \qquad \varepsilon_0 \equiv \frac{3}{2} n T_0 . \tag{18}$$

Among eigenfunctions g_{ip} there are vector $A_p p_n$ and scalar B_p ones with the corresponding eigenvalues λ_u, λ_T

$$\hat{K}A_p p_l = \lambda_u A_p p_l\,, \qquad \hat{K}B_p = \lambda_T B_p \tag{19}$$

i.e. A_p, B_p are scalar functions of p. Quantities A_p, B_p are defined by (19) with an accuracy up to arbitrary factors. Let us choose additional conditions for equations (19) to eliminate this uncertainty

$$\langle A_p \varepsilon_p \rangle = 3n/2\,, \qquad \langle B_p \varepsilon_p \rangle = 3n/2\,, \qquad \langle B_p \rangle = 0 \tag{20}$$

(the last relation is added according to (17)). Therefore expression (16) for g_p can be rewritten in the form

$$g_p = c_n A_p p_n e^{-\lambda_u t} + cB_p e^{-\lambda_T t} + \sum_{i'} c_{i'} g_{i'p} e^{-\lambda_{i'} t} \tag{21}$$

where subs i' enumerate other eigenfunctions and eigenvalues.

Let us assume that relaxation times of the system satisfy the condition that some time τ_0 exits with the property

$$\tau_T, \tau_u >> \tau_0 > \tau_{i'}\,. \tag{22}$$

Then relation (21) at $t >> \tau_0$ takes the form

$$g_p \xrightarrow[t>>\tau_0]{} g_p^{(+)} \equiv c_n A_p p_n e^{-\lambda_u t} + cB_p e^{-\lambda_T t}\,. \tag{23}$$

According to (18), (20) at these times momentum π_n and energy ε densities are given by relations

$$\pi_n \xrightarrow[t>>\tau_0]{} \pi_n^{(+)} \equiv mnc_n e^{-\lambda_u t}\,, \qquad \varepsilon \xrightarrow[t>>\tau_0]{} \varepsilon^{(+)} \equiv \frac{3n}{2} c e^{-\lambda_T t} + \varepsilon_0 \tag{24}$$

and therefore asymptotic function $g_p^{(+)}$ can be written in the form

$$g_p^{(+)} \equiv \pi_n^{(+)} A_p p_n / mn + (\varepsilon^{(+)} - \varepsilon_0) B_p 2/3n\,. \tag{25}$$

Variables $\varepsilon^{(+)}$, $\pi_n^{(+)}$ satisfy the time equations

$$\partial_t \varepsilon^{(+)} = -\lambda_T (\varepsilon^{(+)} - \varepsilon_0)\,, \qquad \partial_t \pi_n^{(+)} = -\lambda_u \pi_n^{(+)} \tag{26}$$

and describe relaxation processes in the electron subsystem. Let us introduce as observable variables instead of $\varepsilon^{(+)}$, $\pi_n^{(+)}$ electron subsystem temperature T and mass velocity u_n

$$\varepsilon^{(+)} = (3nT + mnu^2)/2\,, \qquad \pi_n^{(+)} = mnu_n\,. \tag{27}$$

These variables satisfy the relaxation equations

$$\partial_t u_n = -\lambda_u u_n\,, \qquad \partial_t T = -\lambda_T (T - T_0) + (2\lambda_u - \lambda_T) mu^2/3 \tag{28}$$

and asymptotic electron distribution function takes the form

$$g_p^{(+)} = u_n A_p p_n + (T - T_0 + mu^2/3) B_p\,. \tag{29}$$

In fact this formula expresses the idea of the Bogolyubov functional hypothesis [8] that is the basis of his method of the reduced description of nonequilibrium processes (see a review in [3]). So, the theory developed in the present paper contains a proof of this hypothesis for the case under consideration.

4. Method of the truncated expansion in the Sonine polynomials for spectral problem solution

Here at approximate solution of equations (19) by an expansion in orthogonal polynomials the Sonine polynomials $S_q^{\alpha}(x)$ ($q = 0,1,2,\ldots$, α is a real number) are used (see, for example, [6, 7, 9]). Let us find the functions A_p, B_p in the form of expansions

$$A_p = \sum_{q=0}^{\infty} a_q S_q^{3/2}(\beta\varepsilon_p), \qquad B_p = \sum_{q=0}^{\infty} b_q S_q^{1/2}(\beta\varepsilon_p) \qquad (\beta \equiv T_0^{-1}). \tag{30}$$

The equations for the coefficients a_q and b_q are found by the next way. Substituting the expansions (30) into equations (19), multiplying them by $p_l S_q^{3/2}(\beta\varepsilon_p)$ and $S_q^{1/2}(\beta\varepsilon_p)$, correspondingly, and taking the average value with the distribution function w_p in the both sides gives

$$\sum_{q'=0}^{\infty} A_{qq'} a_{q'} = \lambda_u a_q x_q, \qquad \sum_{q'=0}^{\infty} B_{qq'} b_{q'} = \lambda_T b_q y_q. \tag{31}$$

Here the notations

$$A_{qq'} = \{p_l S_q^{3/2}(\beta\varepsilon_p), p_l S_{q'}^{3/2}(\beta\varepsilon_p)\}, \qquad B_{qq'} = \{S_q^{1/2}(\beta\varepsilon_p), S_{q'}^{1/2}(\beta\varepsilon_p)\}$$

$$x_q \equiv \frac{4mn}{\beta}\frac{\Gamma(q+5/2)}{\pi^{1/2} q!}, \qquad y_q \equiv 2n\frac{\Gamma(q+3/2)}{\pi^{1/2} q!} \tag{32}$$

are introduced and the bilinear forms (brackets) defined by the formula [7]

$$\{\mathrm{f}_{1p}, \mathrm{f}_{2p}\} = \int d^3 p w_p \mathrm{f}_{1p} \hat{K} \mathrm{f}_{2p} = \int d^3 p w_p D_{nl}(p) \frac{\partial \mathrm{f}_{1p}}{\partial p_n} \frac{\partial \mathrm{f}_{2p}}{\partial p_l} \tag{33}$$

are used (f_{1p} and f_{2p} are arbitrary functions).

Additional conditions (20) give the contributions of the first polynomials to expansion (30)

$$a_0 = \beta; \qquad b_0 = 0, \qquad b_1 = -\beta. \tag{34}$$

We will say that the solutions of equations (31) are calculated in the s-polynomial approximation if all the coefficients of the expansions (30) starting from $(s+1)$-th are assumed to be zero: $a_q^{[s_1]} = 0$ ($q = 0,1,2,\ldots$) at $q \geq s$, $b_q^{[s_1]} = 0$ ($q = 1,2,\ldots$) at $q \geq s+1$ and $a_0^{[s]} = \beta$, $b_1^{[s]} = -\beta$ ($s = 1,2,\ldots$). Hereafter index s at a quantity $a^{[s]}$ indicates the number of polynomials used in the considered approximation. At the same time the obtained from (31) equations give the corresponding expressions $\lambda_u^{[s]}$, $\lambda_T^{[s]}$ for relaxation coefficients.

The calculations in the one- and two-polynomial approximations give [7]

$$\lambda_u^{[1]} = A_{00} / x_0 , \qquad \lambda_T^{[1]} = B_{11} / y_1 ;$$

$$\lambda_u^{[2]} = \left\{ (A_{11} + \frac{x_1}{x_0} A_{00}) - [(A_{11} - \frac{x_1}{x_0} A_{00})^2 + 4 \frac{x_1}{x_0} A_{01}^2)]^{1/2} \right\} \frac{1}{2x_1} , \tag{35}$$

$$\lambda_T^{[2]} = \left\{ (B_{22} + \frac{y_2}{y_1} B_{11}) - [(B_{22} - \frac{y_2}{y_1} B_{11})^2 + 4 \frac{y_2}{y_1} B_{12}^2)]^{1/2} \right\} \frac{1}{2y_2} .$$

Coefficients $\lambda_u^{[2]}$, $\lambda_T^{[2]}$ satisfy quadratic equations and in (35) the smallest roots of this equations are given which define the slowest evolution of the system.

A detailed analysis of the next many-polynomial approximations is very cumbersome and can be conducted in some limiting cases [11].

5. Calculation of the relaxation coefficients λ_u, λ_T in one- and two-polynomial approximation

According to (32) we have to calculate the following functions $A_{qq'}$ and $B_{qq'}$

$$\begin{gathered} A_{00} = \{p_l, p_l\}, \qquad A_{01} = \frac{5}{2}\{p_l, p_l\} - \beta\{p_l, \varepsilon_p p_l\}, \\ A_{11} = \frac{25}{4}\{p_l, p_l\} - 5\beta\{p_l, \varepsilon_p p_l\} + \beta^2\{\varepsilon_p p_l, \varepsilon_p p_l\}; \\ B_{11} = \beta^2\{\varepsilon_p, \varepsilon_p\}, \quad B_{12} = -\frac{1}{2}\beta^3\{\varepsilon_p, \varepsilon_p^2\} + \frac{5}{2}\beta^2\{\varepsilon_p, \varepsilon_p\}, \\ B_{22} = \frac{1}{4}\beta^4\{\varepsilon_p^2, \varepsilon_p^2\} - \frac{5}{2}\beta^3\{\varepsilon_p^2, \varepsilon_p\} + \frac{25}{4}\beta^2\{\varepsilon_p, \varepsilon_p\}. \end{gathered} \tag{36}$$

In our paper [7] the formula for calculation of the brackets (33) was obtained

$$\{a_p, b_p\} = \frac{nn_0 m^{1/2} C}{(1+\mu^2)^{1/2} T_0^{1/2}} \int d^3 q_c d^3 q w_{q_c} w_q S_{nl}(q) \frac{\partial a_p}{\partial p_n} \frac{\partial b_p}{\partial p_l}\bigg|_{p \to \frac{(mT_0)^{1/2}}{(1+\mu^2)^{1/2}}(\mu q_c + q)} \tag{37}$$

where

$$w_q \equiv \frac{1}{(2\pi)^{3/2}} e^{-\frac{q^2}{2}}, \qquad \mu \equiv \left(\frac{m}{M}\right)^{1/2}. \tag{38}$$

At calculation of the brackets with the formula (37) is convenient at first to take integrals over q_{cn} and then ones over q_n. By this way we obtain

$$\begin{gathered} \{p_s, p_s\} = \frac{mnT_0}{(1+\mu^2)^{1/2}}\lambda, \qquad \{p_s, \varepsilon_p p_s\} = \frac{mnT_0^2(1+4\mu^2)}{2(1+\mu^2)^{3/2}}\lambda, \\ \{\varepsilon_p p_s, \varepsilon_p p_s\} = \frac{mnT_0^3(55\mu^4 + 36\mu^2 + 8)}{4(1+\mu^2)^{5/2}}\lambda; \qquad \{\varepsilon_p, \varepsilon_p\} = \frac{nT_0^2\mu^2}{(1+\mu^2)^{3/2}}\lambda, \end{gathered} \tag{39}$$

$$\{\varepsilon_p^2,\varepsilon_p\}=\frac{nT_0^3\mu^2(5\mu^2+2)}{(1+\mu^2)^{5/2}}\lambda,\qquad \{\varepsilon_p^2,\varepsilon_p^2\}=\frac{nT_0^4\mu^2(35\mu^4+28\mu^2+8)}{(1+\mu^2)^{7/2}}\lambda$$

where

$$\lambda\equiv\frac{n_0 2^{3/2}C}{\pi^{1/2}m^{1/2}T_0^{3/2}}. \tag{40}$$

In two-polynomial approximation relations (39) give a very cumbersome results. Therefore, here the relaxation constants λ_T and λ_u are calculated at $\mu=1$ that corresponds to the physical sense of $\mu=(m/M)^{1/2}$ where m and M are electron and ion masses. Therefore, all values in (36) are taken with the same accuracy only in the first two approximations

$$\begin{gathered}A_{00}\approx\lambda mnT_0(1-\frac{1}{2}\mu^2),\quad A_{01}\approx\lambda mnT_0 2(1-\frac{5}{4}\mu^2),\quad A_{11}\approx\lambda mT_0\frac{1}{4}(23-\frac{43}{2}\mu^2),\\ B_{11}\approx\lambda n\mu^2(1-\frac{3}{2}\mu^2),\quad B_{12}\approx\lambda n\mu^2\frac{3}{2}(1+\frac{5}{2}\mu^2),\quad B_{22}\approx\lambda n\mu^2\frac{1}{4}(13-\frac{75}{2}\mu^2).\end{gathered} \tag{41}$$

Using formulas (35) gives the relaxation coefficients λ_T and λ_u in one- and two-polynomial approximations with the corresponding accuracy

$$\lambda_u^{[1]}\approx\lambda\frac{1}{3}(1-\frac{1}{2}\mu^2),\qquad \lambda_T^{[1]}\approx\lambda\mu^2\frac{2}{3}(1-\frac{3}{2}\mu^2);$$

$$\lambda_u^{[2]}\approx\lambda\frac{1}{4}[(33-809^{1/2})+\frac{1}{2}(-53+\frac{2029}{809^{1/2}})\mu^2]\frac{1}{15}\approx\lambda(0,076+0,153\mu^2), \tag{45}$$

$$\lambda_T^{[2]}\approx\lambda\mu^2[\frac{1}{2}(9-61^{1/2})+(-\frac{45}{4}+\frac{345}{61^{1/2}})\mu^2]\frac{4}{15}\approx\lambda\mu^2(0,484+14,0\mu^2)$$

These formulas allow comparing of both approximations

$$\frac{\lambda_u^{[2]}-\lambda_u^{[1]}}{\lambda_u^{[1]}}\approx-0,77+0,83\mu^2,\qquad \frac{\lambda_T^{[2]}-\lambda_T^{[1]}}{\lambda_T^{[1]}}\approx-0,484+14,0\mu^2 \tag{55}$$

This result shows that the convergence of the method of expansion in orthogonal Sonine polynomials used by us is at least very slow. There is no argument to expect the convergence of the procedure for the spectral problem because the problem is equivalent to solving the Fredholm integral equation of the second kind (see, for example, [12]).

6. Conclusions

The paper investigates completely ionized plasma in the generalized Lorentz model, which considers the system of ions to be equilibrium one and neglect electron-electron and ion-ion interactions. Component temperature and velocity relaxations are studied in spatially uniform states of the plasma. The investigation is based on the spectral theory of the collision integral operator that is linear one. Relaxation processes are studied without assumption that they are near their completion. The tensor dimensionality of the eigenfunctions of the collision integral operator determines the sense of parameters that describe the plasma evolution. Electron temperature and velocity relaxations are

described by scalar and vector eigenfunctions. Relevant spectral problem is related to the Fredholm integral equations of the second kind. This problem is solved in the paper by the method of truncated expansion of the eigenfunctions in the Sonine orthogonal polynomials. The convergence of this method cannot by proved. Therefore, we calculate relaxation coefficients (eigenfunctions) in one- and two-polynomial approximations and compare them. To avoid cumbersome calculations our investigation is conducted in the limit of small electron-to-ion mass ratio with accuracy up to the first order after the leading contribution included. In is shown that the convergence of the expansion in the Sonine polynomials is at least very slow.

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